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A Review Study on Seismic Analysis of Multistorey Building with Floating Column

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ABSTRACT:

Some Buildings having floating columns is one of the common element in modern multistory architecture in urban India nowadays. In buildings constructed in seismically active areas, such elements are particularly undesirable. The necessity of explicitly noting presence of these types of columns in building analysis is highlighted in this study. To alleviate the irregularity produced by the floating columns, other measures including stiffness of first storey to the above storey have been proposed.To examine structural responses under multiple seismic excitations with various frequency content while maintaining the PGA and time duration factor constant, For 2D multi-story frames having floating columns or not, FEM programmes are developed. The following observation taken as floor displacement as per time history method, shear, drift is estimated for both types of frame.

KEYWORDS: Floating Column, Multistory, FEM, Floor Displacement, Overturning Moment.

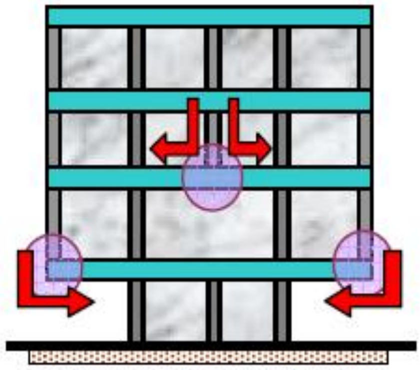
INTODUCTION:

In India, open first stories are an unavoidable feature of many metropolitan multistory buildings. This is commonly utilised on the first level for parking or reception areas. The natural duration of an earthquake determines the overall seismic base shear experienced by a building, while the stiffness and mass distribution along the height dictate the seismic force distribution.

The general shape, scale, and geometry of a building, as well as how earthquake forces are transported to the ground, have a significant impact on its behaviour during earthquakes. Any variation or break in load transfering path degrades the building's performance. Poor building performance is caused by any variation in load transfering channels. The earthquake forces generated at different levels of a building must be transferred down the shortest path possible to the ground. At discontinuity level, buildings with vertical setbacks create jumps in earthquake forces. Buildings having walls per level, particularly those with extremely tall storeys, are more prone to damage or collapse. During 2001 Bhuj earthquake in Gujarat, several buildings which having open ground storey generally built for parking purposes is completely collapsed or seriously damaged. Load transmission discontinuities exist columns with buildings that floating on beams at middle floor but do not extend foundation way.

WHAT IS FLOATING COLUMN

Basically its a vertical structure that begins at the foundation and carries weight to the earth. Due to architectural design/site circumstance, its resting on the beam.The load is then transferred to additional columns below the beams.



Hanging or Floating Columns

Many designs use floating columns, which is basically above G.L. when transfer girders are used, to create greater open space. These open areas can be needed for an assembly hall or parking. Transfer girders, especially in earthquake-prone areas, must be well planned and detailed. Basically column is a focused loading on beam that supports it. In terms of analysis, the column is frequently assumed to be pinned at base and hence treated as point load on the transfer beam. This type of structure can be analysed using STAAD Pro, ETABS, and SAP2000. These types of columns are capable of carrying gravity loads, but transfer girder must be of sufficient size (stiffness) and have very little deflection.

Looking ahead, one will, of course, continue to make structures that are intriguing rather than boring. However, this does not have to expense of poor behaviour or structure safety during earthquakes. Buildings with architectural elements that are harmful to earthquake reaction. Firstly its avoided or reduced to a minimum. When irregular characteristics are used in buildings, the structural design requires significantly more technical effort, and the building may not be as good as one with simple architectural aspects.

As a result, structures built with these types of discontinuous parts are at risk in seismic zones. However, those structures cannot be demolished; instead, studies to strengthen the structure or suggestions for corrective elements can be made. The first-storey columns can be strengthened, their stiffness enhanced through retrofitting, or bracing added to reduce lateral distortion.

LITERATURE REVIEW

Maison and Neuss (1984), ASCE members conducted a computer analysis of an existing forty-four storey steel frame in order to investigate the impact of various modelling parameters on anticipated dynamic characteristics and computed seismic response behaviour. The projected dynamic property compared to genuine qualities of the building, as determined previously through experimental testing. The reaction spectrum (Newmark and ATC spectra) and comparable static load approaches are used to calculate seismic response behaviour.

Maison and Ventura (1991), ASCE members computed the dynamic properties and response behaviours of a thirteen-story building, comparing the results earthquakes, demonstrating that state-of-the-art design dynamic properties.

Arlekar, Jain & Murty (1997) claimed that such elements were highly undesirable in structures erected in seismically active places, as evidenced by countless experiences of violent shaking during previous earthquakes. This acknowledging the presence of open storey in the building's analysis .

Awkar and Lui (1997) Using a computer model, researchers investigated earthquake excitations. In the analyses, connection flexibility or geometrical and material nonlinearities, and the results show that connection flexibility increases upper-story inter-storey drifts while lowering base shears and base overturning moments for multi-story frames.

Balsamoa, Colombo, Manfredi, Negro & Prota (2005) Pseudodynamic experiments were done on Reinforced Concrete structure that had been reconstructed with CFRP laminates. The benefits of using CFRP composites for seismic restoration of RC members were evaluated in the ELSA laboratory using a full-scale dual system that was subjected to pseudodynamic tests. The goal of restore the structural qualities of the frame prior to the seismic events by increasing the deformation capacity of both the columns and joints. Depending on the fundamental mechanism governing each component, the repair was characterised by a variety of fibre textures. It discussed the design concepts for CFRP repairing and experimental tests.

Vasilopoulos and Beskos (2006) Its conducted a rational and economical seismic design technique for planar steel frames utilising modern methods of analysis. This design methodology makes use of an advanced FEM analysis that accounts for geometrical and material nonlinearities, as well as flaws in members and frames. It can adequately capture the structure's limit states, stability, and damage.

Bardakis & Dritsos (2007) evaluated the American and European procedural assumptions for the assessment of the seismic capacity of existing buildings via pushover analyses. The FEMA and the A four-story bare-framed building was assessed using Euro code-based GRECO techniques, and the results were compared to available experimental results.

Mortezaei *et al* (2009) During an earthquake, this pulse can inflict significant damage, especially to structures with natural periods that are close to the pulse's. Recent earthquakes have shown the vulnerability of current RC buildings to pulse-type ground vibrations due to failures of modern designed structures in the near-fault region. This could be because these modern structures were predominantly constructed using the design spectra of accessible standards, which were generated using stochastic processes with relatively lengthy durations that define more distant ground vibrations. In order to operate successfully when subjected to near-fault ground vibrations, many newly planned and constructed buildings may require reinforcement. Due to their ease of installation,least costs, and lack of maintenance requirements, FRP considered as feasible alternative.

Ozyigit (2009) explored the beam is circular in cross section and has straight and curved component. A concentrated mass can also be found at many positions around the frame, each with a different mass ratio. The problem is analysed using FEM.

Williams, Gardoni & Bracci (2009) Using the framework details, we investigated the economic value of a specific refit process. To determine how specific characteristics affecting seismic retrofit was performed. A case study was conducted using a simple retrofit approach for the sample buildings in Memphis and San Francisco.

Garcia *et al* (2010) The study done by shake table testing. Models of steel concrete bond slip and bond-strength degradation under cyclic loading were used to simulate inadequate beam column joints. Using a collection of mild to strong seismic recordings, the analytical models were utilised to estimate CFRP rehabilitation efficiency. The CFRP strengthening intervention improved inadequate beam column joints, resulting in a significant improvement in the damaged RC frame's seismic performance. If the damaged buildings subjected to following the CFRP intervention, it was proven that it would sustain 65 percent less global damage than the original construction.

Niroomandi, Maheri, Maheri & Mahini (2010) web-bonded CFRP was used to retrofit an eight-story structure that had previously been strengthened with bracing system. When the FRP retrofitted frame at joints was compared to the steel X-braced retrofitting method, it was observed similar capacities to enhance the ductility reduction factor and the over-strength factor, with the former surpassing the latter on ductility. The RC frame's steel bracing can be useful. To achieve the needed increases.

Keerthi gowda B. S. (May 2014) earthquakes in various places of the world, it was said, illustrated the dangerous repercussions and fragility of inadequate constructions. Buildings with floating columns are common in India's cities. The seismic inertia forces generated at a building's floor levels must be transferred down the building's height to the ground, irregularity causes poor performance. The negative impact of floating columns in construction. The results show that providing lateral bracing to reduce lateral deformation is an alternate technique that should be implemented. After adding lateral bracing.

Ms. Priyanka D. Motghare (2016) This study is about analytical studies that were done on RCC frame in various floating column positions. The load transfer is disrupted in a building with the help of column it transfer to the beam at an intermediate floor and does not travel all the way to the foundation. STAAD pro was used to do the analysis while considering various placement of this. The influence position was also investigated. All of the floating column examples had higher bending moments. The existence had an impact on the final maximum bending moments values.

Sharma R. K. (June 2016) Floating column buildings were a common element in modern multistory architecture in urban India, according to the study. Building were used for architectural reasons or when additional free space in the ground level was necessary. In seismically active places, such features were particularly undesired. The analysis of G+5, G+7, G+9, G+11, and G+13 storey buildings having floating columns or not was carried out in project studies. The analysis was carried out by Staad Pro V8i programme and Response spectrum analysis. The study discusses the differences in findings in structural displacement, base shear, building seismic weight calculation from manual calculations, and Staad pro V8i. Finding earthquake reaction parameters for buildings and describing what happens when the variation is high or low. The study was conducted to determine whether floating column constructions were safe or hazardous when constructed in seismically prone places, such as whether it was cost-effective or not.

Ms. Waykule S.B. (2017) Sortage of space, population, and aesthetic and practical needs, multi-story structures in metropolitan areas have recently been mandated to include column-free space. Buildings were equipped with floating columns on one or more storeys to facilitate this. In a building constructed in a seismically active area, these floating columns were quite inconvenient. The building's performance suffers as a result of discontinuity. The overall shape, scale, and geometry of a building, as well as how the seismic forces were transported to the ground, and check behaviour during earthquakes.

STATIC ANALYSIS

Plane frame element

Its two-dimensional finite element which having local and global coordinates. Its also have elasticity modulus of E, an inertia moment I, a cross-sectional area A, length L. Each plane frame element contains two nodes and is angled counterclockwise from the positive global X axis, as given below Let C= cosθ and S= sinθ.

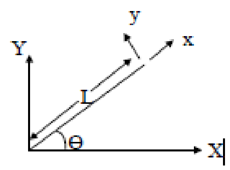


Fig. 3.1 The plane frame element

It has 6 D.O.F. - three at each node (two displacements and a rotation). According to the sign convention, displacements are +ve if point upwards, rotations also +ve if it counterclockwise. As a result, for any structure which have n numbers of nodes, the global stiffness matrix K will be 3n X 3n. K global stiffness is using MATLAB function , which was created expressly for this purpose.

We get the following structure equation if we have k :

[K]{U}={F} (3.1)

Where [K] is stiffness matrix, {U} is global nodal displacement vector and {F} is global nodal force vector. Boundary conditions are manually applied U and F in this stage. After that, partitioning and Gaussian elimination are used for solving matrix problem (3.1). After determining the unknown displacements and responses, the nodal force vector for each element is calculated as follows:

{f} = [k] [R] {u} ....(3.2)

Where {f} is the 6 X 1 nodal force vector in the element and {u} is the 6 X 1 element displacement vector. The matrices [k] and [R] are given by the following:

$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \qquad (3.3)$$

$$[R] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.4)$$

Steps followed for the analysis of frame

1. Discretising the domain: Dividing the element into number of nodes and numbering them globally i;e breaking down the domain into smaller parts.

2. Writing of the Element stiffness matrices: For each element, local stiffness matrix is determined, and K of size 3n x 3n is constructed using these local stiffness matrices.

3. Assembling the global stiffness matrices: The element stiffness matrices are combined globally based on their degrees of freedom values.

4. Applying the boundary condition: The boundary element condition is applied by suitably deleting the rows and columns which are not of our interest.

5. Solving the equation: The equation is solved in MATLAB to give the value of U.

6. Post- processing: The reaction at the support and internal forces are calculated.

Dynamic analysis

The behaviour of a flexible structure subjected to dynamic loading is examined in dynamic structural analysis, which is a subset of structural analysis. Dynamic load fluctuates over time. Wind, live load, earthquake load, and other dynamic loads are examples of dynamic loads. As a result, we may argue that practically all real-world situations can be investigated dynamically.

If dynamic loads fluctuate gradually, static analysis used to estimate the structure's reaction by ignoring inertia forces. However, if the dynamic load changes rapidly, the reaction must be determined using dynamic analysis, in which the inertial force equals mass time of acceleration cannot be ignored.

Mathematically F = M x a

Where F is inertial force, M is inertial mass and ‘a’ is acceleration.

Furthermore, dynamic response (displacement and stresses) are generally higher than corresponding static displacements for same loading amplitudes, especially at resonant conditions.

The real physical structures have many numbers of displacement. As a result, the most important aspect of structural analysis is to construct a computer model that represents the real behaviour of structures by using a finite number of mass-less members and a finite number of node displacements. Another difficult part of dynamic analysis is to calculate energy dissipation and to boundary condition. So it is very difficult to analyze structure for wind and seismic load. This difficulty can be reduced using various programming techniques. For this work we have used finite element analysis and programmed in MATLAB.

Time history analysis

If non-linear behaviour is not present, a linear time history analysis eliminates all of the drawbacks of modal response spectrum analysis. Calculating the response at discrete time needs more computational effort with this method. One appealing feature of this method is that the relative signals of response attributes are preserved in response histories. This is critical when considering interaction effects in stress resultant design.

The dynamic response of the plane frame model to a time history that is compatible with the IS code spectrum and Elcentro (EW) has been investigated in this paper.

In matrix form, the equation of motion for a multi D.O.F. is

$[m]\{\ddot{x}\}+[c]\{\dot{x}\}+[k]\{x\}=-x_g(t)[m]\{I\}$ ....(3.5)

Where,

$[m]$= mass matrix

$[k]$= stiffness matrix

$[c]$= damping matrix

$\{I\}$= unit vector

(t)= ground acceleration

The mass matrix of each element in global direction can be found out using following expression:

m = $[T^T]$ $[m_e]$ [T] ....(3.6)

$$[T]=\begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The solution of equation of motion for any specified forces is difficult to obtain, mainly due to due to coupling variables {x} in the physical coordinate. In mode superposition analysis or a modal analysis a set of normal coordinates i.e principal coordinate is defined, such that, when expressed in those coordinates, motion eq. becomes uncoupled. The physical coordinate {x} may be related with normal or principal coordinates {q} from the transformation expression as,

$\{x\} = [\Phi]\{q\}$

$[\Phi]$ is the modal matrix

Time derivative of {x} are,

$\{\dot{x}\} = [\Phi]\{\dot{q}\}$

$\{xx\} = [\Phi]\{\ddot{q}\}$

Substituting the time derivatives in the equation of motion, and pre-multiplying by [Φ]T results in,

$[\Phi]^T[m][\Phi]\{\ddot{q}\}+[\Phi]^T[c][\Phi]\{\dot{q}\}+[\Phi]^T[k][\Phi]\{q\}=(-x_g(t)[\Phi]^T[m]\{I\})$ ....(3.8)

More clearly it seems as:

$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{P_{eff}(t)\}$... (3.9)

Where,

$[M]= [\Phi][m][\Phi]$

$[C]= [\Phi][c][\Phi] = 2\,\zeta\,[M]\,[\omega]$

$[K]= [\Phi][k][\Phi]$

$\{P_{eff}(t)\} = (-x_g(t)[\Phi]^T[m]\{I\})$

[M], [C] and [K] are the diagonalised modal mass matrix, modal damping matrix and modal stiffness matrix, respectively, and {Peff(t)} is the effective modal force vector.

Newmark's method

Newmark's numerical method has been adopted to solve the equation 3.9. Newmark's equations are given by

$\dot{d}_{i+1} = di + (\Delta t)\,[(1-\gamma)+\gamma d_{i+1}]$.. (3.10)

$d_{i+1} = d_i+(\Delta t)di+(\Delta t)^2\,[(1/2-\beta)di+\beta d_{i+1}]$ ....(3.11)

Where β and γ are parameters chosen by the user. The parameter β is generally chosen between 0 and ¼, and γ is often taken to be ½. For instance, choosing γ = ½ and β = 1/6, are chosen, eq. 4.12 and eq. 4.13 correspond to those foe which a linear acceleration assumption is valid within each time interval. For γ = ½ and β = ¼, it has been shown that the numerical analysis is stable; that is, computed quantities such as displacement and velocities do not become unbounded regardless of the time step chosen.

To find d_{i+1}, we first multiply eq. 4.13 by the mass matrix MM and then substitute the value of d_{i+1} into this eq. to obtain

$M\ddot{d}_{i+1}=M\,di+(\Delta t)M\,di+(\Delta t)^2M\,(1/2-\beta)\,di+\beta(\Delta t)^2[F_{i+1}-Kd_{i+1}]$........... (3.12)

Combining the like terms of eq. 4.14 we obtain

$(M+(\Delta t)^2K)d_{i+1}=(\Delta t)2F_{i+1}+M\,di+(\Delta t)M\,di+(\Delta t)^2M(1/2-\beta)di$ ....(3.13)

Finally, dividing above eq. by $(\Delta tt)2$, we obtain

$K'd_{i+1}=F'_{i+1}$... (3.14)

$K'=K+1/(\Delta t)^2M$... (3.15)

$F'_{i+1}=F_{i+1}+M/(\Delta t)^2\,[di+(\Delta t)di+(1/2-\beta)(\Delta t)^2di]$...................... (3.16)

The solution procedure using Newmark's equations is as follows:

1. Starting at time t = 0, d_0 is known from the given boundary conditions on displacement, and $\dot{d}_0$ is known from the initial velocity conditions.
2. Solve eq. 4.5 at t=0 for $\ddot{d}_0$ (unless d_0 is known from an initial acceleration condition);
 that is,
 $\ddot{d}_0 = M^{-1}(F_0-Kd_0)$
3. Solve eq. 3.16 for $d1$, because F'_{i+1} is known for all time steps and , d_0 ,$\dot{d}_0$,do are known from steps 1 and 2.
4. Use eq. 3.13 to solve for $\ddot{d}_1$ as
 $\ddot{d}1 =1/(\Delta t)^2[d1-d0-(\Delta t)\dot{d}0-(\Delta t)2(1/2-\beta)\ddot{d}_0\,]$
5. Solve eq. 3.12 directly for $\dot{d}1$
6. Using the results of steps 4 and 5, go back to step 3 to solve for $d2$ and then to steps 4 and 5 to solve for $\ddot{d}2$ and $\dot{d}2$. Use steps 3-5 repeatedly to solve for d_{i+1}, d_{i+1} and di_{+1}.

SUMMARY:

The behavior analysis under different earthquake excitation condition. Elcentro earthquake data and a corresponding temporal history were taken into account. The PGA of both earthquakes measured at 0.2g scale, and the excitation duration has been kept constant. Motion of a multi-story frame has been studied using a finite element model. The results of the current finite element code for rest position and freely vibrating position are verified. By changing the column dimension, the dynamic analysis of the frame is investigated. The maximum displacement and inter-storey drift values decrease as the bottom floor column height increases. The base shear or overturning moments fluctuate as column dimensions change.

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