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# **Content Knowledge and Pedagogical Content Knowledge in the Unit Graph of Ninth Grade Mathematics Textbook of West Bengal Board of Secondary Education**

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## **ABSTRACT**

Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) embedded in the ninth grade textbook of West Bengal Board of Secondary Education (WBBSE) are considered for present study. A CK-PCK framework is adopted for analyzing the text and tasks mentioned in the particular unit of the textbook. In this particular framework consistency and clearness of mathematical content, mathematical theoretical aspects, mathematical content development and connections, mathematical representations and their applications, use of proper language, mathematical tasks in the textbooks such as routine exercise, integration of the mathematics concepts and procedures, word problems or real world problems are discussed. It is found that logical sequence of presentation is consistent and knowledge elements are clearly represented. Theoretical background of the knowledge elements formed suitably. Conceptual hierarchy in content development is maintained. Methods and conventions are meaningfully represented and rules are accurately presented. Language of textual presentation is simple and appropriate. Tasks given in the unit should be revised. Some suggestions are made for further improvement of the textual presentation of this unit.

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Keywords: Mathematics Textbook, Ninth Grade, Graph, Content Knowledge, Pedagogical Content Knowledge.

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## **1. Introduction**

Textbooks are permanent, reliable and reusable source of teaching learning. Textbooks are especially helpful for beginning teachers because they cover and design each lesson which are carefully spelled out in detail and provide organized units of work. A textbook gives us all the plans and lessons we need to cover and it provides with a balanced, chronological presentation of information. Textbooks provide teachers and administrators a complete package. Good textbooks may be considered as excellent teaching aids and resource for both teachers and students. It can be undoubtedly opined that a textbook can play a central role in learners' success or failure (Mukundan, Hajimohammadi, and Nimehchisalem, 2011).

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## **2. Literature Review**

Shulman (1987) defined CK and PCK as two of the seven categories to provide a framework for teacher knowledge. Difference between general pedagogical knowledge and subject-matter-specific pedagogical knowledge is explained by Tamir (1988). A study on algebra textbook for CK and PCK was done by Wang Wei Sönnnerhed (2011). According to Huang & Kulm (2012), numerous mistakes were found during survey on knowledge for teaching the concept of function. Depaepe et al. (2013) studied mathematics education research by systematic review of the PCK. Different researchers like McCrory and Stylianides (2014), Bieda et al. (2014), Shepherd and Sande (2014), Otten et al. (2014), Kolloosche (2014) studied PCK in mathematics textbook. CK and PCK in mathematics textbook of WBBSE, West Bengal, India is studied by Sen and Samanta (2015a, 2015b, 2015c, 2015d) and Sen

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(2016). Concept of graph and its use in mathematics is quite useful for further development of knowledge element. Analysis of CK and PCK in textbook is quite helpful for development of strong sense of structural knowledge embedded in textbook. Present work aims to develop the textual presentation of the unit graph.

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### 3. Objectives of the study

The objectives of this study are to explore the CK and the PCK regarding graphical presentation of point and straight line rooted in the ninth grade mathematics textbook used for WBBSE.

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### 4. Methodology

To explore CK-PCK structure of mathematics text, Van Dormolins' (1986) classification of teaching perspectives and learning perspectives of Schmidt et al. (1997) are considered.

CK-PCK overall framework on textual presentation may be considered as follows:

1. Consistency and Clearness of Mathematical Content: Text should be consistent and clear to the reader. Computation and logic must be errorless. Proofs might be incomplete, but not false (Van Dormolen, 1986).
2. Mathematical Theoretical Aspects: Mathematical theorems, rules, definitions, methods and conventions are concerned as knowledge elements in this section. According to Van Dormolen (1986), those are termed as "kernels".
3. Mathematical Content Development and Connections: To explore the embedded teaching trajectory, Schmidt et al. (1997) suggested for investigation of interdependency of mathematical content in the text.
4. Mathematical Representations and Applications: Both formal view and informal view are considered for analysis of text. A set of concepts, rules, theorems and structures are considered as formal view, on the other hand informal view deals with generalizing, classifying, ordering, abstracting, exploring patterns and so on. (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin et al., 2001; Van Dormolen, 1986; Vergnaud, 1987).
5. Language Use: Use of language for explanation and illustration of mathematical theorems, definitions, and rules are considered in this section. To make sense for a student reader, mathematical language pedagogically combined with everyday language.
6. Mathematical tasks in the textbooks (Brändström, 2005) may analyzed according to the following points:
  - A. Routine Exercises: here, newly presented mathematical concepts, rules or algorithmic procedures explained in examples, in order to get acquainted with the content.
  - B. To integrate the mathematics concepts and procedures (Hiebert& Carpenter, 2007; Hiebert&Lefevre, 1986) and to evaluate, analyze and reason mathematically suitable exercise should be incorporated.
  - C. Word Problems or Real World Problems: to bring reality into the texts in mathematics and to create occasions for learning and practicing the different aspects of applied problem solving without the practical contact with the real world situation (Chapman, 2006).

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### 5. Results and Discussions

Name of the ninth grade mathematics text book of WBBSE is 'GanitPrakash (class IX)'. In this book, content of unit III is graph.

Representation of the knowledge element in this unit is as follows:

- At the very beginning of the unit a circular area (playground) is selected to fix the position of a player (locating a point in two dimensional space).
- Name of coordinate geometry and Rene Descartes is mentioned.
- Two perpendicular axes is mentioned but not shown by figure.
- Concept of four quadrants is explored.
- How to locate different points in different quadrant is explained graphically.
- Different points (x, y) are placed in four quadrants.
- Graphical representation of an equation in two variables is discussed. In this context why three points are used to draw a line on graph paper is mentioned.
- Graphical representation of an equation  $ax+by+c=0$  is a straight line is mentioned.
- Feasibility of graphical representation of an equation under certain condition is stated viz number of exercise book or pencil cannot be negative.
- Graphical solution of two simultaneous linear equations in two variables is discussed with suitable examples.
- Graphs like  $x=\text{constant}$  and  $y=\text{constant}$  are drawn with example  $x=0$ ,  $y+7=0$  and  $y=0$  (mentioned only). Graph of the equation  $x-a=0$  is mentioned but not illustrated.
- Concept of x-intercept and y-intercept are illustrated.
- Area of the triangle formed by two axis and graphical presentation of a line is illustrated.

- Linear equation in two variables for which abscissa and ordinate cannot be expressed by integer are discussed (As this concept is not included in the syllabus of class IX, further discussion about this topic is avoided).

#### Consistency and Clearness of Mathematical Content:

Textual presentation of content is consistent and clear to the reader. Logical sequence of presentation is consistent and knowledge elements are clearly represented in the text.

#### Mathematical Theoretical Aspects:

Method and convention of representation in text is quite meaningful. Rules are rightly presented. Theoretical background of the knowledge elements formed properly. So, 'Kernel' is developed rightly.

#### Content Development and Connections:

Content is developed by maintaining conceptual hierarchy. Equations are connected to graphs, graphs are used to represent price, velocity, time etc. with the help of equations.

#### Representations and Applications:

Both formal view and informal view are presented in the text.

#### Language:

Language used in text is simple and easily comprehensible to the students of 9<sup>th</sup> standard. Daily life experiences are transformed into mathematical language using appropriate knowledge elements.

To analyze the mathematical exercise represented in this unit it is observed that

- Routine Exercises: Newly presented mathematical concepts are included in the exercises.
- Reflection of analysis, mathematical reason and evaluation of the Concepts presented in the text: Every point mentioned in these criteria is included in the exercise.
- Sequence of Questions: Questions in the exercise are sequentially arranged.
- Word/Real World Problems: The view of mathematical applications in real-life situation is reflected. There are many problems included in the exercise but actual real world problems are less in number. So, it is suggested that word problems should be increased.

**Table1. Examples and exercises presented in Unit III.**

Concept in Content	Examples	Exercises		
		Routine	Evaluate, Analyze and Reason mathematically	Word Problem
Fixing location in plane	1	0	0	0
Cartesian System	4	3	5	0
Linear equation	2	0	4	4
Linear graph	8	11	0	5
Graphical solution of two simultaneous linear equations	4	11	5	5
Area of triangle	1	2	0	0

Some suggestions are listed below for improvement for the textual representation.

1. Example of playground should be rectangular for formation of concept. A surface is a two dimensional space this idea should be clear. Practically a cricket ground is circular or elliptic but football ground is rectangular. So, it should be more suitable to choose a football ground as example. A discussion on axis of reference may be included. Why one axis is not sufficient to locate a point uniquely in two dimensional spaces is also an important knowledge element and should be discussed.
2. Graph paper is very important teaching learning material for this unit. Why graph paper is necessary is not mentioned. What type of graph paper should be used is not discussed.
3. Pictures of the graphs are not properly drawn. X-axis and Y-axis should be prominent and unwanted bold lines should be omitted. Also, size of the graphs should be large.
4. All possibilities of  $x = \text{constant}$  and  $y = \text{constant}$  are not discussed. Actually  $x = \text{constant}$  may be considered as  $x = 0$ ,  $x = a$  and  $x = -a$  where  $a > 0$ . Here,  $x = -a$  where  $a > 0$  is not mentioned or discussed. Similarly, Actually  $y = \text{constant}$  may be considered as  $y = 0$ ,  $y = b$  and  $y = -b$  where  $b > 0$ . Here,  $y = -b$  where  $b > 0$  is not mentioned or discussed.
5. In case of formation of triangle by two axis and line, at first line is drawn and meeting points of the line, x-axis and y-axis are observed. Here, a simple idea may be incorporated. It may be viewed as putting  $y = 0$  in a particular equation we get x-intercept and similarly putting  $x = 0$  we get

y-intercept.

Let us consider an equation  $3x+4y=24$ :

Putting  $y=0$  we get  $x=8$ , so  $(8, 0)$  is a point on the line as well as on x-axis.

Similarly, putting  $x=0$  we get  $y=6$ , so  $(0, 6)$  is a point on the line as well as on y-axis.

Now length of base = 8 units and

Length of perpendicular =6 units.

So, area =  $\frac{1}{2} \times 8 \times 6 \text{ unit}^2 = 24 \text{ unit}^2$ .

6. Possibilities of other types of triangle should be discussed. Triangle is formed by three lines which are the graphical representation of three different equations. Using this knowledge element an exercise is included but it was not discussed in the text. A clear concept about the knowledge element – ‘a triangle may be formed in four different quadrants’ may be incorporated in text.

Following example may be incorporated:

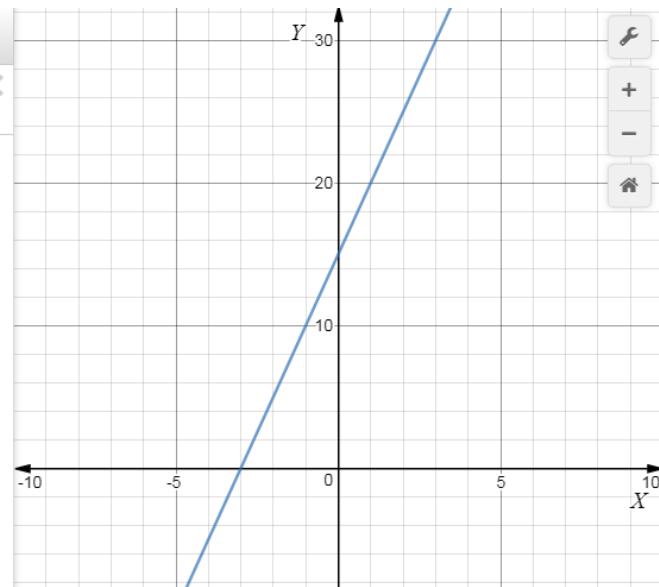


Figure1: Triangle formed by two axis and line  $(y=5x+15)$  in second quadrant

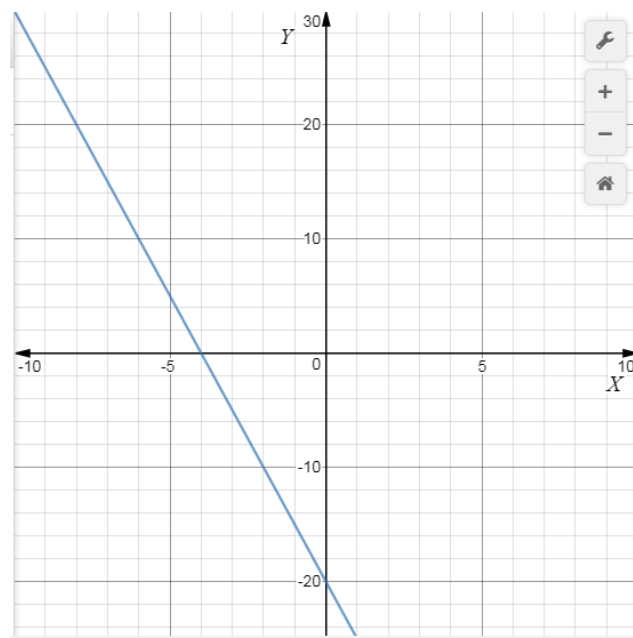


Figure 2: Triangle formed by two axis and line  $(y= -5x-20)$  in third quadrant

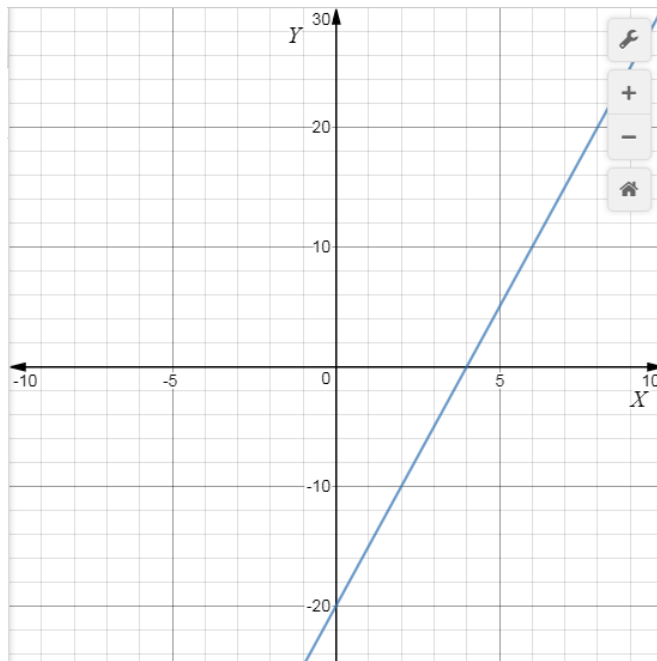


Figure 3: Triangle formed by two axis and line ( $y= 5x-20$ ) in fourth quadrant

These graphs are plotted by Graph Plotter :: An Online Graphing Calculator on 20.07.2021 (<https://www.transum.org/Maths/Activity/Graph/Desmos.asp>)  
 Every line does not form this type of triangle. Conditions of formation of such triangles should be discussed. Especially line parallel to both the axis does not form such triangle. Also in some cases line intersect one the axis at the point which is far away from origin. An example may be sited as below:

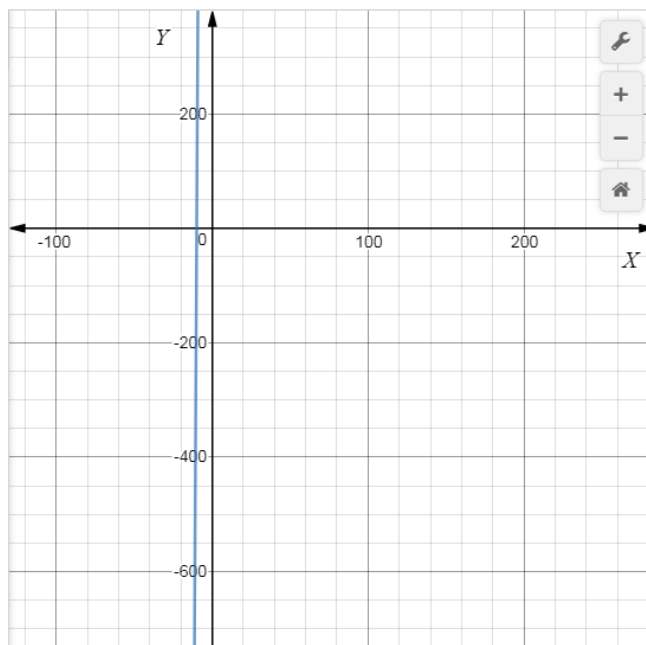


Figure 4: Graph of line  $y=500x+5000$

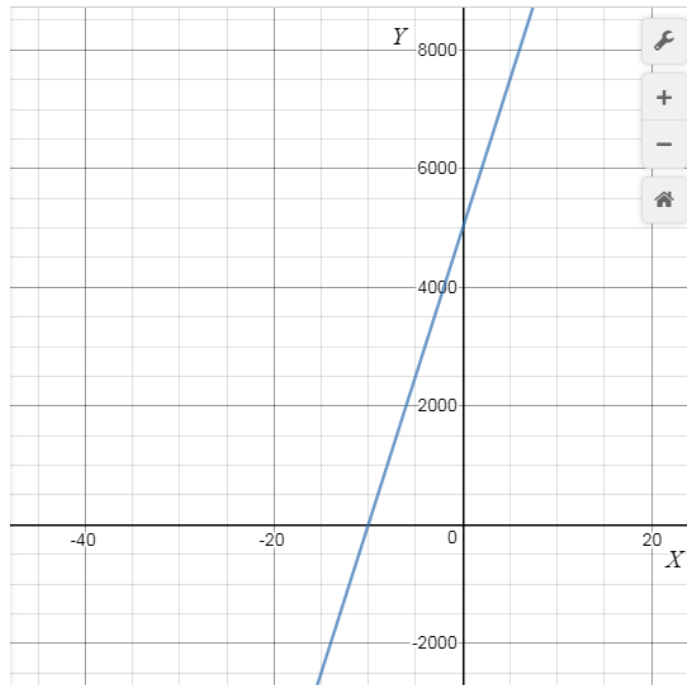


Figure 5: Graph of line  $y=500x+5000$  after choosing the appropriate length of the axis

These graphs are plotted by Graph Plotter :: An Online Graphing Calculator on 20.07.2021 (<https://www.transum.org/Maths/Activity/Graph/Desmos.asp>)

## Concluding remarks

Every book should have a target population of readers. Here, more than ten lakh students and teachers are using this book for transection of knowledge elements in mathematics. As it is a textbook for class IX, importance of this book in the purview of CK-PCK unquestionable. It may be concluded that this book is good because majority of the conditions according to the framework of analysis are satisfactory. For improvement of textual presentation and tasks some suggestions are made.

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