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Vortex, Vortics and Magnetohydrodynamics

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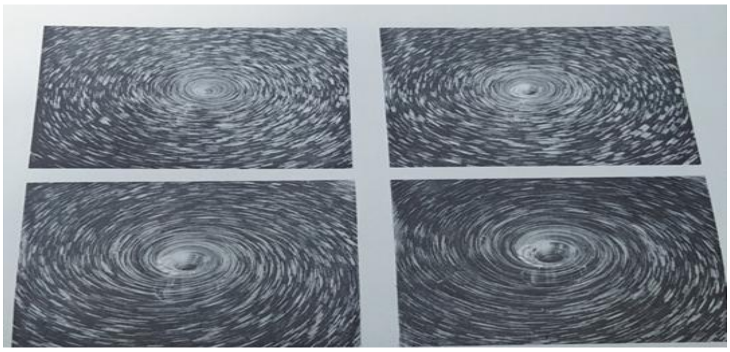
ABSTRACT

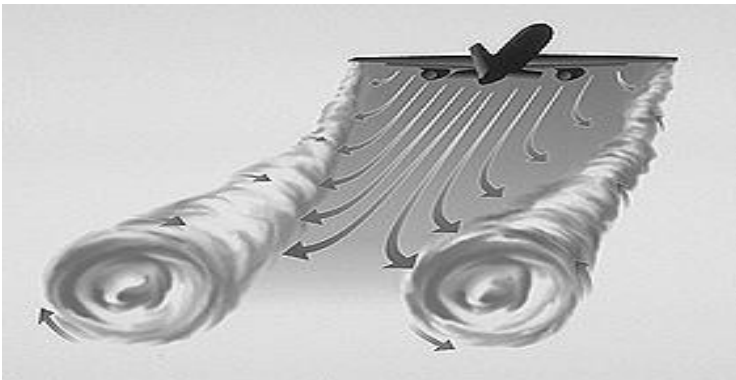
The objective of this work is to present some concepts, cite examples and initiate debates on the theory of vortex, vorticity and their relationship with magnetohydrodynamics (MHD). We also emphasize the importance of the Lorentz and Coriolis forces to analyze the vorticity associated with the MHD. Vortex, vorticity and MHD are closely related through the Lorentz force, and it is very likely that power generation through nuclear fusion will make use of MHD for the magnetic confinement of plasma.

INTRODUCTION

According to LUGT [(1)], vortex are macroscopic structures that occupy a finite space; contrary to vorticities, which are structures with a microscopic scale. The vorticity can be defined as the rotational velocity vector or twice the angular velocity of the fluid (air, water, plasma, conducting fluid, etc). Vortexes are present in nature in various ways, such as: sink vortex, tornadoes, hurricanes, wingtip vortex, a brutal vortex in the take-off of an A-380, etc. The fundamental cause of the vorticity vector is the rotational force: be it viscosity, Lorentz force, Coriolis, etc.



Source - Atlas of Fluid Mechanics by Rui Carlos de Camargo Vieira



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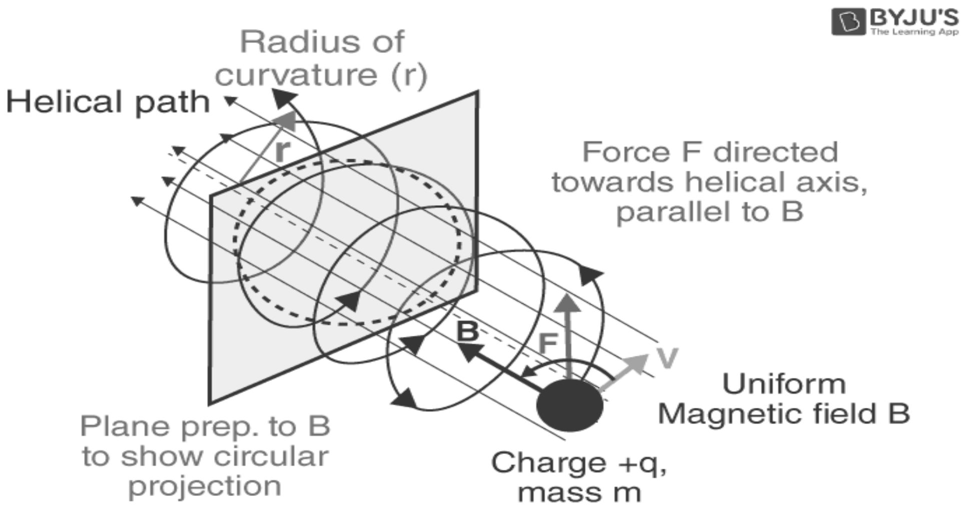
2. IMPORTANCE OF THE LORENTZ FORCE

One might think at first that vortex are closely related to rotation and even vorticity is commonly related to the word rotation, but this cannot mean that a flow has to be curved or have curvature in order for vorticity to be present.



Motion of a charged particle in in a uniform magnetic field

https://byjus.com/physics/motion-charged-particle-magnetic-field/

Just observe what happens in the boundary layer. Within the boundary layer, the flow, even on a flat plate, is highly vorticose. That is, it generates small vortex or eddys. Boundary layer flow is rotational, that is, its main cause comes from a vorticose force: friction. There are also examples where the fluid is rotating like a rigid body, but an element of the fluid does not rotate on its axis, that is, it has no spin. In this region the flow is irrotational despite being in rotation. One objective of this study is to relate these quantities with the MHD. In MHD, the Lorentz force can generate vorticity within the fluid since its rotational is, in general, non-zero. Is it possible to generate vorticity even in an inviscid fluid? Yes, through the Lorentz force, for example. The Lorentz force can be written as:

$$JxB=(B.\nabla)B/\mu - \nabla(B^2/2\mu) \qquad (1)$$

The first term on the right side being the rotational part of the force and the second term the irrotational part, (represented by the magnetic pressure gradient), where Ampére's law was used. We thus see the rotational character of the force and its potential to generate vorticity. The objective is to conceptually discuss these quantities relating them through mathematical relationships and physical explanations. On the other hand, magnetohydrodynamics (MHD) is a theory that combines the equations of conservation of mass, momentum, energy and Maxwell's equations for conducting fluids. Combining the momentum equations and Maxwell's equations, it is easy to deduce the equation of fluid motion (Newton's second law) with the Lorentz force term. (See for example SAFFMAN [2]).

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}\right] = \rho\vec{g} - \vec{\nabla}p - 2\rho\vec{\omega}\times\vec{V} + \frac{1}{\mu}(\nabla\times\vec{B})\times\vec{B} + \nabla.\vec{\tau} \qquad (2)$$

We can neglect the Coriolis term, (it is usually very small, except with respect to the Earth's rotation, for example) third from the right, and the divergent of the tensor, can also take a simpler form. The final term can be written as Laplacian, mainly in the incompressible flow of viscous, conductive fluid. Therefore, there are:

$$\rho[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}] = -\vec{\nabla}p + \rho\vec{g} + (\vec{J}\times\vec{B}) + \mu\,\nabla^2\vec{V} \qquad (3)$$

it is known that w=$\vec{\nabla}\times\vec{V}$is the vorticity and, therefore, applying the rotational operator in the equation of motion eliminates the terms in gradients. With that we arrive at

$$\partial w/\partial t=\vec{\nabla}\times(VxW) +\mu/\rho\nabla.\nabla(W)+ \nabla x\,(JxB)/\rho \qquad (4)$$

That is, the temporal variation of the vorticity vector is equal to the rotational of the vector product of the velocity vector by the vorticity vector, plus the Laplacian term of the vorticity vector, plus the rotational term of the Lorentz force that generates vorticity to the fluid flow. If there is no flow, then the curl of the force is zero. If the fluid is non-viscous, there is still vorticity due to the Lorentz force. Also note that for weakly electrically conducting fluids, the Lorentz force can be obtained with great intensity if a strong external magnetic field is applied to the conducting fluid.This is the case of generation of vortex in sea water with applications for propulsion of boats, submarines, etc. A possible advantage is not having pressure as an unknown function, since the rotational gradient of any scalar function is null (pressure gradient, gravitational potential, electric potential, etc.). As paradoxical as it may seem, pressure being a cause of fluid motion, the problem can be solved more easily by eliminating the pressure by operating the rotational vector in the fluid motion equation (SHERCLIFF [3]).

3.VORTEX GENERATION EXAMPLE

A very interesting real example is the production of vortex using strong magnetic fields applied externally to a cylindrical container filled with sea water. The field is generated by a solenoid with values on the order of 1.0 Tesla. The container, with a diameter of 60 cm and with water at a height of 12 cm, rests on the solenoid base where a radial electric current is applied with a gold electrode in the center of the container and the other in the form of a ring. concentric to the first, being able to move within the fluid along the axis of symmetry. It is a fact that the current follows radial direction due to the strong applied magnetic field. (in a strong magnetic field the electric current flows only in the thin Hartmann layer, this facilitates the increase of the accelerating force). Thus, when the current source is turned on and the current density interacts with the axial field, there is a tangential force that accelerates the fluid. This example illustrates very well the rotational character of the Lorentz force. At first the movement is a little unstable, but after a few seconds the movement generates a spectacular vortex, with an average speed of the order of meters per second.It is also observed that after a few minutes, due to chemical reactions, electrolysis produces a lot of obscurity in the water. It is surprising that sea water can behave this way. Before the experiment I did not imagine such a visible success. The experiment is due to SOMMERIA [(4)], reported in Journal Fluid Mechanics, where he uses mercury as a working fluid, in Electrically driven vortex in a Strong magnetic field. A stationary electric current flows from a central electrode into the plane through the fluid, stimulating it in a circular motion.

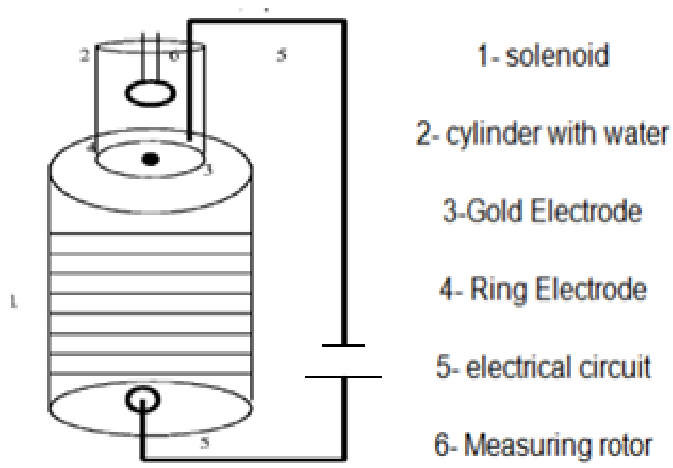


Fig.1: Experimental apparatusof vortex generationof sea water in a cylinder

Source: https://ijaers.com/detail/the-creation-of-a-vortex-in-sea-water-through-the-mhd/

In principle, it neglects the viscosity of the fluid and presents an axisymmetric solution with azimuthal velocity given by:

$$V=\Gamma/r, \text{ and } \Gamma = I/2\pi\ (\nu\rho\mu)^{1/2} \qquad (5)$$

I=current on the order of mA and field on the order of 0.5 T. It should be noted that the rotational Lorentz force is the main cause of the vortex generation.

Let a body of revolution be like a cylinder within a fluid (air) flow. Initially, the fluid flows over the cylinder at rest. It is easy to see that in this case, due to the symmetry of the flow over the cylinder, there is no lift force. Suppose now that the cylinder rotates with an angular velocity in a given direction. Superimposing the movement of the fluid with the rotational movement of the cylinder, an asymmetry is created in the flow over the cylinder, generating a lift force perpendicular to the relative wind. Note that for the cylinder to rotate it must rest on something. Hence the concept of boundary layer and friction (friction). Without circulation there is no support.

ANALOGIES

3.1. An interesting analogy is about the vortex filaments in fluid mechanics and the velocity field that gives rise to the Biot-Savart law and theeven on currents in conducting wires and the magnetic field they cause. It was, however, with the development of MHD during the last century, that the interdisciplinarity of these concepts were based (MOFFAT [(5)]). The Biot-Savart law can also be used in aerodynamic theory to calculate the velocity induced by vortex lines. In MAXWELL's ([6] 1861 paper 'On Physical Lines of Force', the magnetic field strength H was directly equated with pure vorticity (spin), while B was a weighted vorticity that was weighted to the density of the sea of vortex. Maxwell considered the magnetic permeability μ as a measure of the density of the sea of vortex.In aerodynamics, induced air currents form solenoid rings around a vortex axis. An analogy can be made that the vortex axis is playing the role that electric current plays in magnetism. This places the air currents of aerodynamics (fluid velocity field) in the equivalent role of the magnetic induction vector B in electromagnetism. In electromagnetism the B lines form solenoid rings around the source of electrical current, while in aerodynamics, the air currents (velocity) form solenoid rings around the vortex axis of the source. So, in electromagnetism, the vortex plays the role of 'effect', while in aerodynamics, the vortex plays the role of 'cause'. However, when we look at the B lines in isolation, we see exactly the aerodynamic scenario, as B is the vortex axis and H is the circumferential velocity as in Maxwell's 1861 paper.

3.2 Another frequently explored analogy, mentioned by SHAPIRO [7], is between vorticity and magnetic field through the equations below:

$$\partial B/\partial t = \nabla \times (VxB) + \mu \nabla^2 B \quad (6)$$

That is, partial derivative of the magnetic field is equal to the rotational cross product of the velocity vector by the magnetic field plus the viscosity multiplied by the Laplacian of the magnetic field.

$$\partial W/\partial t = (W.\nabla)V + \nu\nabla^2 W \quad (7)$$

Considering the incompressible flow and mass conservation $\nabla.V=0$, we arrive at

$$dW/dt = \nabla \times (u \times W) + \nu\nabla^2 W \quad (8)$$

Note that for inviscid fluids with ν=0, Helmholts theorem guarantees that vorticity can neither be created nor destroyed and according to the above equation, in two-dimensional flow, the vorticity becomes a scalar quantity with dW/dt=0.That is, the partial derivative of the vorticity vector is equal to the curl of the vector product of the velocity vector by the vorticity plus the product of the kinematic viscosity and the Laplacian of the vorticity vector. Note, however, that this analogy is not perfect, given that the vorticity, W, is related to the velocity vector but the magnetic field B is not, despite both being solenoid, that is, zero divergence and, even more, there is a term source in the vorticity equation what does not occur in the induction equation and the fact that vorticity is physically linked to the conservation of angular momentum and the magnetic field to the conservation of magnetic flux, so to speak (DAVIDSON [8]).Just as the magnetic field has associated magnetic field lines, there is an analogue for vorticity called vortex lines.Vortex lines are vorticity field lines that have the same direction as w and whose density in any region is proporcional to the magnitude of vorticity.

3.3 According to HUGHES and BRIGHTON [9], an extremely important problem in fluid dynamics is turbulence. It is directly linked to the formation and interaction of small vortex. But wouldn't the origin of turbulence be at the microscopic level, that is, quantum? In Reynolds tube studies, after a certain number the flow momentarily becomes chaotic, disordered and, therefore, turbulent. But if we could visualize microscopically, surely, at this level, wouldn't we have small vortex? That is, wouldn't turbulence have an origin in quantum mechanics?A question to think about very calmly. Can a strong magnetic field completely suppress the turbulence of a conducting fluid? HARTMANN, cited by HAVERKORT [10], showed in 1937 that it is possible to suppress the vorticity of a flow confined by two parallel planes, except in regions very close to the walls.

3.3.1 HARTMANN’S LAYER

In the presence of a magnetic field, diffusive magnetic layers called Hartmann layers form at the interfaces between materials with different electrical properties. In a layer of standard Hartmann thickness, the Lorentz forces are in equilibrium with the viscous forces. When an incompressible viscous fluid flows laminarly in the space between two unbounded plates separated by a distance d from each other, its velocity profile is known as parabolic. When the fluid is electrically conductive and when a uniform and constant magnetic field acts perpendicular to the channel walls, the structure of the flow changes dramatically. The profile becomes flat in the so-called core as a result of the electromagnetic braking effect.This braking is due to the interaction of the induced electric current with the applied magnetic field. In addition, two boundary layers develop in the vicinity of the walls. These layers were theoretically predicted and experimentally characterized by Julius Hartmann in 1937 and represent one of the most important features of MHD flows. The thickness δ of a Hartmann boundary layer (or "Hartmann layer") which is on the order of

$$\delta = \frac{1}{B}\sqrt{\frac{\rho\nu}{\sigma}} \quad (9)$$

Where B is the strength of the magnetic field, and ρ, ν, σ are the density, kinematic viscosity,and electrical conductivity of the liquid metal. The thickness is independent of the channelwidth and decreases with increasing magnetic field (BRAITHWAITE [11]).

3.4 The big problem to find complete solutions of the MHD system of equations for vortex production is in the coupling of the conservation equations: mass, momentum, energy and Maxwell's equations. Look at Ohm's law: it relates electric fields and magnets with the flow velocity. What does that mean? The interaction of the electric current with the magnetic field generates the Lorentz force that can move the conducting fluid; on the other hand, the fluid flow interacting with the fields produces induction effects by altering the fields. Give the coupling it is not always easy to decouple the equations involved and present analytical solutions to the problem. However, one way adopted to some extent is expansion into small perturbations. This is what can be done to obtain solutions of the Alfvén wave equation in MHD, with vorticity being the unknown function (BATISTA [12]).

4. FINAL CONSIDERATIONS

One of the important conclusions that we can see is that the potential flow is very elegant and almost does not exist in nature, despite having a simpler mathematical treatment. On the other hand, vortex flows are much more abundant, but the treatment is much more complicated and difficult to understand. In MHD flow, the Lorentz force generates vorticity in the sinus or interior of the fluid. So MHD is also more difficult to understand, but at the same time, more fascinating from the point of view of beauty and coupling and interdisciplinarity. The generation of energy by the Sun is the great example. The Lorentz force, because it is rotational, except in particular cases, is largely responsible for the generation of vortex in the middle of the conducting fluid.As stated by Shercliff, this fact makes ordinary fluids behave very differently from conductive fluids, also due to the effects induced by electric currents in the fluid medium. It is a well-known fact that the movement of a fluid of uniform density cannot occur in a closed container without the presence of vorticity, that is, rotational forces are necessary for such movements to take place. Great researchers in the field of fluid dynamics have recognized the importance of vorticity in the explanation of many important phenomena such as: formation and separation of the boundary layer in terms of production, convection and diffusion of vorticity, the lift of a wing is explained by the vorticity of Linked vortex and vortex trail structure, via circulation theory.Vortex, vorticity and MHD are closely related as we saw above, through the Lorentz force. It is very likely that the generation of energy through nuclear fusion will make use of the MHD for the magnetic confinement of the plasma and, as in the Sun, we will see more closely the coupling of these quantities.

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