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Some Path Related Cordial Graphs

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ABSTRACT:

In this research paper, we prove that different families of snake graphs such as triangular snake graph with pendant edges, alternate triangular snake graph with and without pendant edges, quadrilateral snake graph with and without pendant edges, alternate quadrilateral snake graph with pendant edges and double quadrilateral snake graph with and without pendant edges are cordial graphs.

Keywords: Cordial labeling, Cordial graph, Pendant edges, Snake graphs

1. INTRODUCTION

In this paper, we consider only finite, simple and undirected graphs. The field of Graph Theory plays a vital role in various fields. One of the important areas in graph theory is Graph Labeling used in many applications like coding theory, x-ray crystallography, radar, astronomy The concept of cordial labeling was first introduced by Cahit [5] as a weaker version of graceful and harmonious labeling. In the same paper Cahit investigated many classes of cordial graphs and proved that Eulerian graph is cordial graph. Andaret et al. [1, 2] and Hoet et al. [8] have developed new families of cordial graphs. Vaidya and Dani [9, 10] as well as Vaidya and Vihol [11] have investigated many new families of graphs which admit cordial labeling. Vaidya and Shah in 2014 proved that some bistar related graphs are also cordial graphs [12]. Lawrence and Koilraj [7] have discussed cordial labeling for the splitting graph of some standard graphs. Motivated by these authors Babujee and Shobana [3] have introduced the new concepts of cordial languages and cordial numbers. In 2012 Yagnanarayanan, and V., Vaidhyanathan had discussed some interesting and useful results on graph labeling[13].

2. PRELIMINARIES

Definition 2.1

Graph labeling is the procedure of assignment of integers to vertex set or edge set or both.

Definition 2.2

A mapping $f\colon V(G) \to \{0,1\}$ is categorized as cordial labeling if each edge uv is labeled by $|f(u) - f(v)|$ with the following two conditions:

(1) The difference between 0 labeled vertices and 1 labeled vertices is at most one.

(2) The difference between 0 labeled edges and 1 labeled edges is at most one.

Definition 2.3

If a graph admits cordial labeling then this graph is categorized as cordial graph.

Definition 2.4

An edge of a graph is said to be pendant edge if one of its vertices has degree one.

Definition 2.5

Triangular snake graph T_n with pendent edges is formed by vertex set $V(T_n) = \{a_i, b_i; 1 \le i \le n-1\} \cup \{c_i, d_i; 1 \le i \le n\}$ and edge set $E(T_n) = \{a_i b_i; 1 \le i \le n-1\} \cup \{b_i c_i; 1 \le i \le n-1\} \cup \{b_i c_{i+1}; 1 \le i \le n-1\} \cup \{c_i c_{i+1}; 1 \le i \le n-1\} \cup \{c_i d_i; 1 \le i \le n\}$. It has $(5n-4)$ edges and $(4n-2)$ vertices.

Definition 2.6

Alternate triangular snake graph AT_n is formed by vertex set $V(AT_n) = \{a_i; 1 \le i \le n-2\} \cup \{b_i; 1 \le i \le n\}$ and edge set $E(A(T_n)) = \{b_i b_{i+1}; 1 \le i \le n-1\} \cup \{a_i b_i; 1 \le i \le n-2\} \cup \{a_i b_{i+2}; 1 \le i \le n-2\}$. It has $(2n-2)$ vertices and $(3n-5)$ edges.

Definition 2.7

Alternate triangular snake graph AT_n with pendent edges is formed by vertex set $V(AT_n) = \{a_i, b_i; 1 \le i \le n-2\} \cup \{c_i, d_i; 1 \le i \le n\}$ and edge set $E(A(T_n)) = \{a_i b_i; 1 \le i \le n-2\} \cup \{b_i c_i; 1 \le i \le n-2\} \cup \{b_i c_{i+2}; 1 \le i \le n-2\} \cup \{c_i c_{i+1}; 1 \le i \le n-1\} \cup \{c_i d_i; 1 \le i \le n\}$. It has $(4n-4)$ vertices and $(5n-7)$ edges.

Definition 2.8

To obtain a Quadrilateral snake graph let us consider a path graph P_n, $(n > 1)$, if we replace each edge of path graph by a quadrilateral C_4, we get Quadrilateral snake graph QS_n. It can be obtained by vertex set $V(QS_n) = \{a_i; 1 \le i \le 2n-2\} \cup \{b_i; 1 \le i \le n\}$ and edge set can be given as $E(QS_n) = \{b_i b_{i+1}; 1 \le i \le n-1\} \cup \{a_{2i} b_{i+1}; 1 \le i \le n-1\} \cup \{a_{2i-1} b_i; 1 \le i \le n-1\} \cup \{a_{2i-1} a_{2i}; 1 \le i \le n-1\}$.

Definition 2.9

Double Quadrilateral snake graph DQS_n can be obtained from vertex set $V(DQS_n) = \{a_i; 1 \le i \le 2n-2\} \cup \{b_i; 1 \le i \le n\} \cup \{w_i; 1 \le i \le 2n-2\}$ and edge set

$E(DQS_n) = \{b_i b_{i+1}; 1 \le i \le n-1\} \cup \{a_{2i} b_{i+1}; 1 \le i \le n-1\} \cup \{a_{2i-1} b_i; 1 \le i \le n-1\} \cup \{a_{2i-1} a_{2i}; 1 \le i \le n-1\} \cup \{w_{2i-1} w_{2i}; 1 \le i \le n-1\} \cup \{w_{2i} b_{i+1}; 1 \le i \le n-1\} \cup \{w_{2i-1} b_i; 1 \le i \le n-1\}$.

Definition 2.10

Quadrilateral snake graph PQS_n with pendant edges can be obtained from vertex set $V(PQS_n) = \{c_i; 1 \le i \le 2n-2\} \cup \{a_i; 1 \le i \le 2n-2\} \cup \{b_i; 1 \le i \le n\} \cup \{d_i; 1 \le i \le n\}$ and edge set $E(PQS_n) = \{c_i a_i; 1 \le i \le 2n-2\} \cup \{a_{2i-1} b_i; 1 \le i \le n-1\} \cup \{b_i d_i; 1 \le i \le n\} \cup \{b_{i+1} a_{2i}; 1 \le i \le n-1\} \cup \{b_i b_{i+1}; 1 \le i \le n-1\} \cup \{a_{2i-1} a_{2i}; 1 \le i \le n-1\}$.

Definition 2.11

Alternate Quadrilateral snake graph $PAQS_n$ with pendant edges is formed by vertex set $V(PAQS_n) = \{a_i, b_i, c_i, d_i; 1 \le i \le n\}$ and edge set $E(PAQS_n) = \{c_i a_i; 1 \le i \le n\} \cup \{a_i b_i; 1 \le i \le n\} \cup \{b_i d_i; 1 \le i \le n\} \cup \{a_{2i-1} a_{2i}; 1 \le i \le \frac{n}{2}\} \cup \{b_i b_{i+1}; 1 \le i \le n-1\}$. It has $\frac{9n-2}{2}$ edges.

3. MAIN RESULTS

Theorem 3.1 ***Triangular snake graph $T_n, n > 1$ with pendant edges is cordial graph.***

Proof: Let T_n be a triangular snake graph with pendent edges. Now we label all vertices as

$$f(a_i) = \begin{cases} 0, & if\ \ i \equiv 4(mod4), 4 \le i \le n-1 \\ 1, & otherwise \end{cases}$$

$$f(b_i) = \begin{cases} 0, & if\ \ 1 \le i \le n-1, odd \\ 1, & if\ \ 2 \le i \le n-1, even \end{cases}$$

$$f(c_i) = \begin{cases} 0, & if\ 1 \le i \le n \end{cases}$$

$$f(d_i) = \begin{cases} 0, & if\ \ i \equiv 3(mod4), 3 \le i \le n \\ 1, & otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(a_i b_i) = \begin{cases} |a_i - b_i|, & if\ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(b_i c_i) = \begin{cases} |b_i - c_i|, & if\ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(b_i c_{i+1}) = \begin{cases} |b_i - c_{i+1}|, & if\ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(c_i c_{i+1}) = \begin{cases} |c_i - c_{i+1}|, & if\ 1 \le i \le n-1 \end{cases}$$

$$w_t(c_i d_i) = \begin{cases} |c_i - d_i|, & if\ \ 1 \le i \le n \end{cases}$$

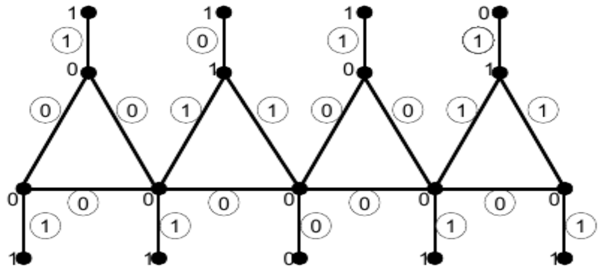

Figure 1: Cordial Labeling on Triangular Snake Graph T_5 with Pendant Edges

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.2 ***Alternate triangular snake graph AT_n, $n \geq 4$ is cordial graph.***

Proof: Let AT_n be an alternate triangular snake graph. Now we label all vertices as:

$$f(a_i) = \begin{cases} 0, & if\ \ 2 \leq i \leq n-2, even \\ 1, & if\ \ i = odd, 1 \leq i \leq n-2 \end{cases}$$

$$f(b_i) = \begin{cases} 0, & if\ \ i \equiv 2(mod4), 2 \leq i \leq n \\ 0, & if\ \ i \equiv 3(mod4), 3 \leq i \leq n \\ 1, & if\ \ i \equiv 1(mod4), 1 \leq i \leq n \\ 1, & if\ \ i \equiv 4(mod4), 4 \leq i \leq n \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(a_i b_i) = \begin{cases} |a_i - b_i|, & if\ \ 1 \leq i \leq n-2 \end{cases}$$

$$w_t(a_i b_{i+2}) = \begin{cases} |a_i - b_{i+2}|, & if\ \ 1 \leq i \leq n-2 \end{cases}$$

$$w_t(b_i b_{i+1}) = \begin{cases} |b_i - b_{i+1}|, & if\ \ 1 \leq i \leq n-1 \end{cases}$$

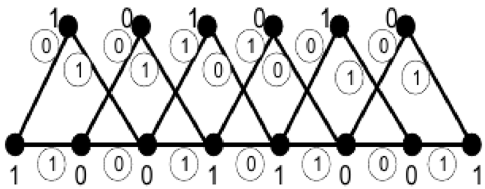

Figure 2: Cordial Labeling on Alternate Triangular Snake Graph AT_8

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.3 ***Alternate triangular snake graph $AT_n, n \geq 4$ with pendant edges is cordial graph.***

Proof: Let AT_n be an alternate triangular snake graph with pendent edges.Now we label all vertices as:

$$f(a_i) = \begin{cases} 0, & if\ \ i \equiv 4(mod4), 4 \leq i \leq n-2 \\ 1, & otherwise \end{cases}$$

$$f(b_i) = \begin{cases} 0, & if\ \ 1 \leq i \leq n-2, odd \\ 1, & if\ \ 2 \leq i \leq n-2, even \end{cases}$$

$$f(c_i) = \begin{cases} 0, & if\ 1 \leq i \leq n, \end{cases}$$

$$f(d_i) = \begin{cases} 0, & if\ \ i \equiv 4(mod4), 1 \leq i \leq n \\ 1, & otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(a_i b_i) = \begin{cases} |a_i - b_i|, & if\ \ 1 \le i \le n-2 \end{cases}$$
$$w_t(b_i c_i) = \begin{cases} |b_i - c_i|, & if\ \ 1 \le i \le n-2 \end{cases}$$
$$w_t(b_i c_{i+2}) = \begin{cases} |b_i - c_{i+2}|, & if\ \ 1 \le i \le n-2 \end{cases}$$
$$w_t(c_i c_{i+1}) = \begin{cases} |c_i - c_{i+1}|, & if\ 1 \le i \le n-1 \end{cases}$$
$$w_t(c_i d_i) = \begin{cases} |c_i - d_i|, & if\ \ 1 \le i \le n \end{cases}$$

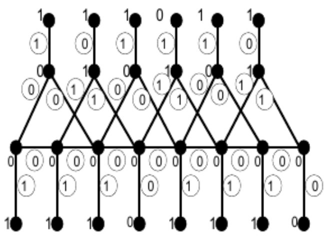

Figure 3: Cordial Labeling on Alternate Triangular Snake Graph AT_8 with Pendant Edges

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.4 ***Quadrilateral snake graph QS_n , $n > 1$ is cordial graph.***

Proof: Now we label all vertices as

$$f(a_i) = \begin{cases} 0, & if\ \ i = 1,2 \\ 1, & if\ \ i \equiv 3(mod4), i \equiv 6(mod4) \\ 0, & otherwise \end{cases}$$

$$f(b_i) = \begin{cases} 1, & if\ \ i = 1 \\ 1, & if\ \ 2 \le i \le n, even \\ 0, & if\ \ 3 \le i \le n, odd \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_i b_{i+1}) = \begin{cases} |b_i - b_{i+1}|, & if\ \ 1 \le i \le n-1 \end{cases}$$
$$w_t(a_{2i} b_{i+1}) = \begin{cases} |a_{2i} - b_{i+1}|, & if\ \ 1 \le i \le n-1 \end{cases}$$
$$w_t(a_{2i-1} b_i) = \begin{cases} |a_{2i-1} - b_i|, & if\ \ 1 \le i \le n-1 \end{cases}$$
$$w_t(a_{2i-1} a_{2i}) = \begin{cases} |a_{2i-1} - a_{2i}|, & if\ \ 1 \le i \le n-1 \end{cases}$$

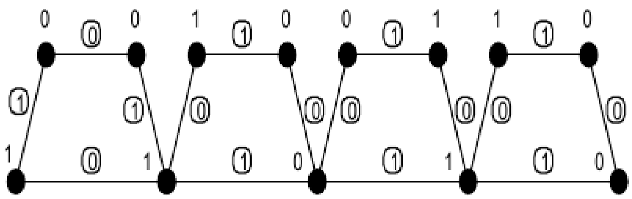

Figure 4: Quadrilateral Snake Graph QS_5

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.5 ***Double quadrilateral snake graph DQS_n, $n > 1$ is cordial graph.***

Proof: Now we label all vertices as

$$f(a_i) = \begin{cases} 0, & if\ \ i = 1,2 \\ 1, & if\ \ i \equiv 3(mod4), i \equiv 6(mod4) \\ 0, & otherwise \end{cases}$$

$$f(b_i) = \begin{cases} 1, & if \ \ i = 1 \\ 1, & if \ \ 2 \le i \le n, even \\ 0, & if \ \ 3 \le i \le n, odd \end{cases}$$

$$f(w_i) = \begin{cases} 1, & if \ \ i = 1, i \equiv 2(mod5), i \equiv 5(mod5) \\ 0, & otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_i b_{i+1}) = \begin{cases} |b_i - b_{i+1}|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(a_{2i} b_{i+1}) = \begin{cases} |a_{2i} - b_{i+1}|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(a_{2i-1} b_i) = \begin{cases} |a_{2i-1} - b_i|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(a_{2i-1} a_{2i}) = \begin{cases} |a_{2i-1} - a_{2i}|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(w_{2i-1} w_{2i}) = \begin{cases} |w_{2i-1} - w_{2i}|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(w_{2i} b_{i+1}) = \begin{cases} |w_{2i} - b_{i+1}|, & if \ \ 1 \le i \le n-1 \end{cases}$$

$$w_t(w_{2i-1} b_i) = \begin{cases} |w_{2i-1} - b_i|, & if \ \ 1 \le i \le n-1 \end{cases}$$

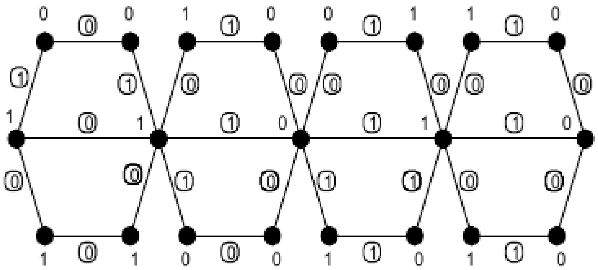

Figure 5: Double quadrilateral snake graph DQA_5

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.6 ***Quadrilateral snake graph*** QS_n, $n > 1$ ***with pendant edges is cordial graph.***

Proof: Now we define labeling for all vertices as

$$f(a_i) = \begin{cases} 0, & if \ \ i = 1,2 \\ 1, & if \ \ i \equiv 3(mod4), i \equiv 6(mod4) \\ 0, & otherwise \end{cases}$$

$$f(b_i) = \begin{cases} 1, & if \ \ i = 1 \\ 1, & if \ \ 2 \le i \le n, even \\ 0, & if \ \ 3 \le i \le n, odd \end{cases}$$

$$f(c_i) = \begin{cases} 0, & if \ \ i = 1 \\ 0, & if \ \ 2 \le i \le 2n-2, even \\ 1, & if \ \ 3 \le i \le 2n-2, odd \end{cases}$$

$$f(d_i) = \begin{cases} 1, & if \ \ 1 \le i \le n, odd \\ 1, & if \ \ i = 2 \\ 0, & if \ \ 4 \le i \le n, even \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(c_i a_i) = \begin{cases} |c_i - a_i|, & if \ \ 1 \le i \le 2n-2 \end{cases}$$

$$w_t(a_{2i-1}b_i) = \begin{cases} |a_{2i-1} - b_i|, & if\ 1 \le i \le n-1 \end{cases}$$

$$w_t(b_i d_i) = \begin{cases} |b_i - d_i|, & if\ 1 \le i \le n \end{cases}$$

$$w_t(b_{i+1}a_{2i}) = \begin{cases} |b_{i+1} - a_{2i}|, & if\ 1 \le i \le n-1 \end{cases}$$

$$w_t(b_i b_{i+1}) = \begin{cases} |b_i - b_{i+1}|, & if\ 1 \le i \le n-1 \end{cases}$$

$$w_t(a_{2i-1}a_{2i}) = \begin{cases} |a_{2i-1} - a_{2i}|, & if\ 1 \le i \le n-1 \end{cases}$$

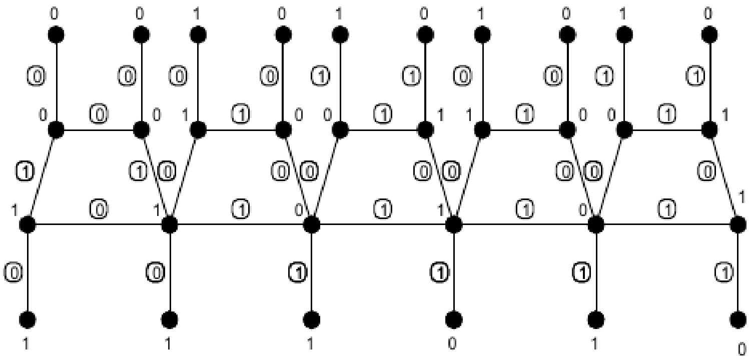

Figure 6: Quadrilateral snake graph with pendent edges QS_6

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

Theorem 3.7 ***Alternate quadrilateral snake graph AQS_n , $n \ge 4$ with pendant edges is cordial graph.***

Proof: Now we define labeling on all vertices as

$$f(a_i) = \begin{cases} 1, & if\ i \equiv 1(mod2), i \equiv 2(mod4) \\ 0, & if\ i \equiv 4(mod4) \end{cases}$$

$$f(b_i) = \begin{cases} 0, & if\ i \equiv 1(mod4), i \equiv 2(mod2) \\ 1, & if\ i \equiv 3(mod4) \end{cases}$$

$$f(c_i) = \begin{cases} 1, & if\ i \equiv 3(mod4) \\ 0, & Otherwise \end{cases}$$

$$f(d_i) = \begin{cases} 0, & if\ i \equiv 4(mod4) \\ 1, & Otherwise \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(c_i a_i) = \begin{cases} |c_i - a_i|, & if\ 1 \le i \le n \end{cases}$$

$$w_t(a_i b_i) = \begin{cases} |a_i - b_i|, & if\ 1 \le i \le n \end{cases}$$

$$w_t(b_i d_i) = \begin{cases} |b_i - d_i|, & if\ 1 \le i \le n \end{cases}$$

$$w_t(a_{2i-1}a_{2i}) = \begin{cases} |b_{2i-1} - a_{2i}|, & if\ 1 \le i \le \frac{n}{2} \end{cases}$$

$$w_t(b_i b_{i+1}) = \begin{cases} |b_i - b_{i+1}|, & if\ 1 \le i \le n-1 \end{cases}$$

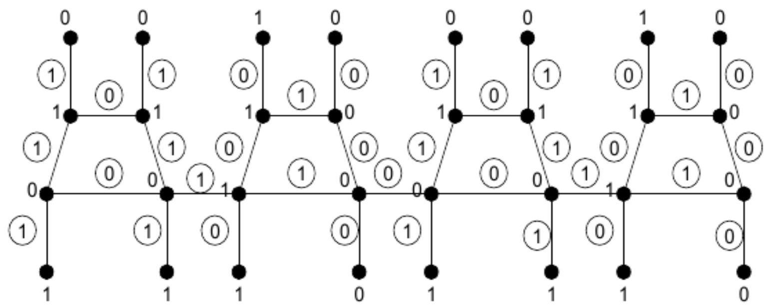

Figure 7: Alternate Quadrilateral Snake Graph with Pendent Edges AQS_8

Above calculations show that the difference between 0 labeled vertices(and edges) and 1 labeled vertices(and edges) is at most one. Hence it is cordial graph.

4. Conclusion

In this research paper, we proved that different families of snake graph such as triangular snake graph with pendant edges, alternate triangular snake graph with and without pendant edges, quadrilateral snake graph with and without pendant edges, alternate quadrilateral snake graph with pendant edges and double quadrilateral snake graph are cordial graphs as all the above mentioned graphs admit cordial labeling.

Conflicts of interest

All the authors declare that they have no conflicts of interest.

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