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Markov Chain: Types and its Applications

A.R.Muralidharan

Assistant Professor, Department of statistics, Debra Berhan University, Ethiopia
Email: mdharan75@gmail.com

ABSTRACT

Today the buzzword is "Data science and its domains". It deals with huge data and to generate this data there are several statistical and mathematical techniques were involved. One such technique is the Markov chain. In this article author deals with some of its importance and applications.Markov chain and its applications are taken a remarkable place in all disciplines in this modern world. It is a special case of a stochastic process on the infinite sequence. Markov chains are stochastic cycles that are portrayed by their memoryless property, where the likelihood of the interaction being in the following condition of the framework relies just upon the present status and not on any of the past states. This property is known as the Markov property.This Markov chain will be very useful in "Bayesian concepts". The main purpose of this article is to make familiarize the scholars in Markovian concepts and its applications.In this article, the author intended to explore the types of Markov chain and their applications. After viewing this article the readers might get some knowledge in Markovian applications

Key words: Stochastic,Markov property, Markov chain, Transition network**,** infinite sequence

Introduction:

A Markoff process is an important resource employed in numerous fields as well as actual science, financial issue, planning, innate characteristics, to mention the smallest amount. it's comprehensively used attributable to its straightforwardness and adaptability.
To do any weak events, there'll be an idea known as "Probability". it's overseen freelance events measures. These cycles relied upon customary applied mathematics nearby experiences. A progression of likelihood investigations shapes a independent starters measure; the perfect outcomes for every outcome square measure identical occur with a comparable likelihood. These square measure key on 2 speculations: Law of huge Numbers and Central limit theory.
Current likelihood speculation helps on credibleness measures that the information on previous outcomes for the figure of future events.. On a serious level, at no matter purpose we tend to notice a gathering of likelihood preliminaries, the add of the past outcomes might have an effect on our conjectures for the incidental to assessment.
A random cycle could be a numerical model for a game-plan of flighty segments. The model got to allow computing the chance of various occasions associated with a self-self-assured surprise.

Markov Chain

A Markov chain is a straightforward a progression of self-assertive characteristics wherein the accompanying worth is some way or another or another ward on the current worth, rather than being absolutely sporadic. To get tests from models, one necessities to address the structure direct and its objectives in a non-obscure way.
A Markov model is a stochastic model used to show unpredictably changing constructions where it is typical that future states rely simply on the current status not on the occasions that happened before it (that is, it recognizes the Markov property). For what it's worth, this suspicion empowers thinking and assessment with the model that would somehow or another be unmanageable
The Markov model is investigated to pick such measures as the likelihood of being in a given state at a given point on schedule, the extent of time a framework is expected to spend in a given state, correspondingly as the regular number of changes between states: for example keeping an eye on the measure of disappointments and fixes.
Markov model is a stochastic model used to show sporadically changing designs where it is ordinary that future states rely simply on the current status not on the occasions that happened before it (that is, it expects the Markov property).
A Markov chain is a fundamental resource used in various fields including actual science, monetary issue, planning, genetic characteristics, to say the least. It is comprehensively used because of its straightforwardness and versatility.
To do any weakness events, there will be a thought called "Probability". It has overseen independent events measures. These cycles relied upon customary probability theory close by experiences. A progression of chance examinations shapes a self-governing starters measure; the ideal outcomes

for each outcome are the identical occur with a comparative probability. These are basic on two speculations: Law of Large numbers and Central limit theory.

Current probability speculation helps on plausibility measures for which the data on previous outcomes for the gauge of future events.. On an essential level, at whatever point we notice a gathering of chance preliminaries, the aggregate of the past outcomes could affect our figures for the accompanying assessment.

A stochastic cycle is a numerical model for a game-plan of unusual parts. The model ought to permit figuring a likelihood of different occasions related to a self-confident wonder.

Definition:

The Markov property

$$P(X_n = i_n \mid X_{n-1} = i_{n-1}) = P(X_n = i_n \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}).$$

A Markov chain is a model of the random motion of an object in a discrete set of possible locations. Two versions of this model are of interest to us: discrete time and continuous time

In discrete time, the position of the object–called the *state* of the Markov chain–is recorded every unit of time, that is, at times 0, 1, 2, and so on. In continuous time, the state is observed at all times $t \geq 0$. One can think of the continuous-time model as being a discrete time model where the time unit is infinitesimally small. The state of the Markov chain changes randomly. In discrete time, there is a die at every location. Every time unit, the Markov chain tosses the die at its current location to decide where to jump next. In that way, the law of the future motion of the state depends only on the present location and not on previous locations. This key property that the Markov chain has of "forgetting" its past locations greatly simplifies the analysis.

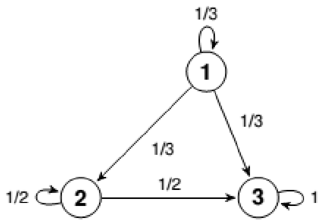

Figure 1: Transition diagram for discrete time Markov chain with three states

A square array is called the Matrix of transition probabilities or the **transition matrix**. It contains probabilities for various kinds of events occurred. The question of determining the probability that the given chain is in state *i* today and in state *j*

- To *completely* describe a Markov chain, we must specify the transition probabilities,
  $p_{ij} = P(X_{t+1}=j \mid X_t=i)$ in a one-step transition matrix, P:

$$P = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0M} \\ p_{10} & p_{11} & \cdots & \cdots \\ \cdots & \cdots & \cdots & p_{(M-1)M} \\ p_{M0} & p_{M1} & \cdots & p_{MM} \end{bmatrix} \quad \rightarrow \quad (1)$$

A Markov chain is a special kind of stochastic interaction, which manages portrayal of successions of arbitrary factors. Extraordinary interest is paid to the dynamic and the restricting practices of the arrangement. A stochastic cycle can be characterized as an assortment of irregular amounts $\{\theta^{(t)} : t \in T\}$for some set T.

The set $\{\theta^{(t)} : t \in T\}$is supposed to be a stochastic interaction with state space S and list (or boundary) set T. All through this book the file set T is taken as countable, characterizing a discrete time stochastic interaction. Without loss of over-simplification, it will be thought to be the arrangement of regular numbers N and will generally address the cycles of a reenactment conspire. The state space will overall be a subset of R^d addressing the help of a boundary vector.

Definition: Transition probabilities

In easy way, a Markov chain is a stochastic process where given the present state, past and future states are independent. This property can be more formally stated through

$$P(\theta^{(n+1)} \in A \mid \theta^{(n)} = x, \theta^{(n-1)} \in A_{n-1, \ldots\ldots,} \quad \theta^{(0)} \in A_0$$

$$= P(\theta^{(n+1)} \in A \mid \theta^{(n)} = x) \rightarrow (2)$$

For all sets $A_0A_1 \dots\dots\dots\dots\dots .. A_{n-1}$,A C S and $x \in S$. The Markovian property (2) can also be established in the equivalent forms:

1. $E[f(\theta^{(n)})|\theta^{(m)}, \theta^{(m-1)}, \dots\dots, \theta^{(0)}] = E[f(\theta^{(n)})|\theta^{(m)}]$ for all bounded functions f and n $> m \geq 0$;
2. P $\left((\theta^{(n+1)} = y|\theta^{(n)} = x,\ \theta^{(n-1)}\right) = x_{n-1,\dots\dots\theta^{(0)}} = x_0$) =p($\theta^{(n+1)} = y|\theta^{(n)} = x$) for all $x_0x_1, \dots\dots.., x_{n-1}$ $x, y \in S$

   The above is appropriate for discrete state spaces. Even it is more appropriate than (2) in this case and is used as defining Markov chains for the initial stages. Commonly the probabilities expressed in (2) depend on x. A and n. When they are not depend on n, the chain is said to be homogeneous. The transition function or kernel P(x,A) in this case will be defined as:

   a. For all $x \varepsilon S$, P(x,.) is a probability distribution over S;
   b. For all A C S, function $x \to P(x, A)$ can be evaluated

It is useful when dealing with discrete state space of identify P(x.{y}) = P(x,y). This function is called a "Transition probability" and it will be satisfy the following conditions:

i. P(x,y) $\geq 0\ \forall x, y \in S$; and
ii. $\sum_{y \in S}$ P(x, y) $= 1 \forall x \in S$;

As any probability distribution P(x.) should.

Applications

1. Consider the discrete-time Markov chain with three states corresponding to the transition diagram on figure1. Assume that the initial distribution of X(0) is given $\emptyset(1) = \emptyset(2) = \frac{1}{2}$.

   Let us compute the following

   a. P(X(0) =1, X(1)=2, X(2)=3)
   b. P(X(2)= i , for i=1,2,3
   c. P (M3=2) consider M3= inf{n>0|X(n)=3} is the time for the first visit to node 3(state).

Solution

a. A discrete time Markov chain on a countable set S is a stochastic process satisfying the Markov property, then $P(X(n) = i_n\, |X(n-1) = i_{n-1,\dots.}\, X(0) = i_0\,) =$

$$P(X(n) = i_n\, |X(n-1) = i_{n-1} \to (3)$$

For any $i_n\ ,i_{n-1}, \dots\dots\dots\dots\dots\dots\dots.\, i_0 \in S\ and\ n \in N.$

This can be expressed, $P_{i,j}\ (n-1) = P(X(n) = j\ |X(n-1) = i$, it is able to observe that a Markov chain the formula for the point probabilities simplifies as:

$$P(X(n) = i_n\, |X(n-1) = i_{n-1,\dots.}\, X(0) = i_0\,) =$$

$$P_{i(n-1),i(n)}(n-1).\ P_{i(n-2),i(n-1)}(n-2)\,.\, P_{i0,i1}(0).\, P(X(0) = i_0 \to \ (4)$$

The final version of the above (4) by considering time-homogeneous Markov chain for which the transition probabilities $P_{i,j}$(n) = P_{ij} do not dependent on the time index n N. For a discrete time and time-homogeneous Markov chain on S have the following expression:

$P(X(n) = i_n\, , \dots\dots\ X(0) = i_0\,) = P_{i(n-1),i(n)}(n-1).\ P_{i(n-2),i(n-1)}(n-2)\,.\, P_{i0,i1}.\, \emptyset(i0) \to\ (5)$

Where the final expression as $\emptyset(i0) = P(\,X(0) = i_0\,)$ for the initial distribution of X(0).

By using (5) for the point probabilities of a discrete time Markov chain, get

$$P(X(0) = 1, X(1) = 2, X(2) = \ 3) = \emptyset(1).\, P_{1,2}P_{2,3} = \frac{1}{2}\frac{1}{3}\frac{1}{2} = \frac{1}{12}$$

b. The distribution of X(2) , is easily computing by using the matrix formula of n-step transition probabilities for a Markov chain on a finite state space , S ={1,2,…..N} , with transition probability matrix P and initial distribution $\bar{\varphi} = (\varphi(1), \dots\dots, \varphi(N))$ as row vector then the distribution of X(n) is given by

   (P(X(n)=1),……P(X(N)) $= \bar{\varphi}P^n \to (6)$

   (P(X(2)=1,P(X(2)=2),P(X(2)=3))

$$= \begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}\begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{2} & \frac{1}{2}\\ 0 & 0 & 1\end{pmatrix}^2 = \begin{pmatrix}\frac{1}{18} & \frac{19}{72} & \frac{49}{72}\end{pmatrix}$$

c. P(m3=2) consider M3= inf{n>0|X(n)=3} is the time for the first visit to node 3(state).

The event M3=2 express that the first visit to state 3 must happen exactly at time n=2, and there were three probabilities for the value of X(0), X(1),X(2) for which the first visit to state 3 happens at time 2

((X(0)=1, X(1)=1,X(2)=3)

((X(0)=1, X(1)=2,X(2)=3)

((X(0)=2, X(1)=2,X(2)=3)

P(M3=2) =P ((X(0)=1, X(1)=1,X(2)=3) + P((X(0)=1, X(1)=2,X(2)=3) +P ((X(0)=2, X(1)=2,X(2)=3)

$$=\varphi(1).P_{1,1}P_{1,3} + \varphi(1).P_{1,2}P_{2,,3} + \varphi(1).P_{2,2}\,P_{2,3}$$

$$=\frac{1}{2}\frac{1}{3}\frac{1}{3}+\frac{1}{2}\frac{1}{3}\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}=\frac{1}{18}+\frac{1}{12}+\frac{1}{8}=\frac{19}{72}$$

The Model

Formally, a Markov chain is a probabilistic automaton. The probability distribution of state transitions is typically represented as the Markov chain's ***transition matrix***. If the Markov chain has **N** possible states, the matrix will be an **N x N** matrix, such that entry **(I, J)** is the probability of transitioning from state **I** to state **J**. Additionally, the transition matrix must be a **stochastic matrix**, a matrix whose entries in each row must add up to exactly 1. This makes complete sense, since each row represents its own probability distribution.

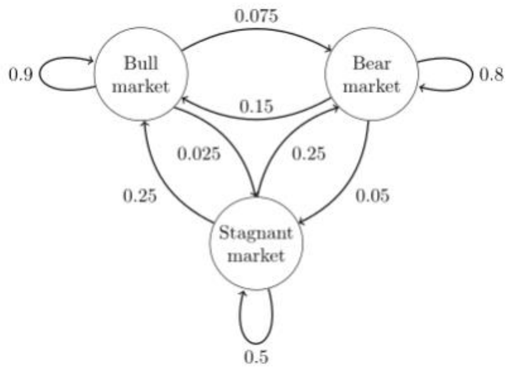

Figure 2: Transition matrix

General view of a sample Markov chain, with states as circles, and edges as transitions

$$P=\begin{bmatrix}0.9 & 0.075 & 0.025\\ 0.15 & 0.8 & 0.05\\ 0.25 & 0.25 & 0.5\end{bmatrix}.$$

Sample transition matrix with 3 possible states

Additionally, a Markov chain also has an ***initial state vector***, represented as an **N x 1** matrix (a vector), that describes the probability distribution of starting at each of the **N** possible states. Entry **I** of the vector describes the probability of the chain beginning at state **I**.

$$x^{(0)}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

Initial State Vector with 4 possible states

These two segments are typically everything expected to address a Markov chain.

We as of now recognize how to acquire the opportunity of changing start with one state then onto the accompanying, yet shouldn't something be said about tracking down the opportunity of that progress happening over different strategies? To formalize this, we as of now need to pick the likelihood of moving from state I to state J over M advances. Suddenly, this is when in doubt easy to discover. Given a progression lattice P, this can be compelled by preparing the appraisal of territory (I, J) of the organization got by raising P to the force of M. For little appraisals of M, this should effortlessly be possible by hand with rehashed development. Regardless, for colossal appraisals of M, in the event that you consider crucial Linear Algebra, a more productive approach to manage raise a framework to a force is to first diagonalizable the structure.

Conclusion

Since you know the basics of Markov chains, you should now have the decision to handily execute them in a language of your decision. In the event that coding isn't your specialty, there are in addition a lot additionally made properties of Markov chains and Markov cycles to plunge into. As it might suspect, the average improvement along the theory course would be toward Hidden Markov Processes or MCMC. Principal Markov chains are the plan squares of other, seriously puzzling, showing methodologies, so with this information, you would now have the alternative to move onto different philosophy inside subjects, for example, conviction appearing and looking at.

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