



Ranking of Trapezoidal Fuzzy Assignment Problem

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Abstract

Assignment problem plays an important role in industry, supply chain management, logistics, commerce etc for selecting a best possible assignment from a number of alternatives. In this paper a trapezoidal fuzzy assignment problem is proposed to solve using a new approach of ranking to find the optimal solution. The efficiency of the method is illustrated by a numerical example.

Keywords: Assignment problem, trapezoidal fuzzy

Subject classification: Operations Research

1.Introduction

In this work we investigate Trapezoidal fuzzy assignment problem. An assignment problem is one that involves the assignment of n different facilities to n different tasks. An assignment problem gives a technique of selecting the best possible assignment of tasks from a number of alternatives. In an assignment problem each resource is assigned exactly one job so that the given measurer of effectiveness is optimized. Many authors presented various approaches for solving Fuzzy Assignment problems and have encountered much application.

Solution of Assignment problem using Hungarian algorithm was developed by Kuhn [1]. A review of some methods for ranking fuzzy subsets, Fuzzy Set and Systems was given by Bortolan and Degani [2]. Chen [3] proposed a fuzzy assignment problem. Long-Shen Huang and Li-pu Zhang [4] proposed a mathematical model for the fuzzy assignment problem and transformed the model as certain assignment problem with restriction of qualification. Chen Lian-Hsuan and Lu Hai-Wen [5] employed a procedure for resolving assignment problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model for optimum efficiency. Kaur, Kumar [6] derived, a new method for solving

fuzzy transportation problems using ranking function. Thorani and Ravishankar [7] proposed Fuzzy assignment problem with generalized Fuzzy Numbers. Sagaya Roseline and Henry Amirtharaj [8] proposed a New Approach to Find the Solution for the Generalized Fuzzy Assignment Problem with Ranking of Generalized Fuzzy Numbers. Agustina and Lukman [9] suggested, a new approach solution for fuzzy assignment problem using the development Zimmermann method. Revathi and Valliathal [10] proposed, a new approach to find optimal solution of fuzzy assignment problem using penalty method for hendecagonal fuzzy number. In this paper a new method is proposed for ranking of Trapezoidal Fuzzy Assignment Problem. This is illustrated by a numerical example.

2.Preliminaries

Definition 2.1 A fuzzy set A is defined on universal set of real numbers R, is said to be a fuzzy number if its membership function satisfies the following conditions:

1. $\mu_A : R \rightarrow [0,1]$ is continuous
2. $\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
3. $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
4. $\mu_A(x) = 1$ for all $x \in [b, c]$ where $a < b < c < d$

Definition 2.2 (Trapezoidal Fuzzy Number) A number $A = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\begin{aligned} \mu_A(x) &= \frac{x-a}{b-a}, & a \leq x \leq b \\ &= 1, & b \leq x \leq c \\ &= \frac{x-d}{c-d}, & c \leq x \leq d \end{aligned}$$

Definition 2.3 (Generalized Trapezoidal Fuzzy Numbers) A fuzzy set A is defined on universal set of real numbers R, is said to be a generalized fuzzy number if its membership function satisfies the following conditions

1. $\mu_A : R \rightarrow [0,1]$ is continuous
2. $\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
3. $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$
4. $\mu_A(x) = \omega$ for all $x \in [b, c]$ where $0 < \omega \leq 1$

Definition 2.4 A number $A = (a, b, c, d, \omega)$ is said to be a Generalized trapezoidal fuzzy number if its membership function is given by

$$\begin{aligned}\mu_A(x) &= \frac{\omega(x-a)}{b-a}, a \leq x \leq b \\ &= \omega, \quad b \leq x \leq c \\ &= \frac{\omega(x-d)}{c-d}, c \leq x \leq d \\ &= 0, \quad \text{otherwise}\end{aligned}$$

3. Ranking of Trapezoidal Fuzzy Numbers

The Centroid of a trapezoid is considered as the balancing point of the trapezoid (Fig.1). We divide the trapezoid into three plane figures. These three plane figures are a triangle (AED), a rectangle (ABCD), and a triangle (BCF) respectively. Let the Centroids of the three plane figures be C_1 , C_2 and C_3 respectively. Each Centroid point is balancing points of each individual plane figure, and the Incentre of these Centroid points is a much more balancing point for a generalized trapezoidal fuzzy number. Let the Centroids of the three plane figures be C_1 , C_2 and C_3 respectively.

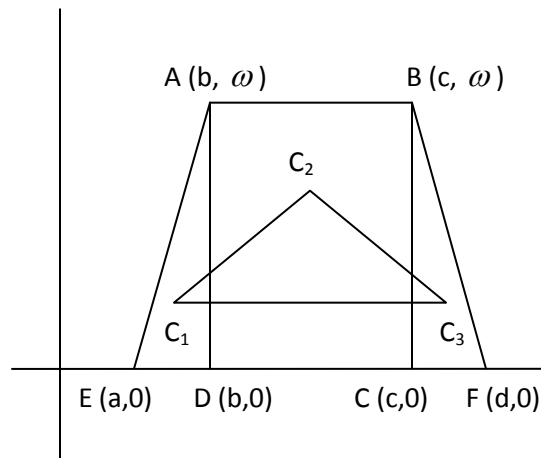


Fig. 1 - Trapezoidal Fuzzy number

The coordinates of the Centroids will be

$$C_1 = \left(\frac{a+2b}{3}, \frac{\omega}{3} \right)$$

$$C_2 = \left(\frac{b+c}{2}, \frac{\omega}{2} \right)$$

$$C_3 = \left(\frac{2c+d}{3}, \frac{\omega}{3} \right)$$

C_1, C_2 and C_3 are non-collinear and they form a triangle. We define the Incentre $C(x_0, y_0)$ of the triangle with vertices C_1, C_2 and C_3 . The coordinates of Incentre will be

$$x_0 = \left(\frac{\alpha \left(\frac{a+2b}{3} \right) + \beta \left(\frac{b+c}{2} \right) + \gamma \left(\frac{2c+d}{3} \right)}{\alpha + \beta + \gamma} \right), \quad y_0 = \left(\frac{\alpha \left(\frac{\omega}{3} \right) + \beta \left(\frac{\omega}{2} \right) + \gamma \left(\frac{\omega}{3} \right)}{\alpha + \beta + \gamma} \right)$$

where

$$\alpha = \frac{\sqrt{(c-3b+2d)^2 + \omega^2}}{6}, \quad \beta = \frac{\sqrt{(2c+d-a-2b)^2}}{3}, \quad \gamma = \frac{\sqrt{(3c-2a-b)^2 + \omega^2}}{6}$$

Therefore Rank of A will be

$$R(A) = \sqrt{x_0^2 + y_0^2}$$

3.1 Steps of the Proposed Method

1. Find the coefficients α, β, γ
2. Find the coordinates of incentre $C(x_0, y_0)$
3. Find the Rank of A
4. Apply Hungarian method to find the Optimal Assignment

4. Numerical Example

Let us consider a Fuzzy assignment problem with rows representing 4 persons A, B, C, D and columns representing the 4 jobs Job1, Job2, Job3 and Job4. The cost matrix is given whose elements are trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

PERSONS	JOBS			
	1	2	3	4
A	(3,5,6,7)	(5,8,11,12)	(9,10,11,15)	(5,8,10,11)
B	(7,8,10,11)	(3,5,6,7)	(6,8,10,12)	(5,8,9,10)
C	(2,4,5,6)	(5,7,10,11)	(8,11,13,55)	(4,6,7,10)
D	(6,8,10,12)	(2,5,6,7)	(5,7,10,11)	(2,4,5,7)

By our Ranking Method we have,

$$\begin{aligned}
R(3,5,6,7) &= 2.096 \\
R(5,8,11,12) &= 3.608 \\
R(9,10,11,15) &= 4.213 \\
R(5,8,10,11) &= 3.414 \\
R(7,8,10,11) &= 3.5 \\
R(3,5,6,7) &= 2.096 \\
R(6,8,10,12) &= 3.5 \\
R(5,8,9,10) &= 3.219 \\
R(2,4,5,6) &= 1.707 \\
R(5,7,10,11) &= 3.262 \\
R(8,11,13,55) &= 4.623 \\
R(4,6,7,10) &= 2.571 \\
R(6,8,10,12) &= 3.5 \\
R(2,5,6,7) &= 2.052 \\
R(5,7,10,11) &= 3.262 \\
R(2,4,5,7) &= 1.75
\end{aligned}$$

Now, by applying Hungarian Method we can find the optimal solution.

The fuzzy optimal solution is

$$\begin{aligned}
A &= (9,10,11,15) + (3,5,6,7) + (2,4,5,6) + (2,4,5,7) \\
&= (16, 23, 27, 35) \\
R(A) &= 9.766
\end{aligned}$$

5. Conclusion

In this paper a simple method of solving trapezoidal fuzzy Assignment problem is introduced by ranking of Trapezoidal fuzzy numbers and found the optimal Assignment. The proposed method is effective to solve such complex fuzzy assignment problems.

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