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On Non-homogeneous Ternary Quadratic Equation

$$5(x^2 - y^2) + 6(x + y) = 6z^2$$

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ABSTRACT :

This paper aims at determining non-zero distinct integer solutions to non-homogeneous ternary quadratic equation $5(x^2 - y^2) + 6(x + y) = 6z^2$. Different sets of integer solutions have been presented through employing linear transformations.

Keywords: Non-homogeneous quadratic , Ternary quadratic , Integer solutions

Introduction:

The Ternary quadratic Diophantine equations offer an unlimited field for research because of their variety [1-3].For an extensive review of various problems, one may refer [4-19] for quadratic equations with two and three unknowns. This communication concerns with yet another interesting ternary quadratic equation $5(x^2 - y^2) + 6(x + y) = 6z^2$ for determining its infinitely many non-zero integral solutions

Method of analysis

The non-homogeneous ternary quadratic equation to be solved for the integer solutions is

$$5(x^2 - y^2) + 6(x + y) = 6z^2 \quad (1)$$

The process of obtaining different sets of integer solutions to (1) are illustrated below:

Set 1

Introduction of the linear transformations

$$x = u + v, y = u - v, z = u\ , u \neq v \neq 0 \quad (2)$$

in (1) leads to the linear Diophantine equation

$$3u - 10v = 6 \quad (3)$$

whose general solution is

$$u = 12 + 10t, v = 3 + 3t \quad (4)$$

In view of (2) , one has

$$x = 15 + 13t, y = 9 + 7t, z = 12 + 10t \quad (5)$$

representing the integer solutions to (1).

Set 2

Introducing the transformations

$$x = uv + 1, y = uv - 1, z = u \quad (6)$$

in (1) , it reduces to

$$16v = 3u \quad (7)$$

which is satisfied by

$$v = 3s, u = 16s \quad (8)$$

Using (2) , the corresponding integer solutions to (1) are given by

$$x = 48s^2 + 1, y = 48s^2 - 1, z = 16s$$

Set 3

Treating (1) as a quadratic in x and solving for x, one obtains

$$x = \frac{-3 \pm \sqrt{30z^2 + (5y-3)^2}}{5} \tag{9}$$

It is possible to choose y, z so that the square-root on the R.H.S. of (9) is removed and the corresponding values of x satisfying (1) are obtained. It is noted that there are two choices of solutions satisfying (1) and they are presented below:

Choice 1:

$x = -10s^2 + 16s - 10, y = 10s^2 - 16s + 4, z = 10s - 8$

Choice 1I:

$x = -10s^2 + 4s - 4, y = 10s^2 - 4s - 2, z = 10s - 2$

Set 4

The substitution

$$x = ky, k \neq 1 \tag{10}$$

in (1) leads to the positive pell equation

$$(5(k-1)y+3)^2 = \frac{30(k-1)}{(k+1)} z^2 + 9 \tag{11}$$

which is solved for y, z when k takes particular values. In view of (10), the corresponding value of x is obtained .A few illustrations are presented below:

Illustration 1

The choice

$$k = 2$$

in (11) leads to the positive pell equation

$$Y^2 = 10z^2 + 9 , \ Y = 5y + 3$$

Employing the standard procedure , the corresponding integer solutions to (1) are found to be

$$x_{n+1}=\frac{57f_n+18\sqrt{10}g_n-6}{5},\ y_{n+1}=\frac{57f_n+18\sqrt{10}g_n-6}{10},\ z_{n+1}=\frac{180f_n+57\sqrt{10}g_n}{20},$$

$$f_n=(19+6\sqrt{10})^{n+1}+(19-6\sqrt{10})^{n+1},\ g_n=(19+6\sqrt{10})^{n+1}-(19-6\sqrt{10})^{n+1},\ n=0,2,4,...$$

Illustration 2

The choice

$$k=3$$

in (11) leads to the positive pell equation

$$Y^2=15z^2+9,\ Y=10y+3$$

Employing the standard procedure , the corresponding integer solutions to (1) are found to be

$$x_{n+1}=\frac{3(12f_n+3\sqrt{15}g_n-6)}{20},\ y_{n+1}=\frac{12f_n+3\sqrt{15}g_n-6}{20},\ z_{n+1}=\frac{15f_n+4\sqrt{15}g_n}{10},$$

$$f_n=(4+\sqrt{15})^{n+1}+(4-\sqrt{15})^{n+1},\ g_n=(4+\sqrt{15})^{n+1}-(4-\sqrt{15})^{n+1},\ n=0,2,4,...$$

Illustration 3

The choice

$$k=5$$

in (11) leads to the positive Pell equation

$$Y^2=20z^2+9,\ Y=20y+3$$

Employing the standard procedure , the corresponding integer solutions to (1) are found to be

$$x_{n+1}=\frac{(27f_n+6\sqrt{20}g_n-6)}{8},\ y_{n+1}=\frac{(27f_n+6\sqrt{20}g_n-6)}{40},\ z_{n+1}=\frac{120f_n+27\sqrt{20}g_n}{40},$$

$$f_n=(9+2\sqrt{20})^{n+1}+(9-2\sqrt{20})^{n+1},\ g_n=(9+2\sqrt{20})^{n+1}-(9-2\sqrt{20})^{n+1},\ n=0,2,4,...$$

Conclusion

In this paper ,an attempt has been made for finding non-zero distinct integer solutions to non-homogeneous ternary quadratic equation $5(x^2-y^2)+6(x+y)=6z^2$. The readers may search for other choices of integer solutions to the quadratic equation in title.

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