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Tuning of Compensators to Control a Coupled Dual Liquid Tank Process

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A B S T R A C T

This paper presents the control of a couple dual liquid tank process using four different compensators for purpose of comparison and selection of appropriate compensator. The MATLAB optimization toolbox is used to tune all the compensators using appropriate objective functions (error-based criterion) with reasonable functional constraints. The process under study has special characteristics leading to non-zero steady state error wit all the studied compensators. The paper presents scientific solution of this problem. The effectiveness of the used compensators is examined through the assignment of the maximum percentage overshoot, settling time and steady state error of the time response of step input tracking. The characteristics of all the compensators used to control the process are compared in graphical and tabulated forms.

Keywords: Coupled dual tank process , feedforward first order compensator, P-D compensator, feedback first order compensator, notch compensator, compensator tuning, control system performance.

1. Introduction

Controlling of the levels 0n liquid tanks has a vital importance because od the too many industries using storage of liquids in tanks and the need to adjust the levels at required levels and maintain them against disturbance. The author investigated the control of coupled dual liquid tanks using a controller from the first generation of PID controllers and five controllers from the second generation of PID controllers [1]. Now, the author investigates the use of control-compensators from the first and second generations for the same purpose of using controllers.

Hassaan (2014) proposed a new feedback Proportional-Derivative (PD) compensator to control underdamped second order processes. He could reduce the settling time associated with the time response of step input tracking to 0.4 s and the maximum percentage overshoot to only 0.1 % [2]. Hassaan (2014) used a feedback PD compensator to control a third order process having 83 % maximum overshoot. Through compensator tuning he could reduce the maximum percentage overshoot of the process to only 5 % and improve the stability of the resulting closed-loop control system [3]. Hassaan (2014) used a 2/2 second order compensator to control a very slow second order process having 200 s settling time. He used a novel tuning technique based on solving a set of nonlinear equations [4].

Hassaan (2015) tuned a first order lag-lead compensator to control a very slow second order process having 200 s settling time. Through tuning he could reduce the settling time of the time response of the step input tracking to only 0.8 s without any overshoot [5]. Hassaan (2015) applied a new third order compensator to control a second order processes having 85.4 and 52.6 % maximum overshoot. Through compensator tuning he could eliminate completely the overshoot associated with the step input tracking [6]. Hassaan (2015) tuned a third order feedforward compensator used to control a delayed integrating process of time delay up to 7.4 s. He could through compensator tuning to achieve a step input tracking time response of 1.2 % maximum percentage overshoot for a process with 7.4 s time delay [7].

Hassaan (2015) tuned a feedback lag-lead first order compensator used to control a fractional time delay double integrating unstable process. He could achieve a step input tracking time response of overshoot between 1 and 4.5 % and settling time ≤ 0.5 s depending on the time delay [8]. Hassaan (2015) tuned a second order compensator to control a highly oscillating second order process having 85.4 % maximum overshoot. Through tuning he could eliminate completely the overshoot associated with the reference input tracking [9]. Hassaan (2021) compared the performance of a feedforward second

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order compensator from the first generation with five compensators from the second generation introduced by the author to control a highly oscillating second order process. The compensators from the second generation were superior in producing a time step response with maximum overshoot between zero and 4.6 % compared with 15.3 % for the compensator from the first generation [10].

2. Process

The controlled process is a two couples liquid tanks with configurations and parameters shown in Figure1 [11].

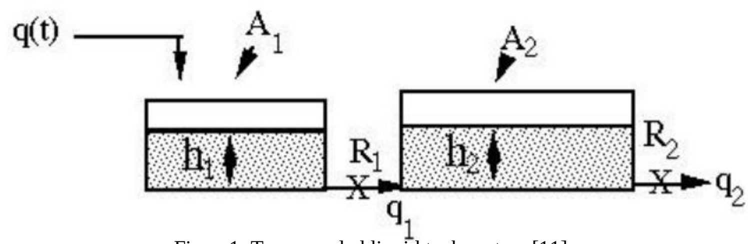

Figure1: Two-coupled liquid tank system [11].

The dynamic model of the two tanks depends on the parameters of the liquid level system shown in Figure 1 and on the operating conditions because the system is extremely nonlinear. The following system parameters and operating conditions of a typical laboratory dual-tank interacting system are used [11]:

Tank area:	13670	mm^2
Input flow rate:	3.833×10^6	mm^3/s
Head in the second tank:	162.5	mm

The interacted two tank system had an identified transfer function, $G_p(s)$ at the assigned operating conditions listed above given by [11]:

$G_p(s) = H_2(s)/Q_i(s)= 0.25 / (1951s^2 + 132.51s + 1)$ (1)

Where: H_2 (s): Laplace transform of the head in the second tank, h_2.

$Q_i(s)$: Laplace transform of the input flow rate to the first tank, q_i.

$G_p(s)$: Process transfer function.

Equation 1 is for a second order dynamic system. It is not written in a standard form of second order systems which takes the form [1]:

$G_p(s) = K_p \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$ (2)

Where: K_p: Process gain

ω_n: Process natural frequency rad/s

ζ: Process damping ratio

Equating the parameters of equations 1 and 2 we get the values of the second order process parameters of the interacting two tank system as:

- Process gain, K_p: 0.25
- Process natural frequency, ω_n: 0.0226 rad/s
- Process damping ratio, ζ: 1.4996

3. Controlling the Process Using a Feed forward first Order Compensator

This compensator belongs to the first generation of compensators. It was presented by D'Azzo and Houpis as first order lag compensator having pole and zero with two parameters and a unit compensator gain [12]. They presented also a first order lead compensator having two parameters and compensator gain less than one [13]. The author used a first order compensator having three parameters and a transfer function $G_c(s)$ given by:

$G_c(s) = K_c(1 + T_z s) / I1 + T_p s)$ (2)

Where:

- K_c is the compensator gain
- T_z is the compensator zero time constant
- T_p is the compensator pole time constant

The author used the closed loop transfer function of the control system obtained using the standard block diagram comprising the controlled process of transfer function given by equation 1 and the compensator of transfer function given by equation 2 to tune the compensator. The compensator is tuned as follows:

- The control and optimization toolboxes of MATLAB are used to assign the tuned three parameters of the controller [9].
- The integral of the time multiplied by the absolute error of the control system (ITAE) is chosen as an objective function for the constrained optimization process.
- The optimization command '*fmincon'* is used to minimise the objective function subjected to constraints on the maximum percentage overshoot and steady state error [14].
- The step response of the closed-loop control system is plotted using the command '*step*' of MATLAB [15].
- The time-based specifications of the closed- loop control system are extracted from the time response of the step tracking time response.
- The tuned parameters of the forward first order compensator are:

$K_c = 46.7509$, $T_z = 18.7957$, $T_p = 0.01419$ (3)

The step input tracking of the control system using the first order compensator is shown in Fig.2. The time based specifications of the control system are as follows:

Maximum percentage overshoot:	0	
Settling time:	38.9	s
Steady-state error:	0.0795	

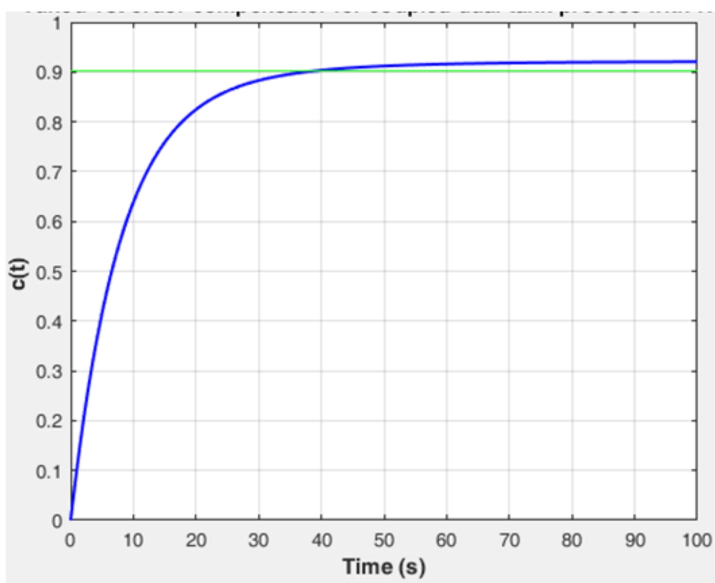

Figure 2 Step input tracking using a feedforward first order compensator.

4. Controlling the Process using a P-D Compensator

The author used a novel P-D compensator having a structure different than this for the feedback PD compensator proposed by him to control underdamped second order processes [2] and third order processes [3]. The new structure was applied by him to control a highly oscillating second order process [16]. The block diagram of the control system incorporating the P-D compensator and the controlled process is shown in Fig.3 [16].

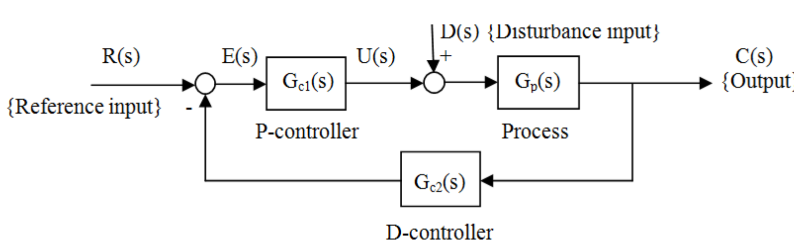

Figure 3 Block diagram of a P-D compensator controlled process.

For reference input tracking, the disturbance input D(s) is set to zero. The process transfer function is given by equation 1 and the compensator transfer function is given by both $G_{c1}(s)$ and $G_{c2}(s)$ as:

$$G_{c1}(s) = K_{pc} \quad \text{and} \quad G_{c2}(s) = K_d s \qquad (4)$$

Using the block diagram in Fig. 3 and Eqs.(1) and (4), the closed loop transfer function of the control system , $H_2(s)/R(s)$ is:

$$H_2(s)/R(s) = b_0 / (a_0 s^2 + a_1 s + a_2 s) \qquad (5)$$

where:

$$\left.\begin{aligned} b_0 &= K_{pc} K_p \omega_n^2 \\ a_0 &= 1 \\ a_1 &= 2\zeta\omega_n + K_{pc}K_pK_d \omega_n^2 \\ a_2 &= \omega_n^2 \end{aligned}\right\} \qquad (6)$$

The time response for step input tracking is evaluated using Eq.(5) and te command '*step*' of the MATLAB program [15]. This time response is used to tune the compensator using the command '*fmincon*' of the optimization toolbox on MATLAB [14] subjected to functional constraints on the maximum percentage overshoot, settling time and steady state error of the closed loop control system incorporating the compensator and the process under control.
It was shown by the author that for the process under control having a small natural frequency of 0.0226 rad/s, the settling time of the closed loop control system time response for step input tracking is function of the system damping ratio [obtained from Eq.(5)] and given by:

$$T_s = -57.197\zeta^2 + 565.6288\zeta - 247.7864 \qquad (7)$$

Using the above tuning approach, the compensator tuned parameters are:

$$K_{pc} = 3.996 \quad , \quad K_d = 0.01 \qquad (8)$$

The compensator parameters in Eq.(8) and the transfer function of the control system in Eq.(5) reveal the time response for step input tracking of the control system shown in Fig.4 compared with that using the feedforward first order compensator

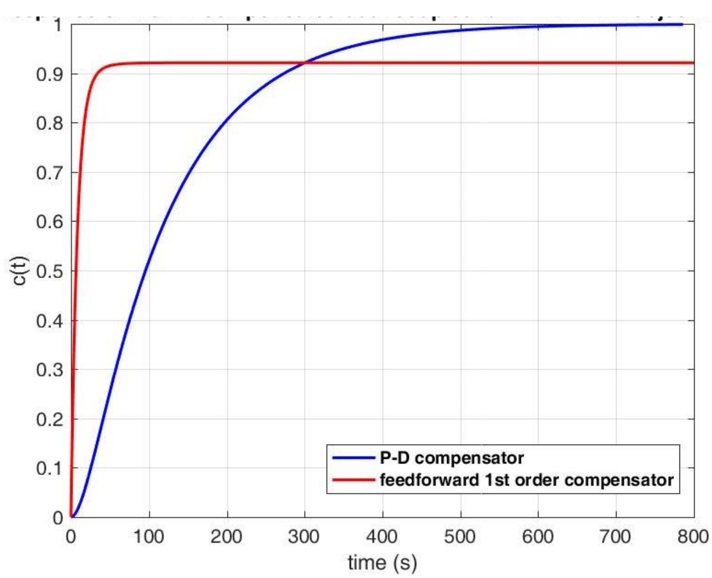

Figure 4 Step input tracking using a P-D compensator.

The time-based specifications of the closed control system incorporating the P-D compensator are as follows:

- Maximum percentage overshoot: 0 %
- Settling time: 452 s compared with 38.9 s for the control system using feedforward first order compensator.
- Steady state error: 0.00009 compared with 0.0795 for the control system using feedforward first order compensator.

5. Controlling the Process Using a Feedback First-Order Compensator

The author proposed a feedback first order compensator to control a simple pole plus double integrators process [17], a highly oscillating second order process [18], a very slow second order process [5] and a fractional time delay double integrating process [8]. In one of his applications he incorporated a

Proportional controller in the forward path just before the process to enhance the performance of the closed loop control system based on using the feedback first order compensator [18]. In the present application for the control of the coupled dual tank process, I propose using PI controller replacing the P-controller in the control scheme of the highly oscillating second order process. The block diagram of the control scheme incorporating the PI controller and the first order compensator is shown in Fig.5.

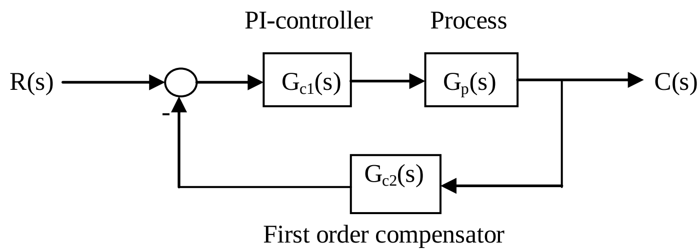

Figure 5 Block diagram of a first-order compensator controlled process.

The PI-controller and the first order compensator have the transfer functions, $G_{c1}(s)$ and $G_{c2}(s)$ given respectively by:

$G_{c1}(s) = K_{pc}+(K_i/s)$, $G_{c2}(s) = K_c(1+T_zs)/(1+T_ps)$ (9)

where:

K_{pc} = proportional gain of the PI controller
K_i = integral gain of the PI controller
K_c = gain of the first order compensator
T_z = zero of the first order compensator
T_p = pole of the first order compensator

The transfer function of the closed loop control is derived using the block diagram in Fig.5. This transfer function is used for:

- Evaluating the time response of the control system to reference input tracking.
- Evaluating the time-based characteristics of the closed loop control system.
- Evaluating the steady state error of the control system.

Investigating the transfer function of the closed loop system of Fig.5 revealed an important feature for the system. That is it is possible to achieve a zero steady state error if the compensator gain K_c is set to a unit value. This leaves four controller and compensator parameters to be adjusted through tuning to maintain good performance for the control system.

The controller-compensator units are tuned using the MATLAB command '*fmincon*' , integral of time multiplied by absolute error (ITAE) as objective function and functional constraints on the maximum percentage overshoot. The tuned controller-compensator parameters are:

$K_{pc} = 99.9247$; $K_i = 5.1393$; $T_z = 14.1131$; $T_p = 3.4247$ (10)

The time response for step input tracking of the control system with the feedback first order compensator and the feedforward PI controller is shown in Fig.6.

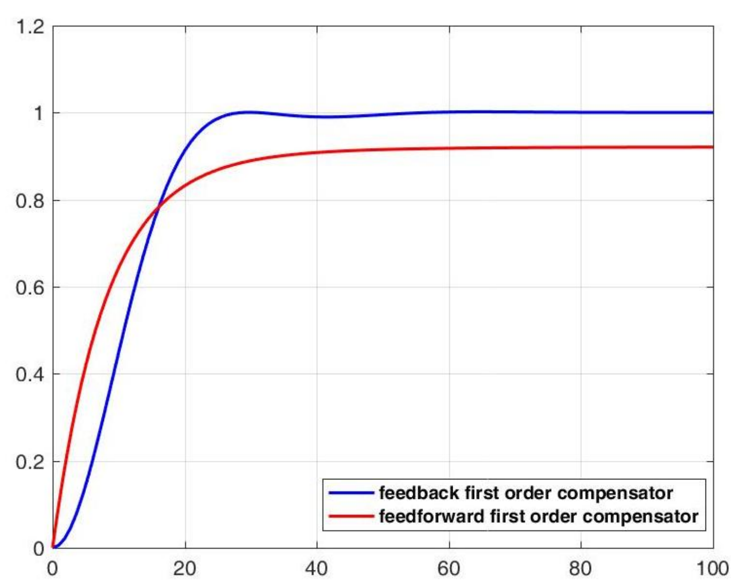

Fig.6 Step input tracking using a feedback first order compensator.

The time-based specifications of the closed control system incorporating the feedback first order compensator and the feedforward PI controller are as follows:

- Maximum percentage overshoot: 0.304 % % compared with zero using the feedforward first order compensator.
- Settling time: 24 s compared with 38.9 s for the control system using feedforward first order compensator.
- Steady state error: zero compared with 0.0795 for the control system using feedforward first order compensator.

6. Controlling the Process Using a Feedforward Notch Compensator

The author proposed a feedforward notch compensator to control a highly oscillating second order process [19] and studies the robustness of the notch compensator and the Sallen-Key compensator when used to control the same process [20]. The notch compensator as suggested by the author has the transfer function, $G_{c1}(s)$ given by [19]:

$$G_{c1}(s) = K_c(s^2+b)/(s^2+as+b) \quad (11)$$

where: K_c = notch compensator gain
a = notch compensator parameter
b = notch compensator parameter

i.e. the compensator has three parameters to be tuned to adjust the performance of the control system.

Because the process under study has a non-unity gain (Eq.2), it is expected that this compensator will produce non-zero steady state error for the closed loop control system incorporating the notch compensator and the process. To overcome this problem an integrator is suggested between the compensator and the process. This will eliminate completely the steady state error of the closed loop control system. The modified compensator transfer function in this case, $G_{c2}(s)$ will be:

$$G_{c2}(s) = K_c(s^2+b)/[s(s^2+as+b)] \quad (12)$$

Now, the block diagram of the control system incorporates the modified compensator and the process in series in the forward path and the transfer function of the closed loop control system can be easily derived.

This transfer function is used to derive the time response of the system step tracking which is to used in the tuning process of the compensator. The MATLAB command '*fmincon*' is used to minimize an integral of square error multiplied by square error (ISTSE) objective function subjected to a functional constraint on the maximum percentage overshoot. The result of this tuning technique is as follows:

Compensator parameters:

$$K_c = 0.0114 \;\; ; \;\; a = 0.0001 \;\; ; \;\; b = 10.4334 \quad (13)$$

Step input tracking:

The tuned compensator parameters are used in the modified compensator transfer function (Eq.12) and the closed loop transfer function of the control system to produce the time response for step input tracking given in Fig.7.

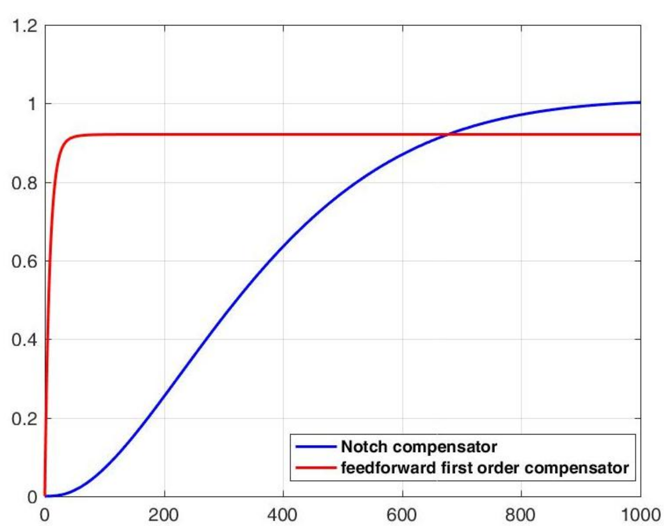

Fig.7 Step input tracking using a feedforward notch compensator.

- Step time response characteristics:
  - Maximum percentage overshoot: 1.02 % compared with zero using the feedforward first order compensator.
  - Settling time: 820 s compared with 38.9 s for the control system using feedforward first order compensator.
  - Steady state error: zero compared with 0.0795 for the control system using feedforward first order compensator.

7. Performance Comparison Using Four Compensators

To examine the effectiveness of using the proposed compensators to control the coupled dual tank process, the reaults of the analysis presented in this paper is compared in graphical and quantitative forms as follows:

- The time response for step input tracking using the feedforward first order compensator, P-D compensator, feedback fitst order compensator and feedforward notch compensator is compared by presentation in one graph shown in Fig.8.

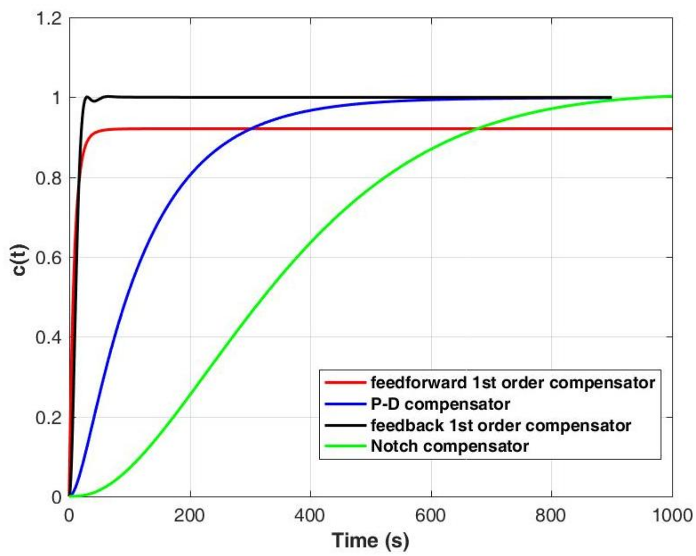

Fig.8 Step input tracking using four compensators.

The characteristics of the time response for step input tracking using the feedforward first order compensator, P-D compensator, feedback first order compensator are compared in Table 1.

Table 1: Characteristics comparison for the time response for step input tracking using four compensators.

Compensator	Feedforward first order compensator	P-D compensator	Feedback first order compensator	Notch compensator
Maximum percentage overshoot (%)	0	0	0.304	1.02
Settling time (s)	38.9	452	24	820
Steady-state error	0.0795	0.00099	0	0

8. Conclusions

- The paper presented the control of a coupled dual tanks process using four control compensators, one from the first generation and three from the second generation.
- The four compensators were tuned using the MATLAB optimization toolbox.
- The time response for step reference input tracking was drawn for each compensator from the second generation with comparison with that fusing the feedforward first order compensator from the first generation.
- The compensators used from the second generation included a P-D compensator, feedback first order compensator and a notch compensator.
- Because the process under control has a non-zero gain, it was essential to find a solution for the problem of non-zero steady state error of the closed loop control system incorporating the compensator and the process.

- The solution was using an integrator and or a Proportional-Integral control in the forward path just before the process. The effect of which was investigated in details.
- The performance of the control system was evaluated through the assignment of the maximum percentage overshoot, settling time and steady state error for all the compensators used.
- The characteristics of the closed loop control system were compared for the four compensators used and compared in graphical and tabulated form.
- It was shown that the control system incorporating a feedback first order compensator had the best performance since it could reduce the settling time to only 24 seconds with zero steady state error and maximum percentage overshoot of only 0.3 %.

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