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Pentapartitioned Neutrosophic Quotient Mappings

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A B S T R A C T

In this paper, we have introduced the concept of a Pentapartitioned Neutrosophic quotient mappings and some of its properties are also discussed. Also the interconnections between the new mappings and pentapartitioned neutrosophic quotient mappings are investigated.

Keywords: neutrosophic set, pentapartitioned neutrosophic set, quotient mappings

Introduction

Fuzzy sets which allows the elements to have a degree of membership in the set and it was introduced by Zadeh in 1965. The degrees of membership lies in the the real unit interval [0,1]. Intuitionistic fuzzy set [IFS] allows both membership and non membership to the elements and this was introduced by Atanassov in1983. By introducing one more core component in IFS set , neutrosophic set was introduced by Smarandache in 1998. Neutrosophic set has three components truth membership function, indeterminancy membership function and falsity membership membership function respectively. This neutrosophic set helps to handle the indeterminate and inconsistent information effectively. Later Wang 2010 introduced the concept of single valued neutrosophic set (SVNS) which is a generalization of classic set, fuzzy set, interval valued fuzzy set and intuitionistic fuzzy set.Pentapartitioned Neutrosophic Pythagorean sets and fermatean quadripartitioned neutrosophic sets was proposed by Radha and Stanis Arul Mary and its properties are introduced. Rama Malik initiated the idea of pentapartitioned neutrosophic set.In this paper we have introduced the concept of pentapartitioned neutrosophic quotient mappings was introduced and its properties were also studied.

2. Preliminaries

2.1 Definition

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A\ , I_A, F_A: U \to [0,1]\ and\ 0 \le T_A(x) + I_A(x) + F_A\,(x) \le 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

2.2 Definition

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A, such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \le 5$$

2.3 Definition

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A\,(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

2.4 Definition

Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are pentapartitioned neutrosophic pythagorean sets. Then

A ∪ B = $< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)),$

$\min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), >$

A ∩ B = $< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), \max(G_A(x), G_B(x))$

$, max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$

2.5 Definition

A PN topology on a nonempty set R is a family of a PN sets in R satisfying the following axioms

1) $0,1 \in \tau$
2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
3) $\cup R_i \in \tau$ for any $R_i : i \in I \subseteq \tau$

The complement R^* of PN open set (PNOS, in short) in PN topological space [PNTS] (R,$\boldsymbol{\tau}$), is called a PN closed set [PNCS].

2.6 Definition

Let (X, τ) be an PNTS and A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ be an PNS in X. Then the interior and the closure of A are denoted by PNInt(A) and PNCl(A) and are defined as follows.

PNCl(A) = ∩{K|K is an PNCS and A ⊆ K} and

PNInt(A) = ∪{G|G is an PNOS and G ⊆ A}

Also, it can be established that PNCl(A) is an PNCS and PNInt(A) is an PNOS, A is an PNCS if and only if PNCl(A) = A and A is an PNOS if and only if PNInt(A) = A. We say that A is PN- dense if PNCl(A) = X.

2.7 Definition

A PN subset S of a PN topological space (R, τ) is called a

(i) PN pre - open set if $S \subseteq PNPInt$ (PNCl(S))
(ii) PN semi - open set if $S \subseteq PNPCl(PNPInt(S))$
(iii) PN α - open set if $S \subseteq PNPInt\left(PNPCl(PNPInt(S))\right)$

Note

We denote the family of all PN αopen sets of PN topological space (R, τ) by τ^α or αO(R) and of all PN semi open sets and of all PN pre open set of (R, τ) by τ^s or SO(R) and τ^p or PO(R) resp.

2.8 Definition

Let $(R, \tau) and\ (S, \sigma)$ be PN topological spaces. A function $f: (R, \tau) \to (S, \sigma)$ is called

(i) PN α - continuous if the inverse image of each PN open set in S is a PN α - open set in R.
(ii) PN semi - continuous if the inverse image of each PN open set in S is a PN semi open set in R
(iii) PN pre continuous if the inverse image of each PN open set in S is a PN pre open set in R

2.9 Definition

Let $(R, \tau) and\ (S, \sigma)$ be PN topological spaces. A function $f: (R, \tau) \to (S, \sigma)$ is called

(i) PN α - open mapping if the image of each PN open set in R is a PN α - open set in S.
(ii) PN semi - open mapping if the image of each PN open set in R is a PN semi - open set in S
(iii) PN pre - open mapping if the image of each PN open set in R is a PN pre - open set in S.

2.10 Theorem

A subset S of a PN topological space (R,τ) is a PN α open set if and only if S is PN semi - open and PN pre - open.

2.11 Theorem

Let $(R,\tau) and\ (S,\sigma)$ be PN topological spaces. A function $f:(R,\tau)\rightarrow(S,\sigma)$ is

(i) PN α - continuous if and only if it is PN semi - continuous and PN pre – continuous.
(ii) PN α - open map if and only if it is PN semi - open and PN pre – open.

2.12 Definition

Let $(R,\tau) and\ (S,\sigma)$ be PN topological spaces. A function $f:(R,\tau)\rightarrow(S,\sigma)$ is called

(i) PN α - irresolute if the inverse image of each PN α -open set in S is a PN α - open set in R.
(ii) PN semi - irresolute if the inverse image of each PN semi - open set in S is a PN semi - open set in R
(iii) PN pre irresolute if the inverse image of each PN pre - open set in S is a PN pre - open set in R

2.13 Definition

Let $(R,\tau) and\ (S,\sigma)$ be PN topological spaces. A function $f:(R,\tau)\rightarrow(S,\sigma)$ is called

(i) PN continuous mapping if $f^{-1}(A)\in\tau$ for each $A\in\sigma$
(ii) PN open mapping if $f(A)\in\sigma$ for each $A\in\tau$

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A=\{<x,T_A(x),I_A(x),F_A(x)>:x\in X\}$$

Where $T_A\ ,I_A,F_A:U\rightarrow[0,1]\ and\ 0\le T_A(x)+I_A(x)+F_A\,(x)\le 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

2.2 Definition

Let R be a universe. A Neutrosophic pythagorean set A with T and F as dependent Neutrosophic Pythagorean components and U as independent component for A on R is an object of the form

$$A=\{<x,T_A,U_A,F_A>:r\in R\}$$

Where $T_A\ +F_A\le 1,\ (T_A)^2+(F_A)^2\le 1$ and

$$(T_A)^2+(U_A)^2+(F_A)^2\le 2$$

Here, $T_A(x)$ is the truth membership, $U_A(x)$ is indeterminancy membership and $F_A(x)$ is the false membership .

2.3 Definition

The complement of a Neutrosophic Pythagorean set $A=\{<x,T_A,U_A,F_A>:r\in R\}$ with dependent Neutrosophic Pythagorean components is

$A^C=\{<x,F_A,1-U_A,T_A>:r\in R\}$.

2.4 Definition

Let $A=\{<x,T_A,U_A,F_A>:r\in R\}\ and\ B=\{<x,T_B,U_B,F_B>:r\in R\}$ are two Neutrosophic Pythagorean sets with dependent Neutrosophic Pythagorean components on the universe R. Then the union and intersection of two sets can be defined by

A $\cup B$ = {max (T_A,T_B), min (U_A,U_B),min (F_A,F_B)}, A $\cap B$ = {min (T_A, T_B), max (U_A, U_B),max (F_A, F_B)}.

2.5 Example

Let R = {a, b} and A ={ (a, 0.4,0.8,0.3),(b,0.5,0.6,0.2)}.

Then τ = {0,1, A} is a topology on R. Then A is a Neutrosophic Pythagorean set.

2.6 Example

Let R = {a, b} and A ={ (a, 0.5,0.6,0.6),(b,0.7,0.6,0.7)}.

Then τ = {0,1, A} is a topology on R. Since $T_A + F_A > 1$,then A is not a Neutrosophic Pythagorean set.

$But\ (T_A)^2 + (U_A)^2 + (F_A)^2 \leq 2$. Hence A is a Pythagorean Neutrosophic set.

3. Pentapartitioned Neutrosophic Pre Quotient Map

3.1 Definition

Let (R,τ) and (S,σ) be two PN topological spaces. Let $f: R \rightarrow S$ be an onto map. Then f is said to be a

(i) PN pre-quotient map if f is PN pre - continuous and $f^{-1}\ V$ is PN open in R implies V is PN pre open set in S.

(ii) PN semi - quotient map if f is PN semi - continuous and $f^{-1}\ V$ is PN open in R implies V is PN pre open set in S.

(iii) PN α - quotient map if f is PN α- continuous and $f^{-1}\ V$ is PN open in R implies V is PN pre open set in S.

3.2 Example:

Let R = {a, b, c} ,S = {p, q},τ = {0,1,A} and σ = {0,1,B} where A = {(a,0.4,0.5,0.6,0.2,0.1),(b,0.3,0.5,0.7,0.1,0.2),(c,0.6,0.4,0.8,0.1,0.2)} and B = {(p,0.3,0.2,0.5,0.3,0.2), (q,0.4,0.3,0.7,0.2,0.4)}. Clearly (R,τ) and (S,σ) are PN topological spaces. Define f: (R,τ) → (S,σ) by f(a) = p, f(b) = f(c) = q. Clearly f is a PN continuous map and a PN pre-continuous map. Also it is clear that when $f^{-1}(V)$ is PN open in R then V is a α −open in S. So f is a PN pre-quotient map.

3.3 Theorem

Let (R,τ) and (S,σ) be two PN topological spaces. If $f: (R, \tau) \rightarrow (S, \sigma)$ is an onto PN pre continuous and PN pre open map, then f is a PN pre quotient map .

3.4 Theorem

Let (R,τ),(S,σ) and (T,δ) be PN topological spaces. Let f: (R,τ) → (S,σ) be an onto PN open and PN pre - irresolute map. Let g: (S,σ) → (T,δ) be a PN pre - continuous. Then $g \circ f$ is a PN pre - quotient map.

Proof

Let V be any PN open set in T. Then $g^{-1}(V)$ is a PN pre - open set as g is a PN pre - quotient map. Since f is PN pre - irresolute, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is a PN pre - open set in R. so $g \circ f$ is a PN pre - continuous map. Suppose $g \circ f^{-1}(V)$ is PN open in R. Then $f^{-1}(g^{-1}(V))$ is PN open in R. Since f is PN open and onto, $(f\left(f^{-1}\left(g^{-1}(V)\right)\right) = (g^{-1}(V)$ is PN open in S. Since G is a PN pre quotient map, V is a PN pre open set T. Hence $g \circ f$ is a PN pre - quotient map.

3.5 Theorem

Let $f: (R, \tau) \rightarrow (S, \sigma)$ be an onto PN open PN irresolute map and $g: (S, \sigma) \rightarrow (T, \delta)$ be a PN semi - quotient map. Then $g \circ f$ is a PN semi - quotient map.

3.6 Theorem

Let $f: (R, \tau) \rightarrow (S, \sigma)$ be an onto PN open PN pre - irresolute map and $g: (S, \sigma) \rightarrow (T, \delta)$ be a PN pre - quotient map. Then $g \circ f$ is a PN pre - quotient map.

3.7 Theorem

Let $(R,\tau),(S,\sigma) and\ (T,\delta)$ be a PN topological space. If $p:(R,\tau)\to(S,\sigma)$ is a PN pre - quotient map and $g:(R,\tau)\to(T,\delta)$ is a PN continuous map such that it is constant on each set $p^{-1}(\{y\})$ for $y\in S$. Then g induces aPN pre continuous map $f:(S,\sigma)\to(T,\delta)$ such that $f\circ p=g$

Proof

Since map g is constant $on\ p^{-1}(\{y\})$ foe each $y\ \in\ S$, the set $g(p^{-1}(y))$ is a one point set in T.

If we let f(y) to denote this point, then it is clear that map f is well defined and for each $x\in R, f\big(p(x)\big)=g(x)$. Now we claim that f is PN pre - continuous. That is $g^{-1}(V)=(f\circ p)^{-1}(V)=p^{-1}(f^{-1}(V))$ is PN open in R. Since p is a PN pre - quotient map, $f^{-1}(V)$ is a PN pre - open set in S.

3.8 Theorem

Let $(R,\tau) and\ (S,\sigma)$ be PN topological space. A function $f:(R,\tau)\to(S,\sigma)$ is a PN α - quotient if and only if it is a PN semi - quotient map and a PN pre - quotient map.

Proof

Let f be a PN α - quotient map. So, $f^{-1}(V)\in\tau\hat{}\alpha$ whenever V is a PN open set in S then $f^{-1}(V)$ is both PN semi - open and PN pre - open. Hence f is both PN semi continuous and PN pre continuous. Let $f^{-1}(V) be\ a$ PN open set in R. Since f is PN α - quotient, $V\in\sigma^{\alpha}$ where $\sigma^{\alpha}=SO(S)\cap PO(S)$. So $V\in SO(S) and\ PO(S)$. This shows that f is both PN semi quotient and PN pre quotient. Conversely let f be a PN semi quotient and PN pre quotient map. We claim that f is a PN α - quotient map. Let V be any PN open set in S. Since f is both PN semi - quotient and PN pre - quotient, $f^{-1}(V)\in SO(R)\cap PO(R)$ so that $f^{-1}(V)\in\tau^{\alpha}$. Hence f is PN α - continuous. Let $f^{-1}(V)$ be PN open in R. so $V\in SO(S)\cap PO(S)$ so that $V\in\sigma^{\alpha}$. Hence f is a PN α - quotient map.

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