



Pentapartitioned Neutrosophic Pythagorean Generalized Precontinuity

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ABSTRACT

A Pentapartitioned neutrosophic set (PNS) is a powerful structure where we have five components Truth, Falsity, Ignorance, Contradiction and unknown. And also it generalizes the concept of fuzzy, intuitionistic and neutrosophic set. In this paper, we applying the idea of generalised Precontinuous function to Pentapartitioned Neutrosophic Pythagorean Sets [PNPS]. Also, we interrelate with other functions and its properties are also studied.

Keywords: Neutrosophic set, PNP set, PNP continuity, PNP generalised precontinuity

1. Introduction

Zadeh introduced the idea of fuzzy sets in 1965 that permits the membership perform valued within the interval $[0,1]$ and set theory its an extension of classical pure mathematics. Intuitionistic Fuzzy set was first introduced by K. T. Atanassov in 1983. After that he introduced, the concept of Intuitionistic sets as generalization of Fuzzy sets. The concept of generalized topological structures in Fuzzy topological spaces using Intuitionistic Fuzzy sets was introduced by D. Coker [3]. D. Coker introduced the concept of Intuitionistic Fuzzy sets, Intuitionistic Fuzzy topological spaces, Intuitionistic topological spaces and Intuitionistic Fuzzy points. R. R. Yager generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean set.

Florentine Smarandache introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) severally. Neutrosophic sets deals with non normal interval of $] -0 \ 1+[$. Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web, electronic database systems, diagnosis, multicriteria higher cognitive process issues etc.,

Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. Further, R. Radha and A. Stanis Arul Mary outlined a brand new hybrid model of Pentapartitioned Neutrosophic Pythagorean sets (PNPS) and Quadripartitioned neutrosophic pythagorean sets in 2021. The Pentapartitioned Neutrosophic Pythagorean topological spaces [9] was introduced and its properties are investigated in 2021.

A. Salma introduced the concept of neutrosophic continuous functions in 2012. In this paper, we have applied the concept of continuous functions in Pentapartitioned Neutrosophic Pythagorean Topological Spaces.

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2. Preliminaries

2.1 Definition

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

2.2 Definition

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ and

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

2.3 Definition

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A , an ignorance membership function G_A , unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

2.4 Definition

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by A^C or A^* and is defined as

$$A^C = \{ \langle x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) \rangle : x \in X \}$$

2.5 Definition

Let $A = \langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) \rangle$ and

$B = \langle x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) \rangle$ are pentapartitioned neutrosophic pythagorean sets. Then

$$\begin{aligned} A \cup B = & \langle x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \\ & \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)) \rangle, \\ A \cap B = & \langle x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)) \\ & , \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) \rangle \end{aligned}$$

2.6 Definition

A PNP topology on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

- 1) $0, 1 \in \tau$
- 2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
- 3) $\cup R_i \in \tau$ for any $R_i: i \in I \subseteq \tau$

The complement R^* of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R, τ) , is called a PNP closed set [PNPCS].

2.7 Definition

Let (X, τ) be an PNPTS and $A = \langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) \rangle$ be an PNPS in X . Then the interior and the closure of A are denoted by $\text{PNPInt}(A)$ and $\text{PNPCl}(A)$ and are defined as follows.

$$\text{PNPCl}(A) = \cap \{ K | K \text{ is a PNPCS and } A \subseteq K \} \text{ and}$$

$$\text{PNPInt}(A) = \cup \{ G | G \text{ is a PNPOS and } G \subseteq A \}$$

Also, it can be established that $\text{PNPCl}(A)$ is an PNPCS and $\text{PNPInt}(A)$ is a PNPOS, A is a PNPCS if and only if $\text{PNPCl}(A) = A$ and A is a PNPOS if and only if $\text{PNPInt}(A) = A$. We say that A is PNP-dense if $\text{PNPCl}(A) = X$.

2.8 Definition

Let (X, τ) and (Y, σ) be any two PNPTSs. A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- 1) PNP continuous if $f^{-1}(V)$ is PNP closed set in (X, τ) , for every PNP closed set V of (Y, σ) .
- 2) PNP Generalized [PNPG] continuous if $f^{-1}(V)$ is PNP generalized closed set in (X, τ) , for every PNP closed set V of (Y, σ) .
- 3) PNP generalized irresolute if $f^{-1}(V)$ is PNP generalized closed set in (X, τ) , for every PNP generalized closed set V of (Y, σ) .

3. Pentapartitioned Neutrosophic Pythagorean Generalized Pre Continuity

3.1 Definition

Let (X, τ) and (Y, σ) be any two PNPTSs. A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be PNP generalized pre continuous (PNPGP continuous) if $f^{-1}(V)$ is PNPGP closed set in (X, τ) for every PNP closed set V of (Y, σ) .

3.2 Example:

Let $X = \{p, q\}$ and $Y = \{a, b\}$, $S = \{(p, 0.1, 0.0, 0.3, 0.4, 0.5), (q, 0.2, 0.1, 0.4, 0.5, 0.6)\}$, $T = \{(p, 0.1, 0.0, 0.2, 0.2, 0.3), (q, 0.4, 0.3, 0.3, 0.5, 0.6)\}$ and $R = \{(p, 0.2, 0.1, 0.3, 0.1, 0.2), (q, 0.9, 0.8, 0.3, 0.0, 1)\}$. Then $\tau = \{0_X, 1_X, S, T\}$ and $\sigma = \{0_Y, 1_Y, R\}$ are PNPTS on X and Y . Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(p) = a$ and $f(q) = b$. Then f is a PNPGP continuous mapping.

3.3 Theorem

Let (X, τ) and (Y, σ) be any two PNPTSs. Then the mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ satisfies the following results

- 1) Every PNP continuous mapping is a PNPGP continuous mapping.
- 2) Every PNPG continuous mapping is a PNPGP continuous mapping.
- 3) Every PNPR continuous mapping is a PNPGP continuous mapping.
- 4) Every $PNP\alpha$ continuous mapping is a PNPGP continuous mapping.
- 5) Every $PNP\alpha G$ continuous mapping is a PNPGP continuous mapping.
- 6) Every PNPP continuous mapping is a PNPGP continuous mapping.
- 7) Every PNPGP continuous mapping is a PNPGSP continuous mapping.
- 8) Every PNPGP continuous mapping is a PNPGSP continuous mapping.

3.4 Theorem

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be PNP continuous mapping. Then the following conditions hold.

- 1) $f(\text{PNPGPCI}(V)) \subseteq \text{PNPCI}(f(V))$, for every PNP set V in X .
- 2) $\text{PNPGPCI}(f^{-1}(T)) \subseteq f^{-1}(\text{PNPCI}(T))$ for every PNP set T in Y .

Proof

(1) Since $\text{PNPCI}(f(V))$ is PNPC set in Y and f is PNPGP continuous mapping, then $f^{-1}(\text{PNPCI}(f(V)))$ is PNPGPC set in X . That is $\text{PNPGPCI}(V) \subseteq f^{-1}(\text{PNPCI}(f(V)))$. Therefore $f(\text{PNPGPCI}(V)) \subseteq \text{PNPCI}(f(V))$, for every PNP set V in X .
 (2) Replacing V by $f^{-1}(T)$ in (1), we get $f(\text{PNPGPCI}(f^{-1}(T))) \subseteq \text{PNPCI}(f(f^{-1}(T))) \subseteq \text{PNPCI}(T)$. Hence $\text{PNPGPCI}(f^{-1}(T)) \subseteq f^{-1}(\text{PNPCI}(T))$, for every PNP set T in Y .

3.5 Theorem

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is PNP continuous mapping if and only if the inverse image of each PNP open set in Y is PNPGPO set in X .

Proof

Necessity: Let V be PNPO set in Y . This implies V^c is PNPC set in Y . Since f is PNPGP continuous mapping, $f^{-1}(V^c)$ is PNPGPC set in X . Since $f^{-1}(V^c) = (f^{-1}(V))^c$, $f^{-1}(V)$ is PNPGPO set in X .
 Sufficiency: The proof is obvious.

3.6 Theorem

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be PNP continuous mapping and let $f^{-1}(V)$ is PNPRC set in X for every PNPC set V in Y . Then f is PNPGP continuous mapping.
 Proof

Let V be PNPC set in Y . Then $f^{-1}(V)$ is PNPRC set in X . Since every PNPRC set is PNPGPC set in X . Hence f is PNPGP continuous mapping.

3.7 Theorem

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be PNPGP continuous mapping and then f is PNP continuous mapping if X is $\text{PNP}_p T_{\frac{1}{2}}$ space.

Proof

Let V be PNPC set in Y . Then $f^{-1}(V)$ is PNPGPC set in X , by hypothesis, since X is $\text{PNP}_p T_{\frac{1}{2}}$ space, $f^{-1}(V)$ is PNPC set in X . Hence f is PNP continuous mapping.

3.8 Theorem

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be PNPGP continuous mapping and then f is PNPP continuous mapping if X is $\text{PNP}_{gp} T_{\frac{1}{2}}$ space.

Proof

Let V be PNPC set in Y . Then $f^{-1}(V)$ is PNPGPC set in X , by hypothesis, since X is $\text{PNP}_{gp} T_{\frac{1}{2}}$ space, $f^{-1}(V)$ is PNPPC set in X . Hence f is PNPP

continuous mapping.

3.9 Theorem

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from PNPTS X into PNPTS Y . Then the following conditions are equivalent if X is $\text{PNP}_{\text{gp}}T_{\frac{1}{2}}$ space.

- 1) f is PNP GP continuous mapping.
- 2) $f^{-1}(T)$ is PNP GPC set in X for every PNPC set T in Y .
- 3) $\text{PNPCI}(\text{PNPInt}(f^{-1}(V))) \subseteq f^{-1}(\text{PNPCI}(V))$ for every PNP set V in Y

Proof

(1) \Rightarrow (2) : It is obvious

(2) \Rightarrow (3) : Let V be PNP set in Y . Then $\text{PNPCI}(V)$ is PNPC set in Y . By hypothesis, $f^{-1}(\text{PNPCI}(V))$ is PNP GPC set in X . Since X is $\text{PNP}_{\text{gp}}T_{\frac{1}{2}}$ space, $f^{-1}(\text{PNPCI}(V))$ is PNPPC set in X . Therefore $\text{PNPCI}(\text{PNPInt}(f^{-1}(\text{PNPCI}(V)))) \subseteq f^{-1}(\text{PNPCI}(V))$. Now $\text{PNPCI}(\text{PNPInt}(f^{-1}(V))) \subseteq \text{PNPCI}(\text{PNPInt}(f^{-1}(\text{PNPCI}(V)))) \subseteq f^{-1}(\text{PNPCI}(V))$.

(3) \Rightarrow (1) : Let V be PNP set in Y . By hypothesis, $\text{PNPCI}(\text{PNPInt}(f^{-1}(V))) \subseteq f^{-1}(\text{PNPCI}(V)) = f^{-1}(V)$. This implies $f^{-1}(V)$ is PNPPC set in X and hence it is PNP GP continuous mapping.

3.10 Theorem

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from PNPTS X into PNPTS Y . Then the following conditions are equivalent if X is $\text{PNP}_{\text{gp}}T_{\frac{1}{2}}$ space.

- 1) f is PNP GP continuous mapping.
- 2) $f^{-1}(T)$ is PNP GPO set in X for every PNPO set T in Y .
- 3) $f^{-1}(\text{PNPInt}(V)) \subseteq \text{PNPInt}(\text{PNPCI}(f^{-1}(V)))$ for every PNP set V in Y

Proof:

(1) \Rightarrow (2) : It is obvious

(2) \Rightarrow (3) : Let V be PNP set in Y . Then $\text{PNPInt}(V)$ is PNPO set in Y . By hypothesis, $f^{-1}(\text{PNPInt}(V))$ is PNP GPO set in X . Since X is $\text{PNP}_{\text{gp}}T_{\frac{1}{2}}$ space, $f^{-1}(\text{PNPInt}(V))$ is PNPPO set in X . Therefore $\text{PNPCI}(\text{PNPInt}(f^{-1}(\text{PNPInt}(V)))) \subseteq f^{-1}(\text{PNPInt}(V))$. Now $f^{-1}(\text{PNPInt}(V)) \subseteq \text{PNPInt}(\text{PNPCI}(f^{-1}(\text{PNPInt}(V)))) \subseteq \text{PNPInt}(\text{PNPCI}(f^{-1}(V)))$.

(3) \Rightarrow (1) : Let V be PNPC set in Y . Then its complement V^c is PNPO set in Y , $\text{PNPInt}(V^c) = V^c$. Now By hypothesis, $f^{-1}(\text{PNPInt}(V^c)) \subseteq \text{PNPInt}(\text{PNPCI}(f^{-1}(V^c)))$. This implies $f^{-1}(V^c) \subseteq \text{PNPInt}(\text{PNPCI}(f^{-1}(V^c)))$. Hence $f^{-1}(V^c)$ is PNPPO set in X . Since every PNPPO set is PNP GPO set, $f^{-1}(V^c)$ is a PNP GPO set in X . Thus $f^{-1}(V)$ is a PNP GPC set in X . Hence f is PNP GP continuous mapping.

3.11 Theorem

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is a PNP GP continuous mapping if $\text{PNPCI}(\text{PNPInt}(\text{PNPCI}(f^{-1}(V)))) \subseteq f^{-1}(\text{PNPCI}(V))$ for every PNP set V in Y .

Proof

Let V be PNPO set in Y . Then its complement V^c is PNPC set in Y . By hypothesis, $\text{PNPCI}(\text{PNPInt}(\text{PNPCI}(f^{-1}(V^c)))) \subseteq f^{-1}(\text{PNPCI}(V^c)) = f^{-1}(V^c)$, since V^c is PNPC set. Now $\text{PNPInt}(\text{PNPCI}(\text{PNPInt}(f^{-1}(V))))^c = \text{PNPCI}(\text{PNPInt}(\text{PNPCI}(f^{-1}(V^c)))) \subseteq f^{-1}(V^c) = (f^{-1}(V))^c$. This implies $f^{-1}(V) \subseteq \text{PNPInt}(\text{PNPCI}(\text{PNPInt}(f^{-1}(V))))$. Hence $f^{-1}(V)$ is PNP α O set and hence it is PNP GPO set in X . Therefore f is PNP GP continuous mapping.

3.12 Theorem

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a PNP GP continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \mu)$ be a PNP continuous mapping. Then $g \circ f : (X, \tau) \rightarrow (Z, \mu)$ is a PNP GP continuous mapping.

Proof:

Let V be PNPC set in Z . Then $g^{-1}(V)$ is PNPC set in Y , by hypothesis. Since f is PNP GP continuous mapping $f^{-1}(g^{-1}(V))$ is PNP GPC set in X . Hence $g \circ f$ is PNP GP continuous mapping.

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