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Adomian Decomposition Method for a class of fractional Nonlinear Programming Problem

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ABSTRACT

In this paper, the solution of mathematical model for a class of fractional nonlinear programming problems is obtained. Adomian Decomposition Method is used to solve the problem.

Keywords: Nonlinear problem, Adomian Decomposition Method

1.Introduction

A fractional nonlinear programming problem is one of the classical optimization problems. Most scientific problem occurs nonlinearly. Adomian [1, 2] introduced, Adomian Decomposition Method for solving nonlinear functional equations. He solved nonlinear functional equations in a series of functions. Polynomials generated from an expansion of analytic function are used to obtain each term of the series. Many authors used Adomian Decomposition Method for solving different types of equations. Abboui and Cherruault [3] solved nonlinear equations using Adomian decomposition method. Dehghan and Tatari [4] used Adomian Decomposition method for solving problems in Calculus of variations. Sanchez [5] applied Adomian Decomposition method for a class of nonlinear problems. Ahmed and Ekhlass [6] used Adomian Decomposition Method with Modified Bernstein Polynomials for Solving Ordinary and Partial Differential Equations. Zeidan, Chau and Lu [7] provide a synthesis toward understanding ADM to resolve the Riemann problem. Li and Pang [8] worked on the Application of Adomian Decomposition method to nonlinear systems. Alkarawi and Inaam [9] used Adomian Decomposition method to solve linear and non-linear partial differential equations such as (the linear wave equation, nonlinear Klein Gordon equation, the non-linear wave equation, the diffusion equation and some non-linear partial differential equations). Dalal Adnan Maturi, Honaida Mohammed Malaikah [10] applied Adomian Decomposition Method for Solving Nonlinear Partial Differential Equation Using Maple.

The objective of this paper is to find the solution of a special class of fractional nonlinear programming problem $\frac{dy}{dx} = \frac{f(x)+e^{-y(x)}}{g(x)}$. We have applied Adomian Decomposition method to solve this special class of problem.

Solution Method

$$\frac{dy}{dx} = \frac{f(x)+e^{-y(x)}}{g(x)}, \qquad y(x_0) = 0 \qquad ...(2.1)$$

Where f(x) and g(x) are continuous functions, defined in an open interval $I \subset R$ and $x_0 \in I$

$$\text{Let } \frac{dy}{dx} = \frac{f(x)+e^{-y(x)}}{g(x)} = R^0$$

Now the problem (2.1) can be written as

$$\frac{dy}{dx} = f(x) + e^{-y(x)} - R^0 g(x)$$

In the operator form, the above problem becomes

$$Ly = f(x) + e^{-y(x)} - R^0 g(x)$$

or

$$y(x) = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} e^{-y(s)}ds - R^0 \int_{x_0}^{b} g(s)ds \qquad ...(2.2)$$

To set up the Adomian Method, consider y in the series form

$$y(x) = y_0 + \sum_{n=1}^{\infty} y_n(x) \qquad ...(2.3)$$

And write the nonlinear function $N(y) = e^{-y}$ as the series of functions

$$N(y) = e^{-y} = \sum_{n=0}^{\infty} N_n(y_0, y_1, ..., y_n) \qquad ...(2.4)$$

$N_n(y_0, y_1, ..., y_n)$ is obtained by

$$N_n = \frac{1}{n!}\frac{d^n}{d\in^n}\left[N\left(\sum_{I=0}^{N} \in^i y_i\right)\right]_{\in=0}$$

Substituting (2.3) and (2.4) in equation (2.2)

$$y_0 + \sum_{n=1}^{\infty} y_n(x) = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \sum_{n=0}^{\infty} N_n(y_0, y_1, ..., y_n)ds - R^0 \int_{x_0}^{b} g(s)ds \qquad ...(2.5)$$

Based on the Adomian Decomposition Method, we construct the solution y(x) as $y = \lim_{n\to\infty} \varphi_n$

where the *n+1* term of the solution is defined as

$$\varphi_n = \sum_{i=0}^{\infty} y_i(x)$$

By applying the Adomian decomposition procedure, we find the series solution of y(x).

$$y_0 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} N(y_0)ds - R^0 \int_{x_0}^{b} g(s)ds$$

$$y_1 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \left(\frac{d}{d\lambda} N(y_0 + \lambda y_1)\right)ds - R^0 \int_{x_0}^{b} g(s)ds$$

$$\Rightarrow y_1 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} y_1 N'(y_0)ds - R^0 \int_{x_0}^{b} g(s)ds$$

$$y_2 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \left(\frac{1}{2!}\frac{d^2}{d\lambda^2} N\left(y_0 + \lambda y_1 + \lambda^2 y_2\right)\right)ds - R^0 \int_{x_0}^{b} g(s)ds$$

$$\Rightarrow y_2 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \left(y_2 N'(y_0) + \frac{1}{2} y_1^2 N''(y_0)\right)ds - R^0 \int_{x_0}^{b} g(s)ds$$

Similarly

$$y_3 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \left(\frac{1}{3!} \frac{d^3}{d\lambda^3} N\left(y_0 + \lambda y_1 + \lambda^2 y_2 + \lambda^3 y_3 \right) \right) ds - R^0 \int_{x_0}^{b} g(s)ds$$

$$\Rightarrow y_3 = \int_{x_0}^{b} f(s)ds + \int_{x_0}^{b} \left(y_3 N'(y_0) + y_1 y_3 + \frac{1}{3!} y_1^3 N'''(y_0) \right) ds - R^0 \int_{x_0}^{b} g(s)ds$$

The solution is given by $\varphi_n = y_0(x) + y_1(x) + y_2(x) + ... + y_n(x) + ...$

Conclusion

In this article, Adomian decomposition method for approximating the solutions for a class of fractional Nonlinear Programming Problem is implemented. The ADM is clearly very efficient and powerful technique in finding the numerical solutions of the nonlinear equation.

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