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Pythagorean Triangle with 2*A/P as Gopa Numbers of the First Kind

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Abstract:

This study deals with the problem of obtaining Pythagorean triangles where, in each Pythagorean triangle, the expression $\frac{2*Area}{Perimeter}$ is represented by Gopa numbers of the first kind. Also, we present the number of primitive and non-primitive Pythagorean triangles and some of the relations among them.

Keywords: Pythagorean triangles, primitive Pythagorean triangle, Non-primitive Pythagorean triangle, Gopa numbers of the first kind.

Introduction:

It is well known that there is a one-to-one correspondence between the polygonal numbers and the sides of polygon. In addition to polygon numbers, there are other patterns of numbers namely Nasty numbers, Harshad Numbers, Dhuruva Numbers, Sphenic Numbers, Jarasandha Numbers, Armstrong Numbers and so on. In particular, refer [1-17] for Pythagorean triangles in connection with each of the above special number patterns. The above results motivated us for searching Pythagorean triangles in connection with a new number pattern. Thus, this paper exhibits Pythagorean triangles such that each Pythagorean triangle with two times the ratio Area/Perimeter is represented by a number known as Gopa numbers of the first kind. A few illustrations with the number of primitive and Non-primitive Pythagorean triangles and some of the properties involving the sides of the Pythagorean triangle are also given.

Definition: Gopa numbers of the First kind

Let N be a non-zero positive integersuch that$N = P \times Q$,where P and Q are distinct primes.

If the relation

Sum of the divisors of N = Product of the sum of the divisors of P, Q

= a perfect square

then, the integer N is referred as Gopa numbers of the first kind

Examples: 22,94,119,214,217,382,497,517,527,679,1177,2101,5029

Method of Analysis:

Let $T(x,y,z)$ be a Pythagorean triangle, where

$$x = 2pq,\ y = p^2 - q^2,\ z = p^2 + q^2, \quad p > q > 0 \tag{1}$$

Denote the area and perimeter of $T(x,y,z)$ by A and P respectively.

The problem under consideration is

$$\frac{2*A}{P} = \alpha \text{, a Gopa number of the first kind} \tag{2}$$

which is equivalent to solving the binary quadratic equation given by

$$q(p-q) = \alpha \tag{3}$$

Given α, it is possible to obtain the values of p and q satisfying (3). Knowing p, q and using (1), one obtains different Pythagorean triangles, each satisfying the relation $\frac{2*A}{P} = \alpha$, a Gopanumbers of the first kind. A few illustrations are presented in the Table 1 below:

Table 1: $\frac{2*A}{P} = \alpha$ **, a Gopa numbers of the first kind**.

α	q	p	x	y	z	$\frac{2*A}{P}$
22	1	23	46	528	530	22
	2	13	52	165	173	
	11	13	286	48	290	
	22	23	1012	45	1013	
94	1	95	190	9024	9026	94
	2	49	196	2397	2405	
	47	49	4606	192	4610	
	94	95	17860	189	17861	
119	1	120	240	14399	14401	119
	7	24	336	527	625	
	17	24	816	287	865	
	119	120	28560	239	28561	
517	1	518	1036	268323	268325	517
	11	58	1276	3243	3485	
	47	58	5452	1155	5573	
	517	518	535612	1035	535613	
1177	1	1178	2356	1387683	1387685	1177
	11	118	2596	13803	14045	
	107	118	25252	2475	25373	
	1177	1178	2773012	2355	2773013	
2101	1	2102	4204	4418403	4418405	2101
	11	202	4444	40683	40925	
	191	202	77164	4323	77285	
	2101	2102	8832604	4203	8832605	

Observations:

1. For even Gopa numbers of the first kind, one obtains, 4 Pythagorean triangles, out of which, two are primitives and two are non- primitives.
2. For odd Gopa numbers of the first kind, one has 4 primitive Pythagorean triangles.
3. $2x - 8\frac{A}{P}$is a perfect square.
4. $x - 4\frac{A}{P} + 2z - 2y$is a square multiple of 6.
5. $4\frac{A}{p} - x + z - y = 0$

Conclusion:

In this paper, we have presented primitive and Non-primitive Pythagorean triangles, where, in each Pythagorean triangle $\frac{2 * Area}{Perimeter}$ is represented by a special Gopanumbers of the first kind with 2, 3 and 4 digits respectively. To conclude one may research for special Pythagorean triangles in connection with higher order Gopanumbers of the first kind.

References:

1. W.Sierpinski(2003), Pythagorean Triangles, Dover publications,INC,Newyork,.
2. M.A.Gopalan and A.Vijaysankar,(2010)"Observations on a Pythagorean problem",ActaCienciaIndica,Vol.XXXVI M.No.4, pp 517-520.
3. M.A.Gopalan,A.Gnanam and G.Janaki,,(2007) 'A Remarkable Pythagorean problem' ActaCienciaIndica,Vol.XXXIII M,No,4, pp 1429-1434 .
4. M.A.Gopalan and A.Gnanam,(2010) 'Pythagorean triangles and Polygonal numbers International Journal of Mathematical Sciences, Vol 9,No 1-2,,pp 211-215.
5. M.A.Gopalan and G.Janaki,(2008) 'Pythagorean triangle with Area Perimeter as a special number' Bulletin of pure and Applied sciences,Vol 27(2),pp 393-402.
6. M.A.Gopalan and G.Janaki,(2008) 'Pythagorean triangle with nasty number as a leg' Journal of Applied Analysis and Applications, Vol 4, No 1-2,,pp 13-17.
7. G.Janaki and R.Radha, (2016) 'Special Pythagorean triangle and six digit Harshad numbers' IJIRSET, Vol. 5, Issue 3, March, pp 3931-3933.
8. G.Janaki and R.Radha,(2016) 'Special pairs of Pythagorean triangle and Harshad numbers' Asian Journal of Science and Technology, volume .7, Issue. 8, August, pp 3397-3399.
9. G.Janaki and P.Saranya,(2016) 'Pythagorean Triangle with Area/Perimeter as a Jarasandha numbers of orders 2 and 4' IRJET ,Volume .3, Issue .7, July, pp 1259-1264.
10. G.Janaki and R.Radha,(2017) 'Pythagorean Triangle with Area/Perimeter as a Harshad number of digits 4,5 and 6' IJRASET, Volume. 5,Issue. 12, December, pp 1754-1762.

11. G.Janaki and P.Saranya,(2016)' Special Pythagorean triangle in connection with triangles Narcisstic Numbers of order 3 and 4' AIJRSTEM, 16-177, , pp 150-153.
12. G.Janaki and P.Saranya,(2016) 'Special pairs of Pythagorean triangles and Narcisstic numbers. IJMRD, Vol. 3, Issue. 4, April, pp 106-108.
13. S. Mallika, (2019)"Pythagorean triangle with 2A/P+H-Leg as a Narcisstic number of orders 3,4 and 5", GJESR, volume 6,Issue 3:March, 1-4.
14. S. Mallika, "A connection between Pythagorean triangle and Sphenic Numbers", IJRASET, Volume 7, Issue III, March 2019, 63-66.
15. A. Kavitha, "A connection between Pythagorean triangle and Harshad Numbers", IJRASET, Volume 7, Issue III, March 2019, 91-101.
16. K.Meena,etal;On Pairs of Rectanges and Gopa-Vidh Numbers, IJESRT, 8(12), 63-72, 2019.
17. S. Vidhyalakshmi, T. Mahalakshmi , M.A. Gopalan, " Pythagorean triangle with $2 \times \left(\frac{A}{P}\right)$ as Gopa − Vidh number, IJRAR, Volume 6, Issue 2, April – June 2019, 59 - 63.