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Some Comb Related Cordial Graphs

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ABSTRACT:

In this research paper, we prove that different families of comb graph such as comb graph Ca_z, comb graph Cd_z, comb graph Ct_z, comb graph Cf_z and comb graph Ch_z are all cordial graphs.

Keywords: Cordial labeling, Cordial graph, Comb Graphs.

1. INTRODUCTION

In this paper, we consider finite, simple and undirected graphs. Various labeling have been developed by many researchers. In [13], Yegnanaryanan et al. discussed many applications of graph labeling. A dynamic survey about different graph labeling can be seen in [6]. Cahit in [5] gave the idea of cordial labeling and investigated several cordial graphs such as Eulerian graph in 1987. Andar et al. in 2002 described many results regarding cordial labeling [1, 2]. Ho and many other researcher investigated in 1989 that generalized petersen graphs and unicyclic graphs are cordial graphs [8]. Vaidya and many other researcher [9-11] in 2010 and 2011 proved that star related graphs and middle graph of some graphs are also cordial graphs. Vaidya et al. in [12] and Lawrence et al. in [7] in 2011 proved that bistar related graphs and splitting graph of some standard graphs respectively admit cordial labeling. In 2011 Babujee and Shobana introduced idea of cordial numbers and cordial languages [3]. In 2022, Muhammad Imran and other authors proved that some path related graphs admit cordial labeling so these graphs are cordial graphs [14].

2. RELIMINARIES

Definition 2.1

Graph labeling is the procedure of assignment of integers to vertex set or edge set or both.

Definition 2.2

A mapping $\psi: V(G) \to \{0,1\}$ is categorized as cordial labeling if weight of each edge uv is labeled by $|\psi(u) - \psi(v)|$ with the following two conditions:

(1) The difference between 0 labeled vertices and 1 labeled vertices must be at most one.

(2) The difference between 0 labeled edges and 1 labeled edges must be at most one.

Definition 2.3

If a graph admits cordial labeling then this graph is categorized as cordial graph.

Definition 2.4

Comb graph Ca_z, $z > 1$ is formed by vertex set $V(Ca_z) = \{b_l^m; 1 \le l \le m+1, 1 \le m \le z\}$ and edge set $E(Ca_z) = \{b_1^m b_1^{m+1}; 1 \le m \le z-1\} \cup \{b_l^m b_{l+1}^m; 1 \le m \le z, 1 \le l \le m\}$. It has $(\frac{z^2+3z}{2})$ vertices and $(\frac{z^2+3z-2}{2})$ edges.

Definition 2.5

Comb graph Cd_z, (for even z) is formed by vertex set $V(Cd_z) = \{b_l^m; 1 \leq m \leq z, 1 \leq l \leq \lfloor\frac{m+3}{2}\rfloor\}$ and edge set $E(Cd_z) = \{b_l^m b_{l+1}^m; 1 \leq m \leq z\ , 1 \leq l \leq \lfloor\frac{m+1}{2}\rfloor\} \cup \{b_1^m b_1^{m+1}; 1 \leq m \leq z-1\}$. It has $(\frac{z^2+6z}{4})$ vertices and $(\frac{z^2+6z-4}{4})$ edges.

Definition 2.6

Comb graph Ct_z, (for even z) is formed by vertex set $V(Ct_z) = \{b_l^m; 1 \leq m \leq z, 1 \leq l \leq 6\}$ and edge set $E(Ct_z) = \{b_l^m b_{l+1}^m; 1 \leq m \leq z, 1 \leq l \leq 5\} \cup \{b_4^m b_3^{m+1}; 1 \leq m \leq z, odd\} \cup \{b_3^m b_4^{m+1}; 2 \leq m \leq z, even\}$. It has $6z$ vertices and $6z-1$ edges.

Definition 2.7

Let Cf_z, $z > 1$ be a comb graph, which is formed by vertex set $V(Cf_z) = \{b_l^m; 1 \leq l \leq 7, 1 \leq m \leq z\}$ and edge set $E(Cf_z) = \{b_4^m b_4^{m+1}; 1 \leq m \leq z-1\} \cup \{b_l^m b_{l+1}^m; 1 \leq m \leq z, 1 \leq l \leq 6\}$. It has $(7z)$ vertices and $(7z-1)$ edges.

Definition 2.8

Comb graph Ch_z, (for even z) is formed by vertex set $V(Ch_z) = \{b_l^m; 1 \leq l \leq 3; 1 \leq m \leq z, odd\} \cup \{b_l^m; 2 \leq m \leq z, even; 1 \leq l \leq 4\}$ and edge set $E(Ch_z) = \{b_l^m b_{l+1}^m; 1 \leq m \leq z, odd; 1 \leq l \leq 2\} \cup \{b_l^m b_{l+1}^m; 1 \leq m \leq z, even; 1 \leq l \leq 3\} \cup \{b_1^m b_1^{m+1}; 1 \leq m \leq z-1\}$. It has $(\frac{7z}{2})$ vertices and $(\frac{7z}{2} - 1)$ edges.

3. MAIN RESULTS

Theorem 3.1 Comb graph $Ca_z, z > 1$ is cordial graph.

Proof: Now we label all vertices as

$$\psi(b_l^m) = \begin{cases} 0, & if\ \ l = 5, m \equiv 4(mod4), m \equiv 5(mod4) \\ 1, & if\ \ l = 5, m \equiv 6(mod4), m \equiv 7(mod4) \end{cases}$$

If m is odd

$$\psi(b_l^m) = \begin{cases} 1, & if\ \ l = 1, l \equiv 2(mod4), l \equiv 7(mod8), l \equiv 13(mod8) \\ 0, & if\ \ l \equiv 4(mod4), l \equiv 3(mod8), l \equiv 9(mod8) \\ where & 1 \leq l \leq m+1, 1 \leq m \leq z \end{cases}$$

If m is even

$$\psi(b_l^m) = \begin{cases} 0, & if\ \ l = 1, l \equiv 2(mod4), l \equiv 7(mod8), l \equiv 13(mod8) \\ 1, & if\ \ l \equiv 4(mod4), l \equiv 3(mod8), l \equiv 9(mod8) \\ where & 1 \leq l \leq m+1, 2 \leq m \leq z \end{cases}$$

Now we evaluate weights for all edges as

$$w_t(b_1^m b_1^{m+1}) = \begin{cases} |b_1^m - b_1^{m+1}|, & 1 \leq m \leq z-1 \end{cases}$$

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & 1 \leq m \leq z, 1 \leq l \leq m \end{cases}$$

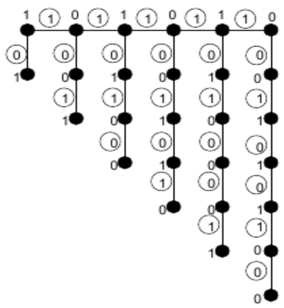

Figure 1: Cordial labeling on comb graph Ca_6

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence comb graph Ca_z is cordial graph.

Theorem 3.2Comb graph Cd_z, (for even z) is cordial graph.

Proof: Now we label all vertices as

If m is even

$$\psi(b_l^m) = \begin{cases} 0, & \text{if } l = 1, l \equiv 2(mod4) \\ 1, & otherwise \\ where & 2 \le m \le z, 1 \le l \le \lfloor \frac{m+3}{2} \rfloor \end{cases}$$

If m is odd

$$\psi(b_l^m) = \begin{cases} 1, & \text{if } l = 1, l \equiv 2(mod4) \\ 0, & otherwise \\ where & 1 \le m \le z, 1 \le l \le \lfloor \frac{m+3}{2} \rfloor \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_1^m b_1^{m+1}) = \begin{cases} |b_1^m - b_1^{m+1}|, & 1 \le m \le z-1 \end{cases}$$

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & 1 \le m \le z, 1 \le l \le \lfloor \frac{m+1}{2} \rfloor \end{cases}$$

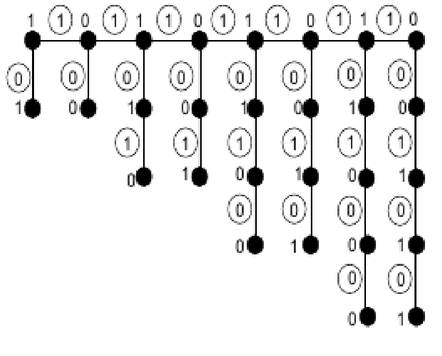

Figure 2: Cordial Labeling on Comb Graph Cd_8

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence comb graph Cd_z is cordial graph.

Theorem 3.3Comb graph Ct_z, (for even z) is cordial graph.

Proof: Now we label all vertices as

If m is odd

$$\psi(b_l^m) = \begin{cases} 0, & \text{if } l \equiv 2(mod4) \\ 1, & otherwise \end{cases}$$

If m is even

$$\psi(b_l^m) = \begin{cases} 0, & \text{if } l = 5, l \equiv 2(mod2) \\ 1, & otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & 1 \le m \le z, 1 \le l \le 5 \end{cases}$$

$$w_t(b_4^m b_3^{m+1}) = \begin{cases} |b_4^m - b_3^{m+1}|, & \text{if } 1 \le m \le z, odd \end{cases}$$

$$w_t(b_3^m b_4^{m+1}) = \begin{cases} |b_3^m - b_4^{m+1}|, & if\ \ 2 \le m \le z, even \end{cases}$$

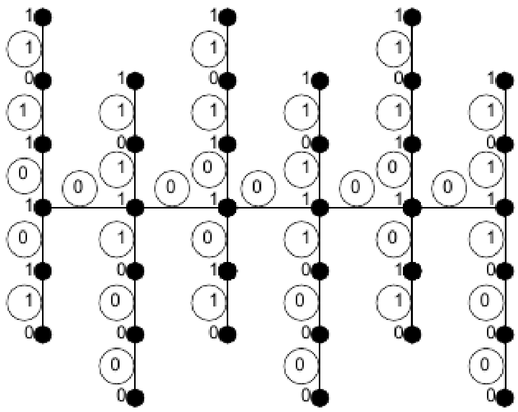

Figure 3: Cordial Labeling on Comb Graph Ct_6

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence comb graph Ct_z is cordial graph.

Theorem 3.4 Comb graph $Cf_z, z > 1$ is cordial graph.

Proof: Now we label all vertices as

If $m = 1$,

$$\psi(b_l^m) = \begin{cases} 1, & if\ \ l = 6, l \equiv 3(mod2) \\ 0, & otherwise \end{cases}$$

If $m \equiv 2(mod2)$

$$\psi(b_l^m) = \begin{cases} 0, & if\ \ l \equiv 3(mod3), l \equiv 4(mod3) \\ 1, & otherwise \end{cases}$$

If $m \equiv 3(mod2)$

$$\psi(b_l^m) = \begin{cases} 1, & if\ \ l \equiv 1(mod5), l \equiv 3(mod4) \\ 0, & otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_4^m b_4^{m+1}) = \begin{cases} |b_4^m - b_4^{m+1}|, & 1 \le m \le z - 1 \end{cases}$$

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & 1 \le l \le 6, 1 \le m \le z \end{cases}$$

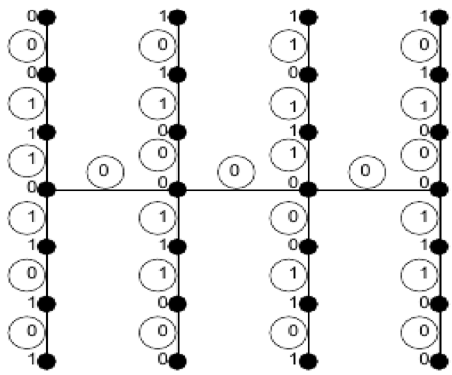

Figure 4: Cordial Labeling on Comb Graph Cf_4

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence comb graph Cf_z is cordial graph.

Theorem 3.5 Comb graph Ch_z, (for even z) is cordial graph.

Proof: Now we label all vertices as

If m is odd

$$\psi(b_l^m) = \begin{cases} 0, & if\ \ l = 1 \\ 1, & Otherwise \end{cases}$$

If $m \equiv 2(mod4)$

$$\psi(b_l^m) = \begin{cases} 1, & if\ \ l = 3 \\ 0, & Otherwise \end{cases}$$

If $m \equiv 4(mod4)$

$$\psi(b_l^m) = \begin{cases} 1, & if\ \ l = 1,2 \\ 0, & Otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & if\ \ 1 \le m \le z, odd; 1 \le l \le 2 \end{cases}$$

$$w_t(b_1^m b_1^{m+1}) = \begin{cases} |b_1^m - b_1^{m+1}|, & if\ \ 1 \le m \le z-1 \end{cases}$$

$$w_t(b_l^m b_{l+1}^m) = \begin{cases} |b_l^m - b_{l+1}^m|, & if\ \ 1 \le m \le z, even; 1 \le l \le 3 \end{cases}$$

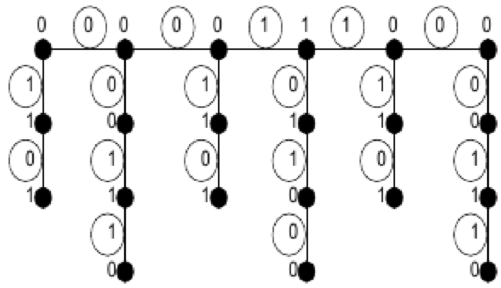

Figure 5: Cordial Labeling on Comb Graph Ch_6

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence comb graph Ch_z is cordial graph.

4. CONCLUSION

In this research paper, we proved that different families of comb graph Ca_z, Cd_z, Ct_z, Cf_z and Ch_z are cordial graphs, as all the graphs admit cordial labeling.

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