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Unsteady Flow Past an Accelerated Vertical Plate through Porous Medium in the Presence of Thermal Radiation with Variable Temperature

I. D. Yale[1*], M. A. T.Uchiri[2]

[1]Departement of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria. yaleibrahim02@gmail.com.

[2]Departement of Mathematics, Kebbi State Polytechnic, Dakingari, Nigeria. kantoma01010@gmail.com.

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ABSTRACT

The one-dimensional unsteady flow past an accelerated vertical plate with the variable temperature and uniform mass diffusion has been carried out through the porous medium in the presence of thermal radiation. The dimensionless governing equations are solved analytically using Laplace transform technique.The flow characteristics of velocity, temperature and concentration are carried for the different physical parameters like thermal Grashof number, modified Grashof number, permeability parameter, radiation parameter, Prandtl number, Schmidt number, and time.The velocity profiles were obtained for the different values of the modify Grashof number Gr, thermal Grashof number Gc, Schmidt numberthermal radiation R and time t. It shows that the velocity decreases with the increasing of the Gr and also the velocity increases with the increasing of the Gc, Schmidt Number Sc, thermal radiation R and time t. The temperature flow profiles for the different values of thermal radiation R and Prandtl number Pr were obtained. It can be observed that the temperature decreases with the increasing of the thermal radiation R and its decreases with the increasing of the prandtl number Pr. The effect of concentration for the different values of thermal radiation has been obtained. It is noted that the concentration decreases with the increasing of thermal radiatio R.

Keywords: Unsteady Flow, Accelerated Vertical Plate, Heat Transfer, Porous Medium and Thermal Radiation.

1. Introduction

The effect of thermal radiation has play an important role in many industries such as glass productionand furnace design and in space technology with applications such as cosmical flight, aerodynamics rocket, propulsion system, plasma physics which operate at high temperature.Effect of heat and mass transfer has a significant role in manufacturing industries for space craft design in cooling of the liquid metal of the nuclear reaction, the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, solar energy collectors, design of chemical processing equipment, satellites and space vehicles. Flow through the porous medium has an important role in engineering problems such as purification of crude oil, the moment of oil and natural gas through oil sand stone reservoirs, underground water resources, flow of blood, paper and pulps industry membrane separation process,are examples of such engineering applications. (Palani and Abbas, 2009) analyzed free convection MHD flow with thermal radiation from an impulsivestarted vertical plate,(Muthucumaraswamy, Lal and Ranganayakulu, 2010) studied the effects of rotation on MHD flow past an accelerated isothermal vertical plate with heat and mass diffusion. (Vijayalakshmi, Paul and Kamalam, 2012) discussed the combined effects of thermal radiation and MHD flow past an accelerated vertical plate in a rotating fluid.(Ahmed and Kalita, 2012)analyzed the Laplace technique on magnetohydrodynamic radiating and chemically reacting fluid over an infinite vertical surface.(Idowu, Dada, Jimoh, and Command, 2013) considered heat and mass transfer of magnetohydrodynamic (MHD) and dissipative fluid flow past a moving vertical porous plate with variable suction.(Ravikumar, Raju, Raju, and Varma, 2013) carried out a study on the magnetic field effect on transient free convection flow through porous medium past an impulsively started vertical plate with fluctuating temperature and mass diffusion.(Uwanta and Yale, 2014) solvedanalyticalitically the unsteady flow past an accelerated vertical plate with constant temperature and variable mass transfer.(Hemamalini and Suresh, 2015) analysed the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion through porous medium.(Deka,Kalita, Paul and Officer, 2015)considered porous medium with exponentially decaying wall temperature.(Vijayalakshmi, 2016)carried outanalytical solution of vertical plate in a rotating fluid with variable temperatureuniform mass diffusion in the presence of MHD and thermal radiasion. (Muthucumaraswamy and Sivakumar, 2016) incorporated the MHD in the flow past a parabolic flow past an infinite isothermal vertical plate with the presence of thermal radiation and chemical reaction, The present study investigatesan unsteady flow of an incompressible viscous fluid with the heat and mass transfer effects on the flow past an accelerated Infinite vertical plate with the variable temperature through the porous medium in the presence of thermal radiation.

2. Formulation of Problem

The unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature through a porous medium is considered. The flow is assumed to be in x-direction which takes vertical plate in the upward direction .The y-axis is taken to be normal to the plate. Initially, the plate and the fluid are in same temperature T′ with the same concentration level C′ at all points. At time t′ > 0 the plate accelerated with velocity $u' = \frac{u_0^2 t'}{v}$ in its own plane. The plate temperature is raised to T'_w and the level of concentration near the plate is raised to C'_w linearly with the time t.

3. The Governing Equation

Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equation.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + V\frac{\partial^2 u'}{\partial y^2} - V\frac{u'}{K'} \tag{1}$$

$$\rho Cp\frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial y^2} + \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

The initial and boundary condition are;

$$u = 0, \quad T' = T'_\infty \; C'_w = C'_\infty \; t' \leq 0 \text{ for all y}$$

$$u = \frac{u_0^3 t'}{v} \; T' = T'_\infty + \theta\left(T'_w - T'_\infty\right)At' \; C' = C'_\infty + C\left(C'_w - C'_\infty\right)At', t' > 0, \; at \;\; y = 0 \tag{4}$$

$$u = 0, \quad T'_w \to T'_\infty \; C'_w \to C'_\infty \quad \text{as } y \to \infty$$

Where $A = \frac{u_0^2}{v}$

The non-dimensional quantities are;

$$u = \frac{u'}{u_0} \quad t = \frac{t' u_0^2}{v} \quad \frac{1}{K} = \frac{u_o K'}{V^2} \quad Pr = \frac{\mu cp}{K} \quad R = \frac{16a^*\delta V^2 T_\infty^3 \theta}{K u_0^2} \quad Sc = \frac{V}{D}$$

$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty} \quad Gr = \frac{g\beta V\left[\left(T'_w - T'_\infty\right)\right]}{u_0^3} \quad Gc = \frac{g\beta^* V\left[\left(C'_w - C'_\infty\right)\right]}{u_0^3} \tag{5}$$

The non-dimensional quantities of equation (5) which analysed (1) to (4),and they lead tothe dimensionless equations as follows;

$$\frac{\partial u}{\partial t} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} - Ku \tag{6}$$

$$\frac{\partial^2 \theta}{\partial y^2} = Pr\frac{\partial \theta}{\partial t} - R\theta \tag{7}$$

$$\frac{\partial c}{\partial t} = \frac{1}{Sc}\frac{\partial^2 c}{\partial y^2} \tag{8}$$

The initial and boundary conditions are reduced to;

$$
\begin{aligned}
&u=0 \quad \theta=0 \quad C=0 \quad \text{for all } y,t \le 0\\
&u=t \quad \theta=t \quad C=t \quad \text{at } y=0,\ t \succ 0\\
&u=0 \quad \theta\to 0 \; C\to 0 \; \text{as } y\to\infty,\ t\succ 0
\end{aligned} \tag{9}
$$

4. Method of Solution

Equations (6) – (8), are solved subjected to the boundary conditions of (9), and the solutions are obtained for concentration, temperature and velocity flow in terms of exponential and complementary error function using the Laplace- transform technique as follows;

$$C = erfc\ \frac{y\sqrt{Sc}}{\sqrt[2]{t}} \tag{10}$$

$$\theta = \frac{1}{2}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}+\sqrt{Rt}\right)+e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr}-\sqrt{Rt}\right)\right\} \tag{11}$$

$$
\begin{aligned}
\text{U}=&\frac{1}{2}\left\{e^{y\sqrt{K}} erfc\left(\sqrt{Kt}+\eta\sqrt{K}\right)+e^{-y\sqrt{K}} erfc\left(-\sqrt{Kt}+\eta\sqrt{K}\right)\right\}\\
&-\frac{Gr}{2a}\left\{\begin{matrix} e^{y\sqrt{K}} erfc\left(\sqrt{Kt}+\eta\sqrt{K}\right)+ \\ e^{-y\sqrt{K}} erfc\left(-\sqrt{Kt}+\eta\sqrt{K}\right)\end{matrix}\right\}-\frac{Gr}{b}\left[erfc(\eta)e^{-\sqrt[y]{K}}\right]\\
&+\frac{Gc}{2f}\left\{\begin{matrix} e^{y\sqrt{K}} erfc\left(\sqrt{Kt}+\eta\sqrt{K}\right)+ \\ e^{-y\sqrt{K}} erfc\left(-\sqrt{Kt}+\eta\sqrt{K}\right)\end{matrix}\right\}-\frac{Gc}{K}\left[erfc(\eta)e^{-\sqrt[y]{K}}\right]\\
&+\frac{Gr}{2a}\left\{\begin{matrix} e^{y\sqrt{\Pr}} erfc\left(\sqrt{Rt}+\eta\sqrt{\Pr}\right) \\ +\, e^{-y\sqrt{\Pr}} erfc\left(-\sqrt{Rt}+\eta\sqrt{\Pr}\right)\end{matrix}\right\}-\frac{Gre^{-y\sqrt{R}}}{2b}\left\{\begin{matrix} e^{y\sqrt{\Pr R}} erfc\left(\sqrt{Rt}+\eta\sqrt{\Pr}\right) \\ +\, e^{-y\sqrt{\Pr R}} erfc\left(-\sqrt{Rt}+\eta\sqrt{\Pr}\right)\end{matrix}\right\}\\
&-\frac{Gct}{f}\left\{\begin{matrix} \left(1+2\eta^2 Sc\right) erfc\left(\eta\sqrt{Sc}\right) \\ -\frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}e^{-\eta^2 Sc}\end{matrix}\right\}-\frac{Gc}{K}\left[erfc(\eta)\sqrt{Sc}\right]
\end{aligned} \tag{12}
$$

$$\text{a} = \left(\Pr-1\right) \quad \text{b} = \left(K-R\right) \quad f = \left(Sc-1\right) \quad \eta = \frac{y}{2\sqrt{t}}$$

5. Results and Discussion

The study of heat and mass transfer on the flow past an accelerated infinite porous plate with the variable temperature through the porous medium in the presence of thermal radiation is formulated and solved analytically. The results for velocity, temperature and concentration are obtained for the parameters such as thermal Grashof number (Gr), modified Grashof number (Gc), Prandtl umber (Pr), Schmidtnumber (Sc), time (t) and permeability (k) and radiation parameter(R). The value of the Prandtl number (Pr) is chosen to represent air (Pr = 0.71) and the values of Schmidt number(Sc) are chosen to represent presence of spices by hydrogen (Sc = 0.22), water vapour (Sc = 0.6), ammonia (Sc = 0.78), carbondioxied (Sc = 0.96). The velocity profile for the different values of the modify Grashof number (Gc = 2,3,4) is shown in the Figure 1. It is shown that the velocity decreases with the increasing of the Gr. The velocity profile for the different values of thermal Grashof number (Gr = 2,3, 4) is shown in Figure 2. It is observed that the velocity increases with the increasing of the Gc. The velocity profiles for the different values of Schmidt number (Sc = 0.45, 0.62, 0.78) is shown in Figure 3. It is ovserved that the velocity increase with the increase in Schmidt Number Sc. The velocity profiles for the different values of thermal radiation (R = 2.5, 3, 3.5) is shown in Figure 4. It is ovserved that the velocity increase with the increase in thermal radiation R. The effect of velocity for the different values of the time (t = 0.2, 0.3, 0.4) is shown in Figure 5. It is notice the slight increase of velocity at result of slite increa of time t. The temperature profiles for the different values of thermal radiation (R = 2, 4, 6) is shown in the Figure 6. It is shows that the temperature decreases with the increasing of the thermal radiation R. The temperature profiles for the different values of Prandtl number (Pr =2, 5, 7.1) is shown in Figure 7. It is noteced that temperature decreases with the increasing of the Prandtl number Pr. The effect of concentration for the different values of thermal radiation (R = 2, 4, 6) is seen in the Figure 8. It is noted that the concentration decreases with the increasing of thermal radiatio R.

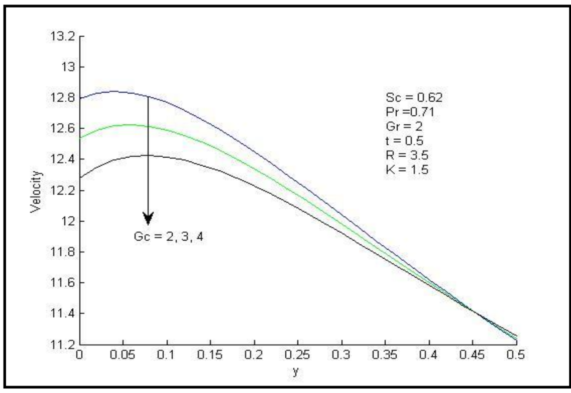

Figure 1: The velocity profiles for the different values of Gc

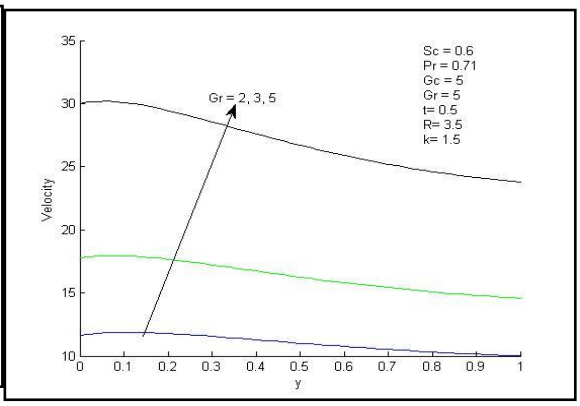

Figure 2: The velocity profiles for the differentvalues of Gr

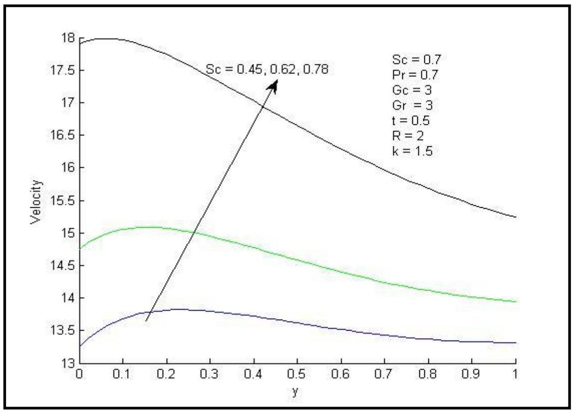

Figure 3: The velocity profiles for the differentvalues of R

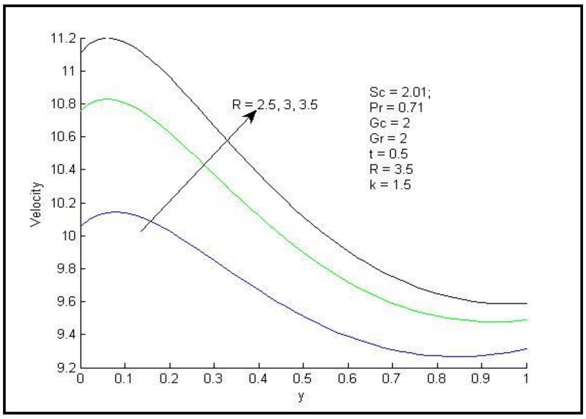

Figure 4: The velocity profiles for the different values of Sc

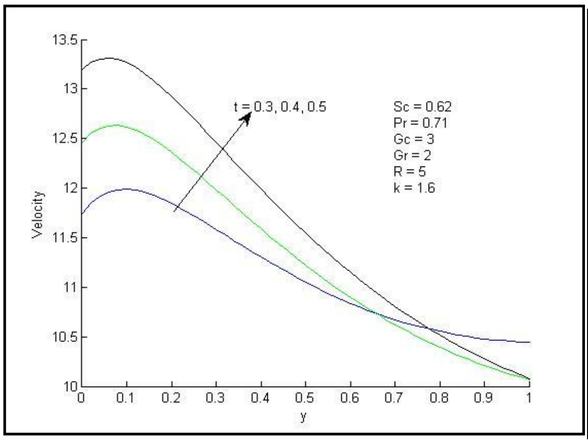

Figure 5: The velocity profiles for the differentvalues of t

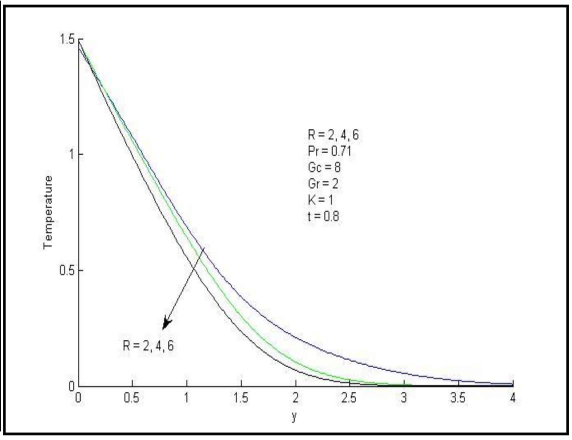

Figure 6: The temperature profiles for thedifferent values of R

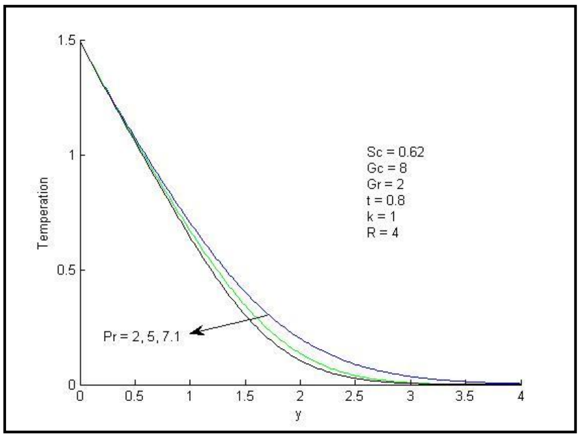

Figure 7: The temperature profiles for theDifferent values of Pr

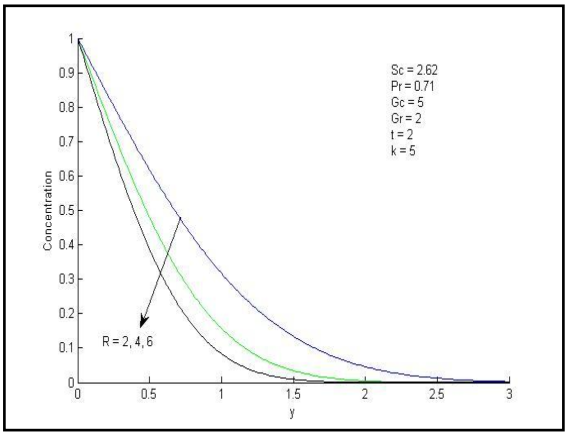

Figure 8: The Concentration profiles for thedifferent values of R

6. Conclusion

Analytical solution of an unsteady flow past an accelerated vertical plate has been carried out through the porous medium in the presence of thermal radiation.The dimensional governing equations are solved by Laplace transform technique and computed for different parameters parameters like Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time. The effect of these parametrs on the flow characteristics of velocity, temperature and concentartion are presented graphically.It is observed that velocity profile increases with increasing parameter like Gr, Sc, t and R and also decreseses with the decrease in Gc. It also notice that temperature decreases with increasing Pr and R. The concentration profiles is observed that the increase in thermal radiation leads to the decreases in concentration profile. In second its observed that problem the velocity profile increase with increase in Gr, K, M, and t and olso decrease with the decreas of R, and Gr. The temperature profile shows a decrease with the decrease in Pr, and t. While the concentration decrease with increase in R.

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