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Observations on the Paper Entitled "On non-homogeneous Octic equation with four unknowns $x^2 = y^3 + z^5 w^3$

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Abstract:

This paper illustrates the process of obtaining different sets of non-zero distinct integer solutions to the non-homogeneous Octic equation with four unknowns given by $x^2 = y^3 + z^5 w^3$

Keywords: Octic non-homogeneous equation, octic equation with four unknowns, integer solutions.

Introduction:

It is known that higher degree Diophantine equations with multiple variables are rich in variety .While searching for the collection of eighth degree Diophantine equations with four unknowns, the authors came across the paper [1] entitled

"On non-homogeneous Octic equation with four unknowns $x^2 = y^3 + z^5 w^3$ "

In the above paper, the authors have presented only a few choices of integer solutions. However, there are many more sets of integer solutions to the considered equation which is the main thrust of this paper.

Method of Analysis

The eighth degree equation with four unknowns to be solved is

$$x^2 = y^3 + z^5 w^3 \tag{1}$$

The process of determining different sets of integer solutions to (1) is illustrated below:

Illustration 1:

The substitution of the transformations

$$\left.\begin{aligned} x &= (2u)^8 p, \\ y &= (2u)^5 (u+v), \\ z &= (2u)^3, \\ w &= u - v \end{aligned}\right\} \tag{2}$$

in (1) gives

$$u^2 + 3v^2 = p^2 = p^2 * 1 \tag{3}$$

Let

$$p = a^2 + 3b^2 \tag{4}$$

Consider the integer 1 on the R.H.S. of (3) as

$$1 = \frac{(1+i\sqrt{3})\,(1-i\sqrt{3})}{4} \tag{5}$$

Substituting (4) & (5) in (3) and employing the method of factorization, consider

$$u + i\sqrt{3}v = \frac{(1+i\sqrt{3})}{2}(a + i\sqrt{3}b)^2 \tag{6}$$

Equating the real and imaginary parts in (6), we get

$$u=\frac{a^2-3b^2-6ab}{2}, v=\frac{a^2-3b^2+2ab}{2} \tag{7}$$

From (4),(7) and (2) ,the corresponding integer solutions to (1) are given by

$$\left.\begin{array}{l} x=(a^2-3b^2-6ab)^8(a^2+3b^2), \\ y=(a^2-3b^2-6ab)^5(a^2-3b^2-2ab), \\ z=(a^2-3b^2-6ab)^3, \\ w=-4ab \end{array}\right\} \tag{8}$$

Note :

In addition to (5), the integer 1 may also be expressed as below:

$$1=\frac{(11+i5\sqrt{3})(11-i5\sqrt{3})}{196},$$

$$1=\frac{(3r^2-s^2+i\sqrt{3}\,2rs)(3r^2-s^2-i\sqrt{3}\,2rs)}{(3r^2+s^2)^2}$$

Following the above procedure, two more sets of integer solutions to (1) are obtained

Illustration 2:

The substitution of the transformations

$$\left.\begin{array}{l} x=(2u)^8p^2, \\ y=(2u)^5(u+v), \\ z=(2u)^3, \\ w=u-v \end{array}\right\} \tag{9}$$

in (1) gives

$$u^2+3v^2=p^4 \tag{10}$$

After a few calculations, it is noted that (10) is satisfied by

$$v=4ab(3a^2-b^2), u=12a^2b^2-(3a^2-b^2)^2, p=3a^2+b^2$$

In view of (9), the corresponding integer solutions to (1) are given by

$$x=(24a^2b^2-2(3a^2-b^2)^2)^8(3a^2+b^2)^2,$$

$$y=(24a^2b^2-2(3a^2-b^2)^2)^5(12a^2b^2-(3a^2-b^2)^2+4ab(3a^2-b^2)),$$

$$z=(24a^2b^2-2(3a^2-b^2)^2)^3,$$

$$w=(12a^2b^2-(3a^2-b^2)^2-4ab(3a^2-b^2))$$

Illustration 3:

The substitution of the transformations

$$\left.\begin{array}{l} x=(2u)^8p^3, \\ y=(2u)^5(u+v), \\ z=(2u)^3, \\ w=u-v \end{array}\right\} \tag{11}$$

in (1) gives

$$u^2+3v^2=p^6 \tag{12}$$

After some algebra, the corresponding integer solutions to (1) are as below:

$$x = 2^8 (m^2 + 3n^2)^{19} (m^2 - 3n^2)^8 ,$$
$$y = 2^5 (m^2 + 3n^2)^{12} (m^2 - 3n^2)^5 (,m^2 - 3n^2 - 2mn) ,$$
$$z = -2^3 (m^2 + 3n^2)^6 (m^2 - 3n^2)^3 ,$$
$$w = -(m^2 + 3n^2)^2 \ (m^2 - 3n^2 + 2mn)$$

Illustration 4:

Taking

$$p = v - h \tag{13}$$

in (3) given by

$$u^2 + 3v^2 = p^2 ,$$

it is written as

$$2v^2 + 2vh + u^2 - h^2 = 0 \tag{14}$$

Treating 14) as quadratic in v and solving for v, we have

$$v = \frac{-h \pm \sqrt{3h^2 - 2u^2}}{2} \tag{15}$$

It is possible to choose h, u so that the square-root on the R.H.S. of (15) is removed.

Knowing the values of h, u, v ; the corresponding solutions to (1) are obtained through employing (13) and (2) . For simplicity and brevity, various sets of solutions thus determined are exhibited below:

Set 1:

$$x = -2^8 (a^2 - 2b^2 + 2ab)^8 (a^2 + 4b^2 + 2ab),$$
$$y = 2^5 (a^2 - 2b^2 + 2ab)^5 (a^2 - 4b^2),$$
$$z = 2^3 (a^2 - 2b^2 + 2ab)^3 ,$$
$$w = a^2 + 4\,ab$$

Set 2:

$$x = -2^8 (a^2 - 2b^2 + 2ab)^8 (2a^2 + 2b^2 - 2ab),$$
$$y = 2^5 (a^2 - 2b^2 + 2ab)^5 (4ab - 2b^2),$$
$$z = 2^3 (a^2 - 2b^2 + 2ab)^3 ,$$
$$w = 2a^2 - 2b^2$$

Set 3:

$$x = -\ (6a^2 - 2b^2)^8 (3a^2 + b^2),$$
$$y = (6a^2 - 2b^2)^5 (3a^2 + 2ab - b^2),$$
$$z = (6a^2 - 2b^2)^3 ,$$
$$w = 3a^2 - b^2 - 2ab$$

Set 4:

$$x = -\ (6a^2 - 2b^2)^8 (6a^2 + 2b^2 + 6ab),$$
$$y = -\ (6a^2 - 2b^2)^5 (4ab + 2b^2),$$
$$z = (6a^2 - 2b^2)^3 ,$$
$$w = 6a^2 + 4ab$$

Set 5:

$$x = -(6a^2 + b^2 + 6ab)^8 (6a^2 + 2b^2 + 6ab),$$
$$y = 2^5 (6a^2 + b^2 + 6ab)^5 (6a^2 + 4ab),$$
$$z = 2^3 (6a^2 + b^2 + 6ab)^3,$$
$$w = 6a^2 + 2b^2 + 8ab$$

Set 6:

$$x = -(6a^2 + b^2 + 6ab)^8 (12a^2 + b^2 + 6ab),$$
$$y = 2^5 (6a^2 + b^2 + 6ab)^5 (b^2 + 4ab),$$
$$z = 2^3 (6a^2 + b^2 + 6ab)^3,$$
$$w = 12a^2 + b^2 + 2ab$$

Set 7:

$$x = -2^8 (2k^2 + 6k + 3)^8 (2k^2 + 6k + 6),$$
$$y = 2^5 (2k^2 + 6k + 3)^5 (2k^2 + 4k),$$
$$z = 2^3 (2k^2 + 6k + 3)^3,$$
$$w = (2k^2 + 8k + 6)$$

Set 8:

$$x = -2^8 (2k^2 + 6k + 3)^8 (4k^2 + 6k + 3),$$
$$y = 2^5 (2k^2 + 6k + 3)^5 (3 + 4k),$$
$$z = 2^3 (2k^2 + 6k + 3)^3,$$
$$w = (4k^2 + 8k + 3)$$

Set 9:

$$x = -2^8 (6k^2 + 6k + 1)^8 (12k^2 + 6k + 1),$$
$$y = 2^5 (6k^2 + 6k + 1)^5 (1 + 4k),$$
$$z = 2^3 (6k^2 + 6k + 1)^3,$$
$$w = (12k^2 + 8k + 1)$$

Set 10:

$$x = -2^8 (6k^2 + 6k + 1)^8 (6k^2 + 6k + 2),$$
$$y = 2^5 (6k^2 + 6k + 1)^5 (6k^2 + 4k),$$
$$z = 2^3 (6k^2 + 6k + 1)^3,$$
$$w = (6k^2 + 8k + 2)$$

Set 11:

$$x = -14 * (26)^8 * k^9, y = 10 * (26)^5 * k^6, z = (26 * k)^3, w = 16k$$

Set 12:

$$x = -19 * (26)^8 * k^9, y = 5 * (26)^5 * k^6, z = (26 * k)^3, w = 21k$$

Set 13:

$$x = -13 * (22)^8 * k^9, y = 7 * (22)^5 * k^6, z = (22 * k)^3, w = 15k$$

Set 14:

$$x = -14 * (22)^8 * k^9, y = 6 * (22)^5 * k^6, z = (22 * k)^3, w = 16k$$

Conclusion:

In this paper, we have obtained different patterns of solutions to Octic equation with four unknowns. As Diophantine equations are rich in variety, one may search for other choices of octic equations with multi variables for obtaining their integer solutions with suitable properties.

Reference

1. M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, " On non-homogeneous Octic equation with four unknowns $x^2 = y^3 + z^5 w^3$ ", IJAIR, Volume 5, Issue1, 2015, Pp 13-15.