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ON SECOND DEGREE EQUATION WITH THREE UNKNOWNS

$$\mathbf{2\left(x^2+y^2\right)-3xy=32z^2}$$

S.Vidhyalakshmi, M.A.Gopalan

Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,Trichy-620 002,Tamil Nadu, India.

ABSTRACT:

The cone represented by the ternary quadratic Diophantine equation $2\left(x^2+y^2\right)-3xy=32z^2$ is analyzed for its patterns of non-zero distinct integral solutions.

Keywords: Ternary quadratic, cone, integral solutions 2010 mathematics subject classification: 11D09

Introduction:

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-11] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation $2\left(x^2+y^2\right)-3xy=32z^2$ representing homogeneous quadratic with three unknowns for determining its infinitely many non-zero integral points.

METHOD OF ANALYSIS:

The ternary quadratic diophantine equation to be solved for its non-zero distinct integral solution is

$$2\left(x^2+y^2\right)-3xy=32z^2 \tag{1}$$

Introduction of the linear transformations $(u \neq v \neq 0)$

$$x=u+v,\ y=u-v \tag{2}$$

in (1) leads to

$$u^2+7v^2=32z^2 \tag{3}$$

We present below different methods of solving (1).

Method I:

We can write 32 as

$$32=(5+i\sqrt{7})(5-i\sqrt{7}) \tag{4}$$

Assume $z=a^2+7b^2$ (5)

where a and b are non-zero distinct integers.

Using (4) and (5) in (3), we get

$$u^2+7v^2=(5+i\sqrt{7})(5-i\sqrt{7})(a+i\sqrt{7}b)^2(a-i\sqrt{7}b)^2$$

Equating the positive and negative factors, the resulting equating are

$$(u+i\sqrt{7}v)=(5+i\sqrt{7})(a+i\sqrt{7}b)^2 \tag{6}$$

$$(u-i\sqrt{7}v)=(5-i\sqrt{7})(a-i\sqrt{7}b)^2 \tag{7}$$

Equating the real and imaginary parts either in (6) or (7), we get

$$u=5a^2-35b^2-14ab$$

$$v=a^2-7b^2+10ab$$

Substituting the values of u and v in (2) we get,

$$x=x(a.b)=6a^2-42b^2-4ab \tag{8}$$

$$y=y(a.b)=4a^2-28b^2-24ab \tag{9}$$

which satisfy (1) along with (5)

NOTE:17

It is seen that 32 on the R.H.S. of (3) is also represented as follows**:**

$$32 = \frac{(11+i\sqrt{7})(11-i\sqrt{7})}{4},$$

$$32 = (2+2i\sqrt{7})(2-2i\sqrt{7})$$

Following the above procedure, one obtains two more sets of integer solutions to (1).

Method II:

Equation (3) is written as

$$u^2-25z^2=7(z^2-v^2) \tag{10}$$

Write (10) in the form of ratio as

$$\frac{u+5z}{z+v}=\frac{7(z-v)}{(u-5z)}=\frac{\alpha}{\beta}\ ,\beta\neq 0 \tag{11}$$

which is equivalent to the system of double equations

$$\beta u-\alpha v+(5\beta-\alpha)z=0 \tag{12}$$

$$-\alpha u-7\beta v+(7\beta+5\alpha)z=0 \tag{13}$$

Applying the method of cross multiplication for solving (12) and (13)

$$u=u(\alpha,\beta)=5\alpha^2-35\beta^2+14\alpha\beta$$

$$v=v(\alpha,\beta)=-\alpha^2+7\beta^2+10\alpha\beta$$

$$z=z(\alpha,\beta)=\alpha^2+7\beta^2 \tag{14}$$

Substituting the values of u and v in (2) we get,

$$\left.\begin{aligned} x&=x(\alpha,\beta)=4\alpha^2-28\beta^2+24\alpha\beta,\\ y&=y(\alpha,\beta)=6\alpha^2-42\beta^2+4\alpha\beta \end{aligned}\right\} \tag{15}$$

Thus,(14) and (15) represent the integer solutions to (1).

NOTE:2

Apart from (11), (10) is also written in the form of ratios as presented below:

(i) $$\frac{u+5z}{7(z-v)}=\frac{z+v}{(u-5z)}=\frac{\alpha}{\beta}$$

(ii) $$\frac{u-5z}{z+v}=\frac{7(z-v)}{(u+5z)}=\frac{\alpha}{\beta}$$

(iii) $$\frac{u-5z}{7(z-v)}=\frac{z+v}{u+5z}=\frac{\alpha}{\beta}$$

The repetition of the above process leads to three different solutions to (1).

METHOD III:

Equation (3) is written as

$$u^2+7v^2=32z^2*1 \tag{16}$$

Write 1 as

$$1=\frac{(1+i3\sqrt{7})(1-i3\sqrt{7})}{64} \tag{17}$$

Using (4), (5) and (17) in (16) we get,

$$(u+i\sqrt{7}v)(u-i\sqrt{7}v)=\left((a+i\sqrt{7}b)^2(a-i\sqrt{7}b)^2(5+i\sqrt{7})(5-i\sqrt{7})\frac{(1+i3\sqrt{7})(1-i3\sqrt{7})}{64}\right)$$

Equating the positive and negative factors, the resulting equations are,

$$(u+i\sqrt{7}v)=(5+i\sqrt{7})\frac{(1+i3\sqrt{7})}{8}(a+i\sqrt{7}b)^2 \tag{18}$$

$$(u-i\sqrt{7}v)=(5-i\sqrt{7})\frac{(1-i3\sqrt{7})}{8}(a-i\sqrt{7}b)^2 \tag{19}$$

Equating real and imaginary part in (18) we get

$$u=-2a^2+14b^2-28ab$$

$$v=2a^2-14b^2-4ab$$

Substituting the values of u and v in (2) ,we get

$$x=x(a,b)=-32ab$$

$$y=y(a,b)=-4a^2+28b^2-24ab$$

which satisfy (1) along with (5)

Note: 3

It is seen that 1 on the R.H.S. of (16) is also represented as follows:

$$1 = \frac{(3 + i\sqrt{7})(3 - i\sqrt{7})}{16},$$

$$1 = \frac{(9 + i\,5\sqrt{7})(9 - i\,5\sqrt{7})}{256},$$

$$1 = \frac{(7\,r^2 - s^2 + i\,2\,r\,s\sqrt{7})(7\,r^2 - s^2 + i\,2\,r\,s\sqrt{7})}{(7\,r^2 + s^2)^2}$$

Following the above procedure, three more sets of integer solutions to (1) are obtained.

CONCLUSION:

In this paper, we have made an attempt to obtain all integer solutions to (1). As (1) is symmetric in x ,y, z, it is to be noted that, if (x, y, z) is any positive integer solution to (1),then the triples $(-x, y, z)$, $(x, -y, z)$, $(x, y, -z)$, $(x, -y, -z)$, $(-x, y, -z)$, $(-x, -y, z)$, $(-x, -y, -z)$ also satisfy (1). To conclude, one may search for integer solutions to other choices of homogeneous cones along with suitable properties.

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