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On Finding Integer Solutions to Non-homogeneous Ternary Cubic Equation $3(x^2-y^2)+x-y=2z^3$

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ABSTRACT:

This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary cubic equation $3(x^2-y^2)+x-y=2z^3$. Different sets of integer solutions are illustrated.

Keywords: non-homogeneous cubic, ternary cubic, integer solutions

I. INTRODUCTION:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-24] for a few problems on ternary cubic equation with 3 unknowns. This paper concerns with yet another interesting ternary cubic diophantine equation with three variables given by $3(x^2-y^2)+x-y=2z^3$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS:

The non-homogeneous ternary cubic equation under consideration is

$$3(x^2-y^2)+x-y=2z^3 \tag{1}$$

Different methods of solving (1) are illustrated below:

Method 1:

Introduction of the linear transformations

$$x=u+v,\, y=u-v,\, z=v,\, u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$(6u+1)=v^2 \tag{3}$$

After performing a few calculations , it is seen that (3) is satisfied by

$$u=6s^2\pm 2s,\, v=6s\pm 1$$

In view of (2) , the corresponding two sets of integer solutions to (1) are exhibited below:

Set 1:

$$x = 6s^2 + 8s + 1, y = 6s^2 - 4s - 1, z = 6s + 1$$

Set 2:

$$x = 6s^2 + 4s - 1, y = 6s^2 - 8s + 1, z = 6s - 1$$

Method 2:

Introduction of the transformations

$$x = u^2 + v, y = u^2 - v, z = v \quad (4)$$

in (1) leads to

$$v^2 = 6u^2 + 1 \quad (5)$$

which is well-known pellian equation whose general solution is given by

$$v_n = \frac{f_n}{2}, u_n = \frac{1}{2\sqrt{6}} g_n \quad (6)$$

where

$$f_n = (5 + 2\sqrt{6})^{n+1} + (5 - 2\sqrt{6})^{n+1}, g_n = (5 + 2\sqrt{6})^{n+1} - (5 - 2\sqrt{6})^{n+1}$$

Substituting (6) in (4) , we have

$$x_n = \frac{1}{24} g_n^2 + \frac{f_n}{2}, y_n = \frac{1}{24} g_n^2 - \frac{f_n}{2}, z_n = \frac{f_n}{2}$$

which represent the integer solutions to (1) .

Method 3:

Introduction of the transformations

$$x = u^k + v^2, y = u^k - v^2, z = v \quad (7)$$

in (1) leads to

$$v = 6u^k + 1$$

In view of (7),the corresponding integer solutions to (1) are given by

$$x = 36u^{2k} + 13u^k + 1, y = -36u^{2k} - 11u^k - 1, z = 6u^k + 1$$

Method 4:

Employing the method of factorization , (1) is written as the system of double equations

as shown in Table 1 below:

Table 1-System of double equations

System	I	II
$3x+3y+1$	z^3	z^2
$x-y$	2	$2z$

Solving each of the above two systems of equations ,the corresponding integer solutions to (1) are obtained and they are presented below:

Solutions from System I:

$$x = 36s^3 - 90s^2 + 75s - 20, y = 36s^3 - 90s^2 + 75s - 22, z = 6s - 5$$

Solutions from System II:

Case 1:

$$x = 6\,s^2 - 4s - 1, y = 6\,s^2 - 16s + 9, z = 6s - 5$$

Case 2:

$$x = 6\,s^2 + 4s - 1, y = 6\,s^2 - 8s + 1, z = 6s - 1$$

Method 5:

Introducing the linear transformation

$$x = y + h, h \neq 0 \quad (8)$$

in (1) ,it it is written as

$$3h^2 + h(6y+1) - 2z^3 = 0 \quad (9)$$

Taking

$$z = 6y + 1 \quad (10)$$

in (9) and treating (9) as a quadratic in h , one has

$$h = \frac{(6y+1)}{6}(-1 + \sqrt{144y + 25}) \quad (11)$$

It is possible to choose y so that the square-root on the R.H.S. of (11) is removed and h is an integer .Substituting the values of y and h in (8) and (10) ,the corresponding values of x and z are obtained. A few examples are given below:

$$x = 15, y = 1, z = 7,$$

$$x = 4264, y = 50, z = 301,$$

$$x = -144421, y = 525, z = 3151$$

III. CONCLUSION:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with three unknowns given by $3(x^2 - y^2) + x - y = 2z^3$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables

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