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Linguistic Neutrosophic Somewhat semi-open and hardly semi-open functions in Linguistic Neutrosophic Topological Spaces

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A B S T R A C T

An entirely new category of functions referred to as somewhat semi-open and nearly semi-open is defined and illustrated with a novel topological space using linguistic neutrosophic numbers. Moreover semi-open set, semi-closed set and semi-sense set are defined in linguistic neutrosophic topological space to carry over the work.

Keywords:Linguistic neutrosophic topology; Linguistic neutrosophic somewhat semi-open function; Linguistic neutrosophic hardly semi-open functions; Linguistic neutrosophic semi-open sets; Linguistic neutrosophic semi-closed sets; Linguistic neutrosophic semi-dense set.

1. Main text

Norman Levine[7] introduce the idea of semi-open sets and semi-continuity in topological spaces. Gentry and Hoyle[6] have introduce somewhat continuous functions in 1971. Recently Caldas[2] have worked on hardly open functions and proved many results. A novel topological space namely linguistic neutrosophic topological space which was invented by Helen and Gayathri[5] in 2021. Throughout the paper, LNCl, LNInt, LNSCl, LNSInt, LNOS, LNCS, LNSOS, LNSCS and LNTS represents linguistic neutrosophic closure, linguistic neutrosophic interior, linguistic neutrosophic semi-closure, linguistic neutrosophic semi-interior, linguistic neutrosophic open set, linguistic neutrosophic closed set, linguistic neutrosophic semi-open set, linguistic neutrosophic semi-closed set and linguistic neutrosophic topological space respectively.

2. Preliminaries

Definition 2.1:[4]Assume that $L = \{l_0, l_1, \ldots\ldots, l_t\}$ is a linguistic term set with odd cardinality $t + 1$. If $e = \langle l_p, l_q, l_r \rangle$ is defined for $l_p, l_q, l_r \in L$ and $p, q, r \in [0, t]$, where l_p, l_q and l_r express independently the truth degree, indeterminacy degree and falsity degree by linguistic terms, respectively, then e is called an LNN.

Definition 2.2:[5]**For a LNTS τ, the collection of linguistic neutrosophic sets(LNSs in short) should satisfy the following:**

1. $0_{LN}, 1_{\mathcal{LN}} \in \tau$
2. $K_1 \cap K_2 \in \tau$ for any $K_1, K_2 \in \tau$
3. $\cup K_i \in \tau, \forall \{K_i : i \in J\} \subseteq \tau$.

We call, the pair (S_{LN}, τ), a Linguistic Neutrosophic Topological Spaces(LNTS in short).

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Definition 2.3:[5]Let (S_{LN},τ) be a LNTS. Then,

- $(S_{LN},\tau)^c$ is the dual LNTS, whose elements are $K^C{}_{LN}$ for $K_{LN}{}_{LN} \in (S_{LN},\tau)$.
- Any open set in τ is known as linguistic neutrosophic open set(LNOS in short).
- Any closed set in τ is known as linguistic neutrosophic closed set(LNCS in short) if and only if it's complement is LNOS.

Definition 2.4:[5]The LN closure and LN interior are given by,

1. $LNInt(K_{LN}) = \cup \ \{O_{LN}/O_{LN}$ is a $LNOS$ in S_{LN} where $O_{LN} \subseteq K_{LN}\}$ and it is the largest LNO subset of K_{LN}.
2. $LNCl(H_{LN}) = \cap \ \{J_{LN}/J_{LN}$ is a $LNCS$ in S_{LN} where $H_{LN} \subseteq J_{LN}\}$ and it is the smallest LNCS containing H_{LN}.

Linguistic Neutrosophic Somewhat Semi-Open Function

Definition 3.1: A function $\boldsymbol{f_{LN}}: (\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}}) \to (\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}})$ is said to be LN somewhat semi open if there exists a non-void LNSO set $\boldsymbol{H_{LN}}$ of $\boldsymbol{T_{LN}}$ such that $\boldsymbol{H_{LN}} \subseteq \boldsymbol{f_{LN}}(\boldsymbol{K_{LN}})$, where $\boldsymbol{K_{LN}}$ is a non-void LNOS in $\boldsymbol{S_{LN}}$.

Example 3.2: Let the universe of discourse be $\boldsymbol{U} = \{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\}$ and let $\boldsymbol{S_{LN}} = \{\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\}$. The set of all LTS be L={no healing($\boldsymbol{l_0}$), deterioting($\boldsymbol{l_1}$), chronic($\boldsymbol{l_2}$), some what chronic($\boldsymbol{l_3}$), extremely chronic($\boldsymbol{l_4}$), very ill($\boldsymbol{l_5}$), ill($\boldsymbol{l_6}$), no healing($\boldsymbol{l_7}$), healing($\boldsymbol{l_8}$), slowly healing($\boldsymbol{l_9}$), fastly healing($\boldsymbol{l_{10}}$)}. Let $\boldsymbol{\tau_{LN}} = \{\boldsymbol{0_{LN}},\boldsymbol{1_{LN}},\boldsymbol{K_{LN}}\}$ and $\boldsymbol{\eta_{LN}} = \{\boldsymbol{0_{LN}},\boldsymbol{1_{LN}},\boldsymbol{E_{LN}},\boldsymbol{F_{LN}}\}$ be two LNTS's with $\boldsymbol{K_{LN}} = (\langle \boldsymbol{b},(\boldsymbol{l_6},\boldsymbol{l_6},\boldsymbol{l_1})\rangle,\langle \boldsymbol{c},(\boldsymbol{l_4},\boldsymbol{l_3},\boldsymbol{l_1})\rangle,\langle \boldsymbol{d},(\boldsymbol{l_9},\boldsymbol{l_8},\boldsymbol{l_4})\rangle),\boldsymbol{E_{LN}} = (\langle \boldsymbol{b},(\boldsymbol{l_5},\boldsymbol{l_6},\boldsymbol{l_1})\rangle,\langle \boldsymbol{c},(\boldsymbol{l_4},\boldsymbol{l_3},\boldsymbol{l_1})\rangle,\langle \boldsymbol{d},(\boldsymbol{l_9},\boldsymbol{l_8},\boldsymbol{l_5})\rangle),\boldsymbol{F_{LN}} = (\langle \boldsymbol{b},\boldsymbol{l_3},\boldsymbol{l_5},\boldsymbol{l_4})\rangle,\langle \boldsymbol{c},(\boldsymbol{l_1},\boldsymbol{l_3},\boldsymbol{l_2})\rangle,\langle \boldsymbol{d},(\boldsymbol{l_7},\boldsymbol{l_5},\boldsymbol{l_8})\rangle)$. Let the mapping $\boldsymbol{f_{LN}}$ be an identity mapping. Here the set $\boldsymbol{K_{LN}}$ is LNO in $(\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}})$ and the set $\boldsymbol{H_{LN}} = (\langle \boldsymbol{b},\boldsymbol{l_4},\boldsymbol{l_5},\boldsymbol{l_2}\rangle,\langle \boldsymbol{c},(\boldsymbol{l_3},\boldsymbol{l_3},\boldsymbol{l_1})\rangle,\langle \boldsymbol{d},(\boldsymbol{l_9},\boldsymbol{l_8},\boldsymbol{l_4})\rangle)$ is LNSO in $(\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}})$. Then the mapping $\boldsymbol{f_{LN}}$ is LN somewhat semi-open.

Theorem 3.3: The composition of any two LN somewhat semi open mappings is need not be a LN somewhat semi open mapping, which can be given in the following example.

Example 3.4: Let $\boldsymbol{S_{LN}} = \boldsymbol{T_{LN}} = \boldsymbol{P_{LN}} = \{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\}$ and Let the set of all LTS be L={quite extremely weak($\boldsymbol{l_0}$), extremely weak($\boldsymbol{l_1}$), slightly weak($\boldsymbol{l_2}$), weak($\boldsymbol{l_3}$), neither weak or nor strong($\boldsymbol{l_4}$), strong($\boldsymbol{l_5}$), slightly strong($\boldsymbol{l_6}$), extremely strong($\boldsymbol{l_7}$), quite extremely strong($\boldsymbol{l_8}$)}. The LNTSs be $(\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}}) = \{\boldsymbol{0_{LN}},\boldsymbol{1_{LN}},\boldsymbol{A_{LN}}\},(\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}}) = \{\boldsymbol{0_{LN}},\boldsymbol{1_{LN}},\boldsymbol{B_{LN}}\},(\boldsymbol{P_{LN}},\boldsymbol{\nu_{LN}}) = \{\boldsymbol{0_{LN}},\boldsymbol{1_{LN}},\boldsymbol{C_{LN}}\}$ with $\boldsymbol{A_{LN}} = (\langle \boldsymbol{w},(\boldsymbol{l_4},\boldsymbol{l_1},\boldsymbol{l_3})\rangle),\boldsymbol{B_{LN}} = (\langle \boldsymbol{w},(\boldsymbol{l_6},\boldsymbol{l_3},\boldsymbol{l_2})\rangle),\boldsymbol{C_{LN}} = (\langle \boldsymbol{w},(\boldsymbol{l_8},\boldsymbol{l_3},\boldsymbol{l_6})\rangle)$. The LN somewhat semi-continuous mappings $\boldsymbol{f_{LN}}: (\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}}) \to (\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}})$ and $\boldsymbol{g_{LN}}: (\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}}) \to (\boldsymbol{P_{LN}},\boldsymbol{\nu_{LN}})$ be defined by $\boldsymbol{f_{LN}}(\boldsymbol{a}) = \boldsymbol{b},\boldsymbol{f_{LN}}(\boldsymbol{b}) = \boldsymbol{a}$ and $\boldsymbol{g_{LN}}(\boldsymbol{a}) = \boldsymbol{c},\boldsymbol{g_{LN}}(\boldsymbol{b}) = \boldsymbol{a},\boldsymbol{g_{LN}}(\boldsymbol{c}) = \boldsymbol{b}$ respectively. The composite mapping is given by $(\boldsymbol{g_{LN}} \circ \boldsymbol{f_{LN}}) = (\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}}) \to (\boldsymbol{P_{LN}},\boldsymbol{\nu_{LN}})$. Let $\boldsymbol{C_{LN}}$ be LNOS in $\boldsymbol{P_{LN}}$. Let $\boldsymbol{K_{LN}} = (\langle \boldsymbol{w},(\boldsymbol{l_3},\boldsymbol{l_1},\boldsymbol{l_3})\rangle)$ be LNSO in $(\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}})$. Then, the composite mapping is not LN somewhat semi-continuous.

Theorem 3.5:A LN function $\boldsymbol{f_{LN}}: (\boldsymbol{S_{LN}},\boldsymbol{\tau_{LN}}) \to (\boldsymbol{T_{LN}},\boldsymbol{\eta_{LN}})$ is LN somewhat semi-open if and only if the inverse image of a LNS dense set in $\boldsymbol{T_{LN}}$ is LND in $\boldsymbol{S_{LN}}$.

Proof: Necessity Part: Let us assume that the set $\boldsymbol{E_{LN}}$ is LND in $\boldsymbol{T_{LN}}$ and $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{E_{LN}})$ is not LND in $\boldsymbol{S_{LN}}$. Then, there exists a LNCS $\boldsymbol{K_{LN}} \subset \boldsymbol{S_{LN}}$ such that $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{E_{LN}}) \subseteq \boldsymbol{K_{LN}} \subset \boldsymbol{S_{LN}}$. Then, $\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}}$ is non-empty LNOS in $\boldsymbol{S_{LN}}$. By assumption, there exists a non-empty set $\boldsymbol{U_{LN}} \in \boldsymbol{LNSO}(\boldsymbol{T_{LN}})$ such that $\boldsymbol{U_{LN}} \subseteq \boldsymbol{f_{LN}}(\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}})$ or $\boldsymbol{T_{LN}} \backslash (\boldsymbol{f_{LN}}(\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}})) \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}}$. Also, $\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}} \subseteq \boldsymbol{S_{LN}} \backslash (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{E_{LN}}) = (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{T_{LN}} \backslash \boldsymbol{E_{LN}})$, then, $\boldsymbol{f_{LN}}(\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}}) \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{E_{LN}}$. Thus, $\boldsymbol{E_{LN}} \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{S_{LN}} \backslash \boldsymbol{K_{LN}}) \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}}$. Thus, it is proved that there exists a LNCS $\boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}}$ in $\boldsymbol{T_{LN}}$ such that $\boldsymbol{E_{LN}} \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}} \subseteq \boldsymbol{T_{LN}}$, which is a contradiction to the definition of $\boldsymbol{E_{LN}}$ as a LNS dense set in $\boldsymbol{T_{LN}}$. Thus, $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{E_{LN}})$ is LND in $\boldsymbol{S_{LN}}$.

Sufficiency Part: Assume that the function $\boldsymbol{f_{LN}}$ is not LN somewhat semi-open, then for every non-empty LNOS $\boldsymbol{A_{LN}}$ in $\boldsymbol{S_{LN}}$, there is no non-empty LNSO set $\boldsymbol{B_{LN}}$ in $\boldsymbol{T_{LN}}$ such that $\boldsymbol{U_{LN}} \subseteq \boldsymbol{f_{LN}}(\boldsymbol{B_{LN}})$. Then, no proper LNSCS $\boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}}$ is such that $\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{B_{LN}}) \subseteq \boldsymbol{T_{LN}} \backslash \boldsymbol{U_{LN}} \subseteq \boldsymbol{T_{LN}}$. Thus, $\boldsymbol{T_{LN}} \backslash \ \boldsymbol{f}(\boldsymbol{B_{LN}})$ is LND in $\boldsymbol{T_{LN}}$. By assumption, $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{B_{LN}}))$ is LND in $\boldsymbol{S_{LN}}$ or $\boldsymbol{S_{LN}} \backslash (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{B_{LN}}))$ is LND in $\boldsymbol{S_{LN}}$. Thus, $\boldsymbol{LNCl}(\boldsymbol{S_{LN}} \backslash (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{B_{LN}}))) = \boldsymbol{S_{LN}}$. Also, $\boldsymbol{B_{LN}} \subseteq (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{B_{LN}}))$, which implies $\boldsymbol{S_{LN}}(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{B_{LN}})) \subseteq \boldsymbol{S_{LN}} \backslash \boldsymbol{B_{LN}}$. Then, $\boldsymbol{S_{LN}} = \boldsymbol{LNCl}(\boldsymbol{S_{LN}} \backslash (\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f}(\boldsymbol{B_{LN}}))) \subseteq \boldsymbol{LNCl}(\boldsymbol{S_{LN}} \backslash \boldsymbol{B_{LN}}) \subseteq \boldsymbol{S_{LN}} \backslash \ \boldsymbol{LNInt}(\boldsymbol{B_{LN}})$ and therefore $\boldsymbol{LNInt}(\boldsymbol{B_{LN}}) = \boldsymbol{\phi}$, which is a contradiction to the fact that $\boldsymbol{B_{LN}}$ is non-empty in $\boldsymbol{S_{LN}}$.

Theorem 3.6: If the LN function $f_{LN}:(S_{LN},\tau_{LN})\to(T_{LN},\eta_{LN})$ is LNO and $g_{LN}:(T_{LN},\eta_{LN})\to(P_{LN},\mu_{LN})$ is LN somewhat semi open mapping,then the composition $(g_{LN}\circ f_{LN}):(S_{LN},\tau_{LN})\to(P_{LN},\mu_{LN})$ is LN somewhat semi open.

Proof: Suppose if A_{LN} is LNOS in S_{LN}, then $f_{LN}(A_{LN})$ is LNOS in T_{LN}, since f_{LN} is LNO mapping. Then, as the mapping g_{LN} is LN somewhat semi open, there exists a non-empty set $B_{LN}\in LNSO(P_{LN},\mu_{LN})$ such that $B_{LN}\subseteq(g_{LN}\circ f_{LN})(A_{LN})$. Thus, the composition is LN somewhat semi open mapping.

Theorem 3.7: Let LN function $f_{LN}:(S_{LN},\tau_{LN})\to(T_{LN},\eta_{LN})$ is one to one and onto, then f_{LN} is LN somewhat semi open if and only if every LNCS A_{LN} in S_{LN} such that $f_{LN}(A_{LN})\neq T_{LN}$, there exists a LNSCS K_{LN} such that $f_{LN}(A_{LN})\subseteq K_{LN}$.

Proof: Necessity Part: Let $A_{LN}\in LNC(S_{LN})$ such that $f_{LN}(A_{LN})\neq T_{LN}$. Then, $S_{LN}\backslash A_{LN}$ is non-empty LNOS in S_{LN}. By assumption, there exists $U_{LN}\in LNSO(T_{LN})$ such that $U_{LN}\subseteq f_{LN}(S_{LN}\backslash A_{LN})$ or $T_{LN}\backslash f_{LN}(S_{LN}\backslash A_{LN})\subseteq T_{LN}\backslash U_{LN}$. As the mapping f_{LN} is one to one and onto, $f_{LN}(A_{LN})\subseteq T_{LN}\backslash U_{LN}$. If $K_{LN}=T_{LN}\backslash U_{LN}$, then $K_{LN}=\phi, K_{LN}\in LNC(T_{LN})$ such that $f_{LN}(A_{LN})\subseteq K_{LN}$.
Sufficiency Part: If V_{LN} is any non-empty set in S_{LN}, then $S_{LN}\backslash V_{LN}$ is a proper LNCS in S_{LN}. If $f_{LN}(S_{LN}\backslash V_{LN})=T_{LN}$, then it is clear that, $V_{LN}=\phi$, which is a contradiction. Thus, $f_{LN}(S_{LN}\backslash V_{LN})\neq T_{LN}$. From the assumption, there exists a LNCS D_{LN} in T_{LN} such that $f_{LN}(S_{LN}\backslash V_{LN})\subseteq D_{LN}$, (i.e) $T_{LN}\backslash D_{LN}\subseteq T_{LN}\backslash f_{LN}(S_{LN}\backslash V_{LN})=f_{LN}(V_{LN})$, where $T_{LN}\backslash D_{LN}\neq\phi,(T_{LN}\backslash D_{LN})$ which is LNSO set. Therefore, f_{LN} is LN somewhat semi-open mapping. A LN subset A_{LN} of (S_{LN},τ_{LN}) is called LNS dense if $S_{LN}=LNSCl(A_{LN})$.

Definition 3.8: A LN function $f_{LN}:(S_{LN},\tau_{LN})\to(T_{LN},\eta_{LN})$ is LN hardly semi-open if for each LNSDS A_{LN} in T_{LN} which is contained in a proper LNSOS in T_{LN}, $(f_{LN})^{-1}(A_{LN})$ is LN semi-dense in S_{LN}.

Example 3.9: Let the universe be $U=\{a,b,c,d,e\}$ and let the LTS be L={ very salty, salty, very sour, sour, bitter, sweety, very sweety } where, $L=\{l_0,l_1,l_2,l_3,l_4,l_5,l_6\}$. And let the set $I_{LN}=\{\langle a,(l_4,l_5,l_2)\rangle,\langle b,(l_3,l_6,l_1)\rangle,\langle c,(l_9,l_6,l_8)\rangle\}$. Let $S_{LN},\tau_{LN})=\{0_{LN},1_{LN},A_{LN}\}$ with $A_{LN}=\{\langle a,(l_5,l_6,l_1)\rangle,\langle b,(l_4,l_3,l_1)\rangle,\langle c,(l_9,l_8,l_5)\rangle\}$ and the set $B_{LN}=\{\langle a,(l_2,l_4,l_5)\rangle,\langle b,(l_1,l_1,l_2)\rangle,\langle c,(l_6,l_5,l_8)\rangle\}$ is LNSOS in S_{LN}. Now, I_{LN} is LNSDS in (T_{LN},η_{LN}) where $\eta_{LN}=\{0_{LN},1_{LN},J_{LN}\}$, with $J_{LN}=\{\langle a,(l_3,l_5,l_4)\rangle,\langle b,(l_1,l_1,l_2)\rangle,\langle c,(l_6,l_5,l_8)\rangle\}$. The LNSO set in T_{LN} is $K_{LN}=\{\langle a,(l_4,l_6,l_2)\rangle,\langle b,(l_4,l_7,l_1)\rangle,\langle c,(l_9,l_6,l_8)\rangle\}$. Then the mapping f_{LN} is LN hardly semi-open.

Remark 3.10: The composition of any two LN hardly semi open mappings is need not be a LN hardly semi open mapping, which can be given in the following example.

Example 3.11: Let the universe and LTS are as in example (3.9). Let the topologies and the composite mapping be defined as $(S_{LN},\tau_{LN})=\{0_{LN},1_{LN},A_{LN}\},(T_{LN},\eta_{LN})=\{0_{LN},1_{LN},B_{LN}\},(P_{LN},\nu_{LN})=\{0_{LN},1_{LN},C_{LN}\}$ and $(g_{LN}\circ f_{LN}):(P_{LN},\nu_{LN})\to(S_{LN},\tau_{LN})$ respectively with $A_{LN}=(\langle c,(l_6,l_2,l_1)\rangle),B_{LN}=(\langle c,(l_6,l_4,l_5)\rangle),C_{LN}=(\langle c,(l_4,l_3,l_3)\rangle)$. The function f_{LN} and g_{LN} are LN hardly semi open. The sets $K_{LN}=(\langle c,(l_7,l_6,l_2)\rangle)$ and $H_{LN}=(\langle c,(l_6,l_5,l_3)\rangle)$ are LNSO in (S_{LN},τ_{LN}) and (P_{LN},ν_{LN}) respectively. Here the composite mapping is not LN hardly semi open as $(g_{LN}\circ f_{LN})^{-1}(H_{LN})$ is not LNSDS in (S_{LN},τ_{LN}).

Theorem 3.12: The LN function $f_{LN}:(S_{LN},\tau_{LN})\to(T_{LN},\eta_{LN})$ is LN hardly semi open if and only if $LNInt((f_{LN})^{-1}(A_{LN}))=\phi$, for every LN subset A_{LN} of T_{LN} such that $LNSInt(A_{LN})=\phi$ contains a non-empty LNCS.

Proof: Necessity Part: Let f_{LN} be LN hardly semi open and $LNSInt(A_{LN})=\phi$ for every LN subset A_{LN}, where $A_{LN}\subseteq T_{LN}$ and a non-empty LNCS E_{LN} in T_{LN} such that $E_{LN}\subseteq A_{LN}$. Then, $LNSCl(T_{LN}\backslash A_{LN})=T_{LN}\backslash LNSInt(A_{LN})=T_{LN}$. As $E_{LN}\subseteq A_{LN}, T_{LN}\backslash A_{LN}\subseteq T_{LN}\backslash E_{LN}\neq T_{LN}$. Thus, $T_{LN}\backslash ALN$ is LNS dense in T_{LN} which is contained in a proper LNOS $T_{LN}\backslash E_{LN}$. By assumption, $(f_{LN})^{-1}(T_{LN}\backslash A_{LN})$ is LNS dense in S_{LN}. Thus, $S_{LN}=LNSCl((f_{LN})^{-1}(T_{LN}\backslash A_{LN}))=S_{LN}\backslash LNInt((f_{LN})^{-1}(A_{LN}))$. Therefore, $S_{LN}\backslash LNSInt((f_{LN})^{-1}(A_{LN}))=S_{LN}$ and thus $LNSInt((f_{LN})^{-1}(A_{LN}))=\phi$.

Sufficiency Part: If D_{LN} is any LNS dense in T_{LN} such that $D_{LN}\subset LNO(T_{LN})$, let it be U_{LN}. As $U_{LN}\neq\phi, T_{LN}\backslash U_{LN}$ is LNCS contained in $T_{LN}\backslash D_{LN}$ and it is non-empty. From the assumption, $LNSInt((f_{LN})^{-1}(T_{LN}\backslash D_{LN}))=\phi$. Then, $S_{LN}\backslash((f_{LN})^{-1}(D_{LN}))=\phi$ and $LNSCl((f_{LN})^{-1}(D_{LN}))=S_{LN}$, (i.e) $(f_{LN})^{-1}(D_{LN})$ is LNS dense set in S_{LN}.

Theorem 3.13: Let $f_{LN}:(S_{LN},\tau_{LN})\to(T_{LN},\eta_{LN})$ be any LN function. Let A_{LN} be a LN subset of S_{LN} having the property that $LNSInt(A_{LN})\neq\phi$ and $LNSInt(f_{LN}(A_{LN}))\neq\phi$. Also, there exists LNCS $B_{LN}(\neq\phi)$ in S_{LN} such that $(f_{LN})^{-1}(B_{LN})\subseteq A_{LN}$, then f_{LN} is LN hardly semi open.

Proof: Let $D_{LN}\subset U_{LN}$, where D_{LN} is any LNS dense set in T_{LN} and $U_{LN}\in LNO(T_{LN})$. As $U_{LN}\neq\phi, T_{LN}\backslash U_{LN}\neq\phi$ and hence $T_{LN}\backslash U_{LN}$ is a non-empty LNCS contained in $T_{LN}\backslash D_{LN}$. If $A_{LN}=(f_{LN})^{-1}(T_{LN}\backslash D_{LN}), B_{LN}=T_{LN}\backslash U_{LN}$, then $(f_{LN})^{-1}(B_{LN})\subseteq A_{LN}$. Also, $LNSInt(f_{LN}(A_{LN}))=LNSInt(f_{LN}((f_{LN})^{-1}(T_{LN}\backslash D_{LN})))\subseteq LNSInt(T_{LN}\backslash D_{LN})=\phi$. From the assumption, $\boldsymbol{LNSInt}(\boldsymbol{A_{LN}}) = \boldsymbol{\phi}$, (i.e) $\boldsymbol{LNSInt}((\boldsymbol{f_{LN}})^{-1}((\boldsymbol{f_{LN}})^{-1}(\boldsymbol{T_{LN}} \backslash \boldsymbol{D_{LN}}))) = \boldsymbol{\phi}$. Thus, $\boldsymbol{S_{LN}} \backslash \boldsymbol{LNSCl}((\boldsymbol{f_{LN}})^{-1}(\boldsymbol{D_{LN}})) = \boldsymbol{\phi}$ and therefore, $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{D_{LN}}) = \boldsymbol{S_{LN}}$. Hence $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{D_{LN}})$ is LNS dense set in $\boldsymbol{S_{LN}}$ and hence $\boldsymbol{f_{LN}}$ is LN hardly semi open.

Theorem 3.14: If the function $\boldsymbol{f_{LN}}: (\boldsymbol{S_{LN}}, \boldsymbol{\tau_{LN}}) \to (\boldsymbol{T_{LN}}, \boldsymbol{\eta_{LN}})$ is LN hardly semi open, then $\boldsymbol{LNSInt}(\boldsymbol{f_{LN}}(\boldsymbol{A_{LN}})) \neq \boldsymbol{\phi}$ for each LN subset $\boldsymbol{A_{LN}}$ of $\boldsymbol{S_{LN}}$ with $\boldsymbol{LNSInt}(\boldsymbol{A_{LN}}) \neq \boldsymbol{\phi}$ and $\boldsymbol{f_{LN}}(\boldsymbol{A_{LN}})$ contains a non-void LNCS.

Proof: If $\boldsymbol{A_{LN}}$ is any LNS such that $\boldsymbol{LNSInt}(\boldsymbol{A_{LN}}) \neq \boldsymbol{\phi}$ and $\boldsymbol{B_{LN}}$ be any LNCS in $\boldsymbol{T_{LN}}$ such that $\boldsymbol{B_{LN}} \subseteq \boldsymbol{f_{LN}}(\boldsymbol{A_{LN}})$. If $\boldsymbol{LNSInt}(\boldsymbol{f_{LN}}(\boldsymbol{A_{LN}})) \neq \boldsymbol{\phi}$, then $\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{A_{LN}})$ is LNSdense in $\boldsymbol{T_{LN}}$. Then, $\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{A_{LN}}) \subset \boldsymbol{T_{LN}} \backslash \boldsymbol{B_{LN}}$. As the mapping $\boldsymbol{f_{LN}}$ is LN hardly semi open, $(\boldsymbol{f_{LN}})^{-1}(\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{A_{LN}}))$ is LNSdense in $\boldsymbol{S_{LN}}$, (i.e) $\boldsymbol{LNSCl}((\boldsymbol{f_{LN}})^{-1}(\boldsymbol{T_{LN}} \backslash \boldsymbol{f_{LN}}(\boldsymbol{A_{LN}}))) = \boldsymbol{S_{LN}}$ or $\boldsymbol{S_{LN}} \backslash \boldsymbol{LNSInt}((\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{A_{LN}}))) = \boldsymbol{S_{LN}}$. Thus, $\boldsymbol{LNSInt}((\boldsymbol{f_{LN}})^{-1}(\boldsymbol{f_{LN}}(\boldsymbol{A_{LN}}))) = \boldsymbol{\phi}$ and it implies $\boldsymbol{LNSInt}(\boldsymbol{A_{LN}}) = \boldsymbol{\phi}$, which is a contradiction to the assumption. Hence, $\boldsymbol{LNSInt}(\boldsymbol{A_{LN}}) \neq \boldsymbol{\phi}$.

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