International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Pythagorean Triangle in Connection with Nasty Numbers

***Dr.N.Thiruniraiselvi*[1], *Dr.M.A.Gopalan*[2]**

[1]Assistant Professor, Department of Mathematics, Nehru Memorial College, Affiliated to Bharathidasan University, Trichy-621 007, Tamil Nadu, India.
email:drntsmaths@gmail.com
[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.
email: mayilgopalan@gmail.com

ABSTRACT

This paper deals with the problem of obtaining Pythagorean triangles such that ,in each Pythagorean triangle , $\frac{Area}{Perimeter}$ = Nasty number

Key words : Pythagorean triangle ,area, perimeter ,nasty number

INTRODUCTION

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt.

The Pythagorean numbers play a significant role in the theory of higher arithmetic as they come in the majority of indeterminate problems; had a marvelous effect on a credulous people and always occupy a remarkable position due to unquestioned historical importance. The method of obtaining three non-zero integers *x,y* and H under certain relations satisfying the relation $x^2 + y^2 = H^2$ has been a matter of interest to various Mathematicians [1]-[4]. In [5]-[13], special Pythagorean problems are studied. In this communication, we search for Pythagorean triangles in which ,each Pythagorean triangle has the property that its ratio of area to perimeter is represented by a nasty number expressed as a square multiple of six .

Method of analysis:

Let m, n be two non-zero distinct positive integers such that $m \geq n \geq 0$.Considering m, n

to be the generators of a Pythagorean triangle ,its legs x, y and hypotenuse H are taken as

$$x = 2mn, y = m^{2^2} - n^2, H = m^2 + n^2, m \geq n, \geq 0 \quad (1)$$

Denoting the area and the perimeter of the above Pythagorean triangle as A, P respectively,one has

$$A = mn(m^2 - n^2) \ , P = 2m(m+n) \quad (2)$$

Also ,a number N is said to be a nasty number if the sum (or the difference) of any two non-zero distinct divisors of N is equal to the difference (or the sum) of other two non-zero divisors of N.

It is well-known that 6 is the smallest nasty number and a number given by the square multiple of 6 is also a nasty number. Thus ,in view of (2), the problem under consideration is mathematically equivalent to

$$n(m-n) = 12\alpha^2 \quad (3)$$

The values of m, n satisfying (3) are presented in Table:1 below:

Table:1 values of m,n

S.No	m	n
1	13α	α
2	8α	2α
3	7α	3α
4	7α	4α
5	8α	6α
6	13α	12α
7	$\alpha^2 + 12$	α^2
8	$2\alpha^2 + 6$	$2\alpha^2$
9	$3\alpha^2 + 4$	$3\alpha^2$
10	$4\alpha^2 + 3$	$4\alpha^2$
11	$6\alpha^2 + 2$	$6\alpha^2$
12	$12\alpha^2 + 1$	$12\alpha^2$
13	$12\alpha^2 + 1$	1
14	$6\alpha^2 + 2$	2
15	$4\alpha^2 + 3$	3
16	$3\alpha^2 + 4$	4
17	$2\alpha^2 + 6$	6
18	$\alpha^2 + 12$	12

In view of (1),the sides of the Pythagorean triangle corresponding to the pair of values of m and n in Table:1 are exhibited in Table:2 below:

Table:2 Sides of Pythagorean triangle

S.No	x	y	H
1	$26\alpha^2$	$168\alpha^2$	$170\alpha^2$
2	$32\alpha^2$	$60\alpha^2$	$68\alpha^2$
3	$42\alpha^2$	$40\alpha^2$	$58\alpha^2$
4	$56\alpha^2$	$33\alpha^2$	$65\alpha^2$
5	$96\alpha^2$	$28\alpha^2$	$100\alpha^2$
6	$312\alpha^2$	$25\alpha^2$	$313\alpha^2$
7	$2\alpha^4+24\alpha^2$	$12(2\alpha^2+12)$	$2\alpha^4+24\alpha^2+144$
8	$8\alpha^4+24\alpha^2$	$6(4\alpha^2+6)$	$8\alpha^4+24\alpha^2+36$
9	$18\alpha^4+24\alpha^2$	$4(6\alpha^2+4)$	$18\alpha^4+24\alpha^2+16$
10	$32\alpha^4+24\alpha^2$	$3(8\alpha^2+3)$	$32\alpha^4+24\alpha^2+9$
11	$72\alpha^4+24\alpha^2$	$2(12\alpha^2+2)$	$72\alpha^4+24\alpha^2+4$
12	$288\alpha^4+24\alpha^2$	$(24\alpha^2+1)$	$288\alpha^4+24\alpha^2+1$
13	$24\alpha^2+2$	$12\alpha^2(12\alpha^2+2)$	$144\alpha^4+24\alpha^2+2$
14	$24\alpha^2+8$	$6\alpha^2(6\alpha^2+4)$	$36\alpha^4+24\alpha^2+8$
15	$24\alpha^2+18$	$4\alpha^2(4\alpha^2+6)$	$16\alpha^4+24\alpha^2+18$
16	$24\alpha^2+32$	$3\alpha^2(3\alpha^2+8)$	$9\alpha^4+24\alpha^2+32$
17	$24\alpha^2+72$	$2\alpha^2(2\alpha^2+12)$	$4\alpha^4+24\alpha^2+72$
18	$24\alpha^2+288$	$\alpha^2(\alpha^2+24)$	$\alpha^4+24\alpha^2+288$

Also ,treating (3) as a quadratic in n and solving for n ,one obtains

$$n=\frac{m\pm\sqrt{m^2-48\alpha^2}}{2} \qquad (4)$$

After performing some algebra ,the values of m,n satisfying (4) are given

in Table:3 below:

Table:3 values of $\boldsymbol{m,n}$

S.No	m	n
1	$48r^2+s^2$	$48r^2,s^2$
2	$6r^2+8$	$6r^2,8$
3	$2r^2+24$	$2r^2,24$
4	$49r$	$48r,r$
5	$19r$	$16r,3r$

In view of (1),the sides of the Pythagorean triangle corresponding to the pair of values of m and n in Table:3 are exhibited in Table:4 below:

Table:4 Sides of Pythagorean triangle

S.No	x	y	H
1	$96r^2(48r^2+s^2)$	$s^2(96r^2+s^2)$	$4608r^4+96r^2s^2+s^4$
	$2s^2(48r^2+s^2)$	$s^2(48r^2+2s^2)$	$2304r^4+96r^2s^2+2s^4$
2	$12r^2(6r^2+8)$	$8(12r^2+8)$	$72r^4+96r^2+64$
	$16(6r^2+8)$	$6r^2(6r^2+16)$	$36r^4+96r^2+128$
3	$4r^2(2r^2+24)$	$24(4r^2+24)$	$8r^4+96r^2+576$
	$48(2r^2+24)$	$2r^2(2r^2+48)$	$4r^4+96r^2+1152$
4	$98*48r^2$	$97r^2$	$(49^2+48^2)r^2$
	$98r^2$	$2400r^2$	$2402r^2$
5	$38*16r^2$	$105r^2$	$617r^2$
	$114r^2$	$352r^2$	$370r^2$

Conclusion:

In this paper ,an attempt has been made to obtain Pythagorean triangles in which,each Pythagorean triangle has the property that its ratio of area to perimeter is represented by a nasty number expressed as a square multiple of six .By definition, the nasty numbers are rich in variety and the researchers in the field of diophantine equations may search for Pythagorean triangles satisfying the above property with other choices of nasty numbers.

REFERENCES:

1. Dickson.L.E., History of Theory of numbers, vol.2:Diophantine Analysis, New York, Dover, 2005.
2. Mordell L.J., Diophantine Equations, Academic press, London (1969).

3. Andre weil, Number theory: An approach through history from hammurapi to legendre/Andre weil:Boston (Birkahasuser boston), 1983.
4. Nigel P.Smart , The algorithmic Resolutions of Diophantine equations,Cambridge University press, 1999.
5. Gopalan M.A. and Leelavathi.S, “Pythagorean triangle with 2(Area/Perimeter) as a cubic integer”, Bulletin of Pure and Applied Sciences,vol.27 E(2), pp. 197-200, 2007
6. Gopalan M.A. and Janaki, G, “Pythagorean triangle with Area/Perimeter as a special polygonal number”,Bulletin of Pure and Applied Sciences,vol.27 E(2), pp. 393-402,2008.
7. Gopalan M.A. and Sangeetha.G, “Pythagoream triangles with perimeter as triangular number”,The Global Journal of Applied Mathematics and Mathematical Sciences,vol.3(1-2),2010,pp.93-97, 2010.
8. Gopalan M.A. and Geetha V., “Pythagorean triangle with Area/Perimeter as a special polygonal number”,International Refereed Journal of Engineering and Science,vol.2(7), pp.28-34, 2013.
9. Gopalan M.A., ManjuSomanath and Geetha .K, “Pythagorean triangle with Area/Perimeter as a special polygonal number”,IOSR-JM,vol.7(3),pp52-62,2013.
10. Gopalan M.A., ManjuSomanath and Sangeetha .V, “Pythagorean triangles and pentagonal number”,Cayley J.Math,,vol.2(2),2013,pp.151-156,2013.
11. Gopalan M.A., ManjuSomanath and Sangeetha.V, “Pythagorean triangles and special pyramidal numbers”, IOSR-JM, vol.7(4), 2013,pp.21-22,2013.
12. Gopalan, M.A., Vidhyalakshmi, S., Thiruniraiselvi, N. Pythagorean triangle with $hypotenuse - 4\left(\frac{Area}{Perimeter}\right)$ is 3 times a square integer, International Journal of Innovative Science and Modern Engineering, Vol.3(6),Pp.1-5, May 2015.
13. Gopalan, M.A., Vidhyalakshmi, S., Thiruniraiselvi, N. Pythagorean triangle with $hypotenuse - 4\left(\frac{Area}{Perimeter}\right) = (4k^2 - 4k + 3)\alpha^2$, International Journal of Multidisciplinary Research and Modern Engineering, Vol.1(1),Pp.123-128, 2015.