



Generalized Quadripartitioned Neutrosophic Set

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ABSTRACT

In this paper, we have introduced the concept of Generalized Quadripartitioned Neutrosophic Set and its properties were discussed.

Keywords: Neutrosophic set, quadripartitioned neutrosophic set, generalized neutrosophic set

1. Introduction

Fuzzy sets were introduced by Zadeh in 1965 that permits the membership perform valued within the interval $[0,1]$ and set theory it's an extension of classical pure mathematics. Fuzzy set helps to deal the thought of uncertainty, unclearness and impreciseness that isn't attainable within the cantorian set. As Associate in Nursing extension of Zadeh's fuzzy set theory intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1986, that consists of degree of membership and degree of non membership and lies within the interval of $[0,1]$. IFS theory wide utilized in the areas of logic programming, decision-making issues, medical diagnosis, clustering analysis etc.

Florentine Smarandache introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of $] -0 \ 1+[$. Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web, electronic database systems, diagnosis, multicriteria higher cognitive process issues etc.,

To method the unfinished data or imperfect data to unclearness a brand new mathematical approach i.e., To deal the important world issues, Wang (2010) introduced the idea of single valued neutrosophic sets (SVNS) that is additionally referred to as an extension of intuitionistic fuzzy sets and it became a really new hot analysis topic currently.

A. A.Salama and Albawi introduced the concept of generalized neutrosophic set and generalized neutrosophic topological spaces in 2012. Rajesh Chatterjee[5] initiated the concept of quadripartitioned neutrosophic single valued neutrosophic set. In this paper we introduced the concept of generalized quadripartitioned neutrosophic set and its properties are studied.

2. Preliminaries

2.1 Definition

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Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

2.2 Definition

Let X be a universe. A quadripartitioned neutrosophic set A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, U_A, F_A \rangle : x \in X \}$$

$$T_A + C_A + U_A + F_A \leq 4$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership.

2.3 Definition

Let X be a universe. A Generalized neutrosophic set A on X is an object of the form

$A = \{ \langle x, T_A, I_A, F_A \rangle : x \in X \}$ satisfying the following conditions

$$T_A + I_A + \text{and } T_A \wedge I_A \wedge F_A \leq 0.5$$

Here, $T_A(x)$ is the truth membership,

$I_A(x)$ is Indeterminacy,

$F_A(x)$ is the false membership

2.4 Definition

The complement of a quadripartitioned neutrosophic set A on R Denoted by A^c or A^* and is defined as

$$A^c = \{ \langle x, F_A(x), U_A(x), C_A(x), T_A(x) \rangle : x \in X \}$$

2.5 Definition

Let $A = \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle$ and

$B = \langle x, T_B(x), C_B(x), U_B(x), F_B(x) \rangle$ are quadripartitioned neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)) \rangle,$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) \rangle$$

3. Generalized Quadripartitioned Neutrosophic Set

3.1 Definition

Let X be a universe. A Generalized quadripartitioned neutrosophic set (GQN)A on X is an object of the form

$A = \{ \langle x, T_A, C_A, U_A, F_A \rangle : x \in X \}$ satisfying the following conditions

$$T_A + C_A + U_A + F_A \leq 4 \text{ and } T_A \wedge C_A \wedge U_A \wedge F_A \leq 0.5$$

Here, $T_A(x)$ is the truth membership,

$C_A(x)$ is contradiction membership,

$U_A(x)$ is ignorance membership,

$F_A(x)$ is the false membership

3.2 Example

Let $R = \{a, b, c\}$, and $A = \{(a, 0.3, 0.6, 0.4, 0.7), (b, 0.5, 0.2, 0.5, 0.3), (c, 0.2, 0.4, 0.5, 0.4)\}$. Then A is a Generalized quadripartitioned neutrosophic set on R.

Generalized quadripartitioned neutrosophic set A

R	$T_A(x)$	$C_A(x)$	$U_A(x)$	$F_A(x)$	$T_A \wedge C_A \wedge U_A \wedge F_A$
a	0.3	0.6	0.4	0.7	0.3
b	0.5	0.2	0.5	0.3	0.2
c	0.2	0.4	0.5	0.4	0.2

3.3 Definition

A Generalized quadripartitioned neutrosophic set A is contained in another Generalized quadripartitioned neutrosophic set B (i.e) $A \subseteq B$ if $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$

3.4 Example

Let $R = \{a, b, c\}$, and $A = \{(a, 0.3, 0.6, 0.4, 0.7), (b, 0.5, 0.2, 0.5, 0.3), (c, 0.2, 0.4, 0.5, 0.4)\}$ and $B = \{(a, 0.5, 0.6, 0.2, 0.5), (b, 0.6, 0.4, 0.3, 0.2), (c, 0.5, 0.6, 0.3, 0.2)\}$ are Generalized quadripartitioned neutrosophic sets on R . Then $A \subseteq B$.

3.5 Definition

The complement of a Generalized quadripartitioned neutrosophic A on X denoted by $(A)^c$ and is defined as $A^c(x) = \{ \langle x, F_A, U_A, C_A, T_A \rangle : x \in X \}$

3.6 Example

Let $X = \{a, b, c\}$, and $A = \{(a, 0.3, 0.6, 0.4, 0.7), (b, 0.5, 0.2, 0.5, 0.3), (c, 0.2, 0.4, 0.5, 0.4)\}$ is a Generalized quadripartitioned neutrosophic sets on R . Then $A^c = \{(a, 0.7, 0.4, 0.6, 0.3), (b, 0.3, 0.5, 0.4, 0.6), (c, 0.2, 0.3, 0.4, 0.2)\}$

3.7 Definition

Let X be a non-empty set, $A = \langle x, T_A, C_A, U_A, F_A \rangle$ and $B = \langle x, T_B, C_B, U_B, F_B \rangle$ are two Generalized quadripartitioned neutrosophic sets. Then $A \cup B = \langle x, \max(T_A, T_B), \max(C_A, C_B), \min(U_A, U_B), \min(F_A, F_B) \rangle$
 $A \cap B = \langle x, \min(T_A, T_B), \min(C_A, C_B), \max(U_A, U_B), \max(F_A, F_B) \rangle$

3.8 Example

Let $R = \{a, b, c\}$, and $A = \{(a, 0.2, 0.5, 0.3, 0.6), (b, 0.4, 0.1, 0.4, 0.2), (c, 0.1, 0.3, 0.4, 0.3)\}$ and $B = \{(a, 0.4, 0.5, 0.1, 0.4), (b, 0.5, 0.3, 0.2, 0.1), (c, 0.4, 0.5, 0.2, 0.1)\}$ are Generalized quadripartitioned neutrosophic sets on R . Then $A \cup B = \{(a, 1.5, 0.5, 0.1, 0.1), (b, 0.5, 0.3, 0.2, 0.1), (c, 0.4, 0.5, 0.2, 0.1)\}$

3.9 Definition

A Generalized quadripartitioned neutrosophic set A over the universe X is said to be empty Generalized quadripartitioned neutrosophic set \emptyset with respect to the parameter A if

$T_A = 0, C_A = 0, U_A = 1, F_A = 1, \forall x \in X, \forall e \in A$. It is denoted by \emptyset or 0

3.10 Definition

A Generalized quadripartitioned neutrosophic set A over the universe X is said to be Δ universe Generalized quadripartitioned neutrosophic set with respect to the parameter A if

$T_A = 1, C_A = 1, U_A = 0, F_A = 0$

It is denoted by Δ or 1

3.11 Definition

Let A and B be two Generalized quadripartitioned neutrosophic sets on X then $A \setminus B$ may be defined as

$A \setminus B = \langle x, \min(T_A, F_B), \min(C_A, U_B), \max(U_A, C_B), \max(F_A, T_B) \rangle$

3.12 Theorem

Let A, B and C are three Generalized quadripartitioned neutrosophic sets over the universe X . Then the following properties hold true.

- ❖ Commutative law
 - a) $A \cup B = B \cup A$
 - b) $A \cap B = B \cap A$
- ❖ Associative law

$$c) (A \cup B) \cup C = A \cup (B \cup C)$$

$$d) (A \cap B) \cap C = A \cap (B \cap C)$$

❖ Distributive law

$$e) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$f) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

❖ Absorption law

$$g) A \cup (A \cap C) = A$$

$$f) A \cap (A \cup C) = A$$

❖ Involution law

$$i) (A^c)^c = A$$

❖ Law of contradiction

$$j) A \cap A^c = \emptyset$$

❖ De Morgan's law

$$k) (A \cup B)^c = A^c \cap B^c$$

$$l) (A \cap B)^c = A^c \cup B^c$$

3.12 Theorem

Let K and L are two Generalized quadripartitioned neutrosophic sets over the universe X . Then the following are true.

$$(i) K \subseteq L \text{ iff } K \cap L = K$$

$$(ii) K \subseteq L \text{ iff } K \cup L = L$$

3.13 Theorem

Let K be Generalized quadripartitioned neutrosophic set over the universe X . Then the following are true.

$$(i) (\emptyset)^c = X$$

$$(ii) (X)^c = \emptyset$$

$$(iii) K \cup \emptyset = K$$

$$(iv) (ii) K \cup X = X$$

$$(v) (i) K \cap \emptyset = \emptyset$$

$$(vi) (ii) K \cap X = K, A$$

Proof: It is obvious

3.14 Definition

A Generalized quadripartitioned neutrosophic topology on a non-empty set X is a τ of Generalized quadripartitioned neutrosophic sets satisfying the following axioms.

$$i) 0_M, 1_M \in \tau$$

ii) The union of the elements of any sub collection of τ is in τ

iii) The intersection of the elements of any finite sub collection τ is in τ

The pair (X, τ) is called an Generalized quadripartitioned neutrosophic Topological Space over X .

Note :

1. Every member of τ is called a GQN open set in X .

2. The set A_M is called a GQN closed set in X if $A_M \in \tau^c$, where $\tau^c = \{A_M^c : A_M \in \tau\}$.

3.15 Example :

Let $M = \{b_1, b_2\}$ and Let A, B, C be Generalized quadripartitioned neutrosophic sets where

$$A = \{ \langle b_1, 0.5, 0.1, 0.7, 0.2 \rangle \langle b_2, 0.7, 0.5, 0.2, 0.1 \rangle \langle b_3, 0.6, 0.5, 0.4, 0.3 \rangle \}$$

$$B = \{ \langle b_1, 0.6, 0.7, 0.1, 0.2 \rangle \langle b_2, 0.2, 0.3, 0.4, 0.7 \rangle \langle b_3, 0.5, 0.6, 0.1, 0.3 \rangle \}$$

$$C = \{ \langle b_1, 0.6, 0.7, 0.1, 0.2 \rangle \langle b_2, 0.7, 0.5, 0.2, 0.1 \rangle \langle b_3, 0.6, 0.6, 0.1, 0.3 \rangle \}$$

$\tau = \{A, B, C, 0_M, 1_M\}$ is an Generalized quadripartitioned neutrosophic topology on M .

3.16 Proposition

Let (M, τ_1) and (M, τ_2) be two Generalized quadripartitioned neutrosophic topological space on M, Then $\tau_1 \cap \tau_2$ is an Generalized quadripartitioned neutrosophic topology on M where

$$\tau_1 \cap \tau_2 = \{A_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$$

Proof :

Obviously $0_M, 1_M \in \tau$.

Let $A_M, B_M \in \tau_1 \cap \tau_2$

Then $A_M, B_M \in \tau_1$ and $A_M, B_M \in \tau_2$

We know that τ_1 and τ_2 are two Generalized quadripartitioned neutrosophic topological space M.

Then $A_M \cap B_M \in \tau_1$ and $A_M \cap B_M \in \tau_2$

Hence $A_M \cap B_M \in \tau_1 \cap \tau_2$.

Let τ_1 and τ_2 are two Generalized quadripartitioned neutrosophic topological spaces on X.

Denote $\tau_1 \vee \tau_2 = \{A_M \cup B_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

$\tau_1 \wedge \tau_2 = \{A_M \cap B_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

3.17 Example

Let A_M and B_M be two Generalized quadripartitioned neutrosophic topological space on X.

Define $\tau_1 = \{0_M, 1_M, A_M\}$

$$\tau_2 = \{0_M, 1_M, B_M\}$$

Then $\tau_1 \cap \tau_2 = \{0_M, 1_M\}$ is a Generalized quadripartitioned neutrosophic topological space on M.

But $\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}$,

$$\tau_1 \vee \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\} \text{ and}$$

$\tau_1 \wedge \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}$ are not Generalized quadripartitioned neutrosophic topological space on X.

3.18 Definition

Let (M, τ) be a Generalized quadripartitioned neutrosophic topological space on M and let A_M belongs to Generalized quadripartitioned neutrosophic set on M. Then the interior of A_M is denoted as $\text{GQNInt}(A_M)$. It is defined by $\text{GQNInt}(A_M) = \cup \{B_M \in \tau : B_M \subseteq A_M\}$

3.19 Definition

Let (M, τ) be a Generalized quadripartitioned neutrosophic topological space on M and let A_M belongs to G quadripartitioned neutrosophic set M. Then the closure of A_M is denoted as $\text{GQNCl}(A_M)$. It is defined by $\text{GQNCl}(A_M) = \cap \{B_M \in \tau^c : A_M \subseteq B_M\}$

3.20 Theorem

Let (M, τ) be a Generalized quadripartitioned neutrosophic topological space over M. Then the following properties are hold.

- i) 0_M and 1_M are Generalized quadripartitioned neutrosophic closed sets over M
- ii) The intersection of any number of Generalized quadripartitioned neutrosophic closed set is a Generalized quadripartitioned neutrosophic closed set over M.
- iii) The union of any two Generalized quadripartitioned neutrosophic closed set is Generalized quadripartitioned neutrosophic set over M.

Proof

It is obviously true.

3.21 Theorem

Let (M, τ) be a Generalized quadripartitioned neutrosophic topological space over M and Let $A_M \in$ Generalized quadripartitioned neutrosophic topological space. Then the following properties hold.

- (i) $\text{GQNInt}(A_M) \subseteq A_M$
- (ii) $A_M \subseteq B_M$ implies $\text{GQNInt}(A_M) \subseteq \text{GQNInt}(B_M)$.
- (iii) $\text{GQNInt}(A_M) \in \tau$.
- (iv) A_M is a GQN open set implies $\text{GQNInt}(A_M) = A_M$.
- (v) $\text{GQNInt}(\text{GQNInt}(A_M)) = \text{GQNInt}(A_M)$
- (vi) $\text{GQNInt}(0_M) = 0_M, \text{GQNInt}(1_M) = 1_M$.

3.22 Theorem

Let (M, τ) be a Generalized quadripartitioned neutrosophic topological space over M and Let A_M is in the Generalized quadripartitioned neutrosophic topological space. Then the following properties hold.

- (i) $A_M \subseteq \text{GQNCl}(A_M)$
- (ii) $A_M \subseteq B_M$ implies $\text{GQNCl}(A_M) \subseteq \text{GQNCl}(B_M)$.
- (iii) $\text{GQNCl}(A_M)^c \in \tau$.
- (iv) A_M is a GQN closed set implies $\text{GQNCl}(A_M) = A_M$.
- (v) $\text{GQNCl}(\text{GQNCl}(A_M)) = \text{GQNCl}(A_M)$
- (vi) $\text{GQNCl}(0_M) = 0_M, \text{GQNCl}(1_M) = 1_M$.

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