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SOME TYPES OF $\delta\omega$ -CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets called $\delta\omega$-closed sets in topological spaces. This class lies between the class of δ-closed sets and the class of δg-closed sets.

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1. INTRODUCTION

In 1963 Levine introduced the notion of semi-open sets. Velicko introduced the notion of δ-closed sets and it is well known that the collection of all δ-closed sets of a topological space forms a topology and is denoted by $\tau\delta$. Levine also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful.

After the advent of g-closed sets, Arya and Nour, Sheik John and Dontchev introduced gs-closed sets, ω-closed sets and gsp-closed sets respectively.

In this paper, we introduce a new class of sets called $\delta\omega$-closed sets in topological spaces. This class lies between the class of δ-closed sets and the class of δg-closed sets.

2. PRELIMINARIES

Throughout this thesis (X, τ) and (Y, σ) (or X and Y) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c or X | A denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1

A subset A of a space (X, τ) is called:

(i) semi-open set if $A \subseteq cl(int(A))$;

(ii) preopen set if $A \subseteq int(cl(A))$;

(iii) α-open set if $A \subseteq int(cl(int(A)))$;

(iv) β-open set (= semi-preopen) if $A \subseteq cl(int(cl(A)))$;

(v) regular open set if $A = int(cl(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

The preclosure (resp. semi-closure , α-closure , semi-pre-closure of a subset A of X, denoted by pcl(A) (resp. scl(A), αcl(A), spcl(A)), is defined to be the intersection of all preclosed (resp. semi-closed, α -closed, semi-preclosed) sets of (X, τ) containing A. It is known that pcl(A) (resp. scl(A), αcl(A), spcl(A)) is a preclosed (resp. semi-closed, α -closed, semi-preclosed) set.

Definition 2.2

A point x of a space X is called a θ-adherent point of a subset A of X if $cl(U) \cap A \neq \phi$, for every open set U containing x. The set of all θ-adherent points of A is called the θ-closure of A and is denoted by cl_θ(A). A subset A of a space X is called θ-closed if and only if A = cl_θ(A). The complement of a θ-closed set is called θ-open. Similarly, the θ-interior of a set A in X, written int_θ (A), consists of those points x of A such that for some open set U containing x, $cl(U) \subseteq A$. A set A is θ-open if and only if A = int_θ (A), or equivalently, X \ A is θ-closed.

A point x of a space X is called a δ-adherent point of a subset A of X if $int(cl(U)) \cap A \neq \phi$, for every open set U containing x. The set of all δ-adherent points of A is called the δ-closure of A and is denoted by cl_δ (A). A subset A of a space X is called δ-closed if and only if A = cl_δ (A). The complement of a δ-closed set is called δ-open. Similarly, the δ-interior of a set A in X, written int_δ (A), consists of those points x of A such that for some regularly open set U containing x, U $\subseteq$ A. A set A is δ-open if and only if A = int_δ (A), or equivalently, X \ A is δ-closed.

The family of all θ-open (resp. δ-open) subsets of (X, τ) forms a topology on X and is denoted by τ_θ (resp. τ_δ). From the definitions it follows immediately that $\tau_\theta \subseteq \tau_\delta \subseteq \tau$. [9].

Definition 2.3

A point x ∈ X is called a semi θ-cluster [9] point of A if A ∩ scl(U) ≠ ϕ for each semi-open set U containing x.

The set of all semi θ-cluster points of A is called the semi-θ-cluster of A and is denoted by sclθ (A). Hence, a subset A is called semi-θ-closed if sclθ (A) = A. The complement of a semi-θ-closed set is called semi-θ-open set.

Recall that a subset A of a space (X, τ) is said to be δ-semi-open [20] if A ⊆ cl(intδ (A)).

Definition 2.4

A subset A of a space (X, τ) is called:

(i) a generalized closed (briefly, g-closed) set if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(ii) a generalized semi-closed (briefly, gs-closed) set if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(iii) an α-generalized closed (briefly, αg-closed) set if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(iv) a generalized semi-preclosed (briefly, gsp-closed) set if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(v) a generalized preclosed (briefly, gp-closed) set if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(vi) a regular generalized closed (briefly, rg-closed) set if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).

(vii) a δ-generalized closed (briefly, δg-closed) set if cl_δ (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(viii) a ĝ-closed set (= ω-closed set) if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).

The complement of $\hat{g}$ −closed set is called $\hat{g}$ −open set. The collection of all $\hat{g}$ −open sets is denoted by $\hat{G}O(X)$.

Remark 2.5

The collection of all δg-closed (resp. ω-closed, g-closed, δ-closed, α-closed, semi-closed) sets of X is denoted by δGC(X) (resp. ωC(X), GC(X), δC(X), αC(X), SC(X)).

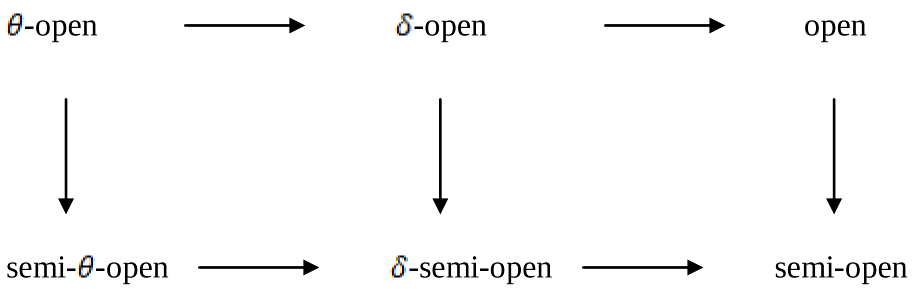
We denote the power set of X by P(X).

Definition 2.6

A space (X, τ) is said to be sub weakly T_2 if cl_δ ({x}) = cl({x}) for each x ∈ X.

Remark 2.7

We have the following diagram in which the converses of the implications need not be true.



Theorem 2.8

Let (X, τ) be a space. The following hold.

(i) Every δ-closed set is δg-closed.

(ii) Every δg-closed set is g-closed and hence αg-closed, gs-closed, gsp-closed and rg-closed.

Remark 2.9

δg-closed sets and ω-closed sets are independent.

Definition 2.10

A space (X, τ) is called semi-regular if $\tau_\delta = \tau$.

Definition 2.11

A space X is called $\tau\omega$ if ω-closed set in X is closed in X.

Proposition 2.12

Let (X, τ) be a space. If A ⊆ X is preopen then cl(A) = αcl(A) = cl_δ (A).

Lemma 2.14

In any space, a singleton is δ-open if and only if it is regular open.

3. $\delta\omega$-CLOSED SETS

We introduce the following definition.

Definition 3.1

A subset A of X is called a $\delta\omega$-closed set if cl_δ (A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ). The complement of $\delta\omega$-closed set is called $\delta\omega$-open set.

The collection of all $\delta\omega$-closed sets of X is denoted by $\delta\omega$C(X).

Proposition 3.2

Every δ-closed set is $\delta\omega$-closed.

Proof

Let A be a δ-closed set and G be any semi-open set containing A. Since A is δ-closed, cl_δ (A) = A for every subset A of X. Therefore cl_δ (A) $\subseteq$ G and hence A is $\delta\omega$-closed set.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and δC(X) = {ϕ, X}. We have A = {b, c} is $\delta\omega$-closed but not δ-closed set in (X, τ).

Proposition 3.4

Every $\delta\omega$-closed set is g-closed.

Proof

Let A be a $\delta\omega$-closed set and G be any open set containing A. Since every open set is semi-open and A is $\delta\omega$-closed, cl_δ (A) $\subseteq$ G. Since cl(A) $\subseteq$ cl_δ (A) $\subseteq$ G , cl(A) $\subseteq$ G and hence A is g-closed.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and GC(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. We have A = {a, b} is g-closed but not $\delta\omega$-closed set in (X, τ).

Proposition 3.6

Every $\delta\omega$-closed set is ω-closed.

Proof

Let A be a $\delta\omega$-closed and G be any semi-open set containing A. Since cl(A) $\subseteq$ cl_δ (A) $\subseteq$ G and hence A is ω-closed.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

Let X = {a, b, c} with τ = {ϕ, {a}, {a, b}, X}. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and ωC(X) = {ϕ, {c}, {b, c}, X}. We have A = {c} is ω-closed but not $\delta\omega$-closed set in (X, τ).

Proposition 3.8

Every $\delta\omega$-closed set is δg-closed.

Proof

Let A be a $\delta\omega$-closed set and G be any open set containing A. Since every open set is semi-open and A is $\delta\omega$-closed, cl_δ (A) ⊆ G. Therefore cl_δ (A) ⊆ G and G is open. Hence A is δg-closed.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and δGC(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. We have A = {a, c} is δg-closed but not $\delta\omega$-closed set in (X, τ).

Remark 3.10

The following examples show that $\delta\omega$-closedness is independent of closedness, semi-closedness and α-closedness.

Example 3.11

Let X and τ be as in the Example 3.3. Then $\delta\omega$C(X) = {ϕ, {b, c}, X} and αC(X) = SC(X) = {ϕ, {b}, {c}, {b, c}, X}. We have A = {b} is α-closed as well as semi-closed in (X, τ) but it is not $\delta\omega$-closed set in (X, τ).

Example 3.12

Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then $\delta\omega$C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and αC(X) = SC(X) = {ϕ, {c}, X}. We have A = {a, c} is $\delta\omega$-closed but it is neither α-closed set nor semi-closed set in (X, τ).

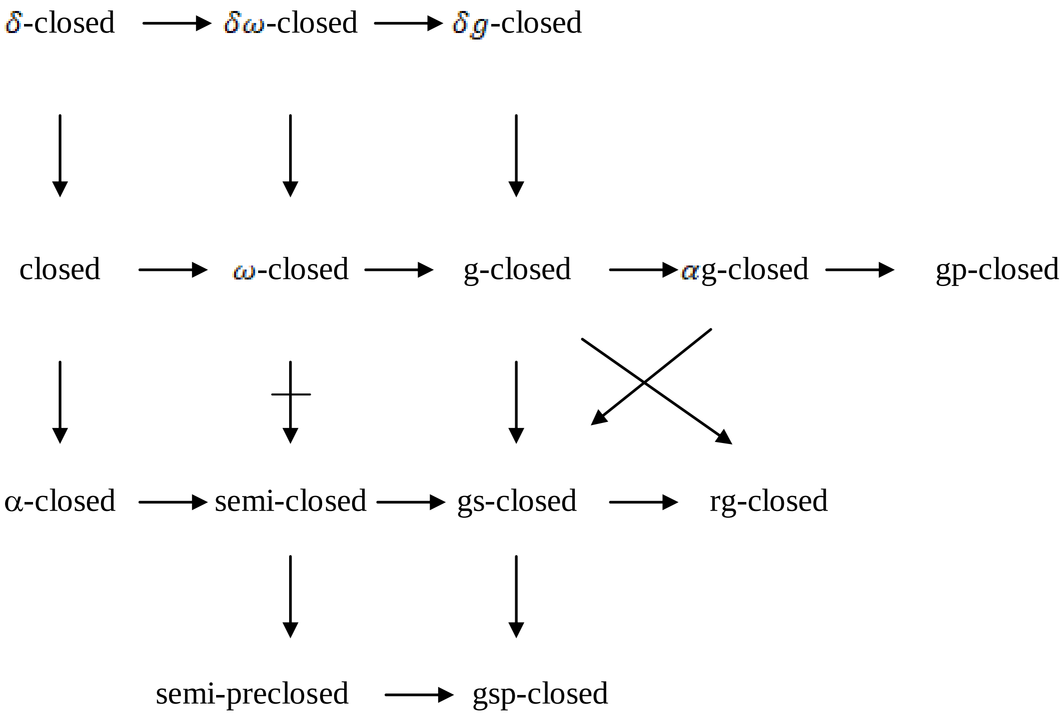
Example 3.13

In Example 3.7, {c} is closed but not $\delta\omega$-closed set.

In Example 3.12, {b, c} is $\delta\omega$-closed but not closed set.

Remark 3.14

From the above discussions and known results in [9, 10, 21, 24], we obtain the following diagram, where A $\rightarrow$ B (resp. A $\nleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).



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