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On The Ternary Quadratic Diophantine Equation $2x^2 - 3xy + 2y^2 = 14z^2$

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ABSTRACT

The homogeneous ternary quadratic diophantine equation represented by $2x^2 - 3xy + 2y^2 = 14z^2$ is studied for finding its non – zero distinct integer solutions.

Keywords: Homogeneous Ternary Quadratic, Integral solutions

INTRODUCTION

Ternary quadratic equations are rich in variety [1-4, 17-19]. For an extensive review of sizable literature and various problems, one may refer [5-16]. In this communication, we consider yet another interesting homogeneous ternary quadratic equation $2x^2 - 3xy + 2y^2 = 14z^2$ and obtain infinitely many non-trivial integral solutions.

METHOD OF ANALYSIS:

Let x, y, z be any three non-zero distinct integers such that

$$2x^2 - 3xy + 2y^2 = 14z^2 \qquad (1)$$

Substitution of the linear transformations

$$x = u + v \ , y = u - v \ , u \neq v \neq 0 \tag{2}$$

in (1) leads to

$$u^2 + 7v^2 = 14z^2 \tag{3}$$

We present below different methods of solving (3) and thus, obtain different patterns of integral solutions to (1).

Method:1

Let

$$z = a^2 + 7b^2 \tag{4}$$

Write 14 on the R.H.S. of (3) as

$$14 = \frac{(7 + i\sqrt{7})(7 - i\sqrt{7})}{4} \tag{5}$$

Using (4) & (5) in (3) and employing the method of factorization,

define

$$u + i\sqrt{7}v = \frac{(7 + i\sqrt{7})}{2}(a + i\sqrt{7}b)^2 \tag{6}$$

On equating the real and imaginary parts,one obtains

$$u = \frac{7a^2 - 49b^2 - 14ab}{2} \ , v = \frac{a^2 - 7b^2 + 14ab}{2}$$

In view of (2),the values of x,y are given by

$$x = 4a^2 - 28b^2 \ , y = 3a^2 - 21b^2 - 14ab \tag{7}$$

Thus,(4) and (7) give the integer solutions to (1).

Method:2

Write (3) as

$$u^2+7v^2=14z^2*1 \tag{8}$$

Represent 1 on the R.H.S. of (8) as

$$1=\frac{(3+i\sqrt{7})(3-i\sqrt{7})}{16} \tag{9}$$

Using (4) , (5) & (9) in (8) and employing the method of factorization, define

$$u+i\sqrt{7}v=\frac{(7+i\sqrt{7})}{2}\frac{(3+i\sqrt{7})}{4}(a+i\sqrt{7}b)^2 \tag{10}$$

Following the procedure similar to Method:1,the values of x and y are found to be

$$x=3(a^2-7b^2)-14ab\ ,\ y=\frac{(a^2-7b^2)}{2}-21ab \tag{11}$$

It is noted that (4) and (11) represent the integer solutions to (1) provided a and b are of the same parity.

Note:1

Observe that 1 on the R.H.S. of (8) may also be taken as

$$1=\frac{(1+i3\sqrt{7})(1-i3\sqrt{7})}{64}$$

$$1=\frac{(7r^2-s^2+i\sqrt{7}2rs)(7r^2-s^2-i\sqrt{7}2rs)}{(7r^2+s^2)^2}$$

and thus ,one obtains two more sets of integer solutions to (1).

Method:3

Introduction of the linear transformations

$$u = X + 7T \ , v = X - T \ , z = 2w \tag{12}$$

in (3) leads to

$$X^2 + 7T^2 = 7w^2 \tag{13}$$

which is satisfied by

$$w = 7r^2 + s^2 \ , T = 7r^2 - s^2 \, , X = \pm 14rs \tag{14}$$

In view of (12) and (2),the integer solutions to (1) are given by

$$x = 42r^2 - 6s^2 \pm 28rs \ , y = 56r^2 - 8s^2 \ , z = 14r^2 + 2s^2$$

Note:2

Instead of (12) ,suppose we have the transformations

$$u = X - 7T \ , v = X + T \ , z = 2w \quad ,$$

then the corresponding integer solutions to (1) are found to be

$$x = -42r^2 + 6s^2 \pm 28rs \ , y = -56r^2 + 8s^2 \ , z = 14r^2 + 2s^2$$

Method:4

Rewrite (3) as

$$14z^2 - 7v^2 = u^2 * 1 \tag{15}$$

Let

$$u = 14a^2 - b^2 \tag{16}$$

Assume 1 on the R.H.S. of (15) as

$$1 = \frac{(2\sqrt{14} + \sqrt{7})(2\sqrt{14} - \sqrt{7})}{49} \tag{17}$$

Following the procedure as in Method:2,the corresponding integer solutions to (1) are given by

$$x = 16a^2 - 6b^2 + 8ab \ , y = 12a^2 - 8b^2 - 8ab \ , z = 4a^2 + 2b^2 + 2ab$$

Note:3

Observe that 1 on the R.H.S. of (15) may also be taken as

$$1 = \frac{(4\sqrt{14} + 5\sqrt{7})(4\sqrt{14} - 5\sqrt{7})}{49}$$

and thus, one obtains a different set of integer solutions to (1).

OBSERVATIONS:

1. If (x, y, z) is any given solution to (1),then ,the triple given by

   $(u, v, w) = (19x - 8y + 56z, 10x - 5y + 28z, 5x - 2y + 15z)$ also satisfies (1).

2. 4w-u=x+4z
3. 2w-v=y+2z
4. 2v-u=x-2y
5. 6w-2u+v=2x-y+6z

Conclusion:

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the ternary quadratic diophantine equation $2x^2 - 3xy + 2y^2 = 14z^2$. As there are varieties of ternary quadratic equations, the readers may search for other forms of ternary quadratic equations to obtain their integer solutions .

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