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Solution of Travelling Salesman Problem with Intuitionistic Triskaidecagonal Fuzzy Numbers

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ABSTRACT:

In this paper, A travelling Salesman problem with its Cost or distance or time between two cities is taken as intuitionstic triskaidecagonal fuzzy numbers . The problem is solved by two methods here first by a new method and second by a new ranking function method. Finally the two methods are compared.

Key words: Instuitionistic fuzzy number, Triskaidecagonal, membership, non-membership function, Travelling Salesman Problem.

2010 Mathematics subject classification: 90 C70

1. INTRODUCTION:

Instuitionistic fuzzy set is one of the more realistic situation of fuzzy sets. In fuzzy set theory only truthfulness of a statement is considered. In the instuitionistic fuzzy set theory the truthfulness and falseness both statements are considered, which is more realistic situation. At present Instuitionistic fuzzy sets are being applied and used in various sector of science.

Prof. Zadeh [10] pioneered set in 1965, since then researchers established pentagonal, hexagonal, octagonal, decagonal fuzzy numbers, these increasing fuzziness was used to cope up the vulnerability of data. Even then dodecagonal, pentadecagonal fuzzy numbers were taken to more accuracy. Instead of structural expansion of fuzzy sets Prof. Atanassov [3] in 1986 manifested the thought of instuitionistic fuzzy set which is the mixture of membership and non membership function, After that Lium and Yuan [7] proposed triangular I.F.S., Ye [9] explained the design of trapezoidal I.F.S. Amutha B. and Uthara G. [1] gave defuzziness of symmetric octagonal I.F.S.

Travelling salesman problem is a classical problem of combinatorial optimization of Operations Research. It deals with finding the shortest tour of a salesman that visits each city in a given list exactly once and then comes back to the starting city. The cost of travelling from location 'i' to 'j' is denoted by C_{ij}. If the $C_{ij} = C_{ji}$ then this problem is called symmetric TSP and it $C_{ij} \neq C_{ji}$ then this problem is called asymmetric TSP. This work is done over symmetric travelling salesman problem with instuitionistic triskaidecagonal fuzzy numbers.

2. BASIC DEFINITIONS:

Definition 2.1: **(Instuitionistic Fuzzy Number)**

Let x denote a universal set, then the Instuitionistic fuzzy set is $\tilde{p}$ *in* X is given by

$$\tilde{p} = \{x; [\psi(x), \omega(x)] : x \in X = universal\, set\}$$

Where $\psi(x) : X \to [0,1]$ is termed as membership function,

$\omega(x) : X \to [0,1]$ is termed as non membership function

and $0 \le \psi(x) + \omega(x) \le 1.$

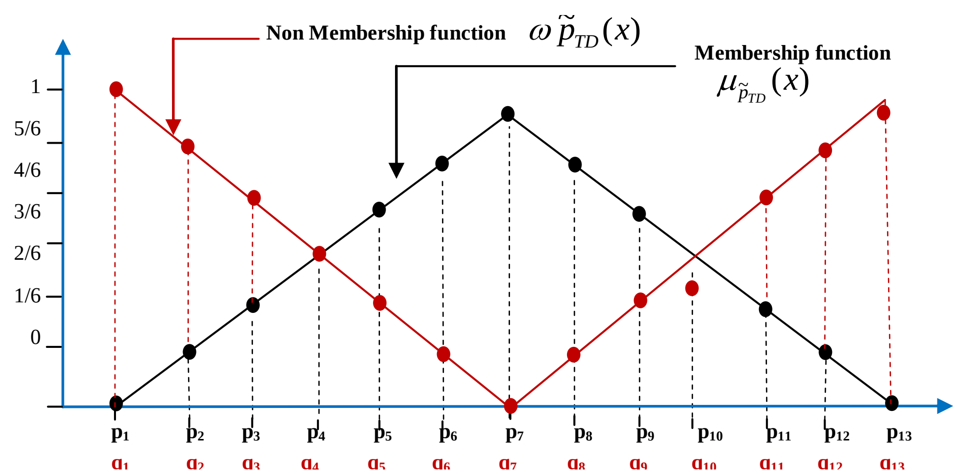
Definition 2.2: Instuitionistic Triskaidecagonal Fuzzy Number: An intuitionstic TFN is of type

$$\tilde{p}_{ITD} = \{(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}); (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13})\} \text{ where } p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}\, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13} \text{ are real numbers.}$$

So non membership functions and the membership function of the above Inuinistic triskaidecagonal fuzzy number are as follows –

$$
\omega \tilde{p}_{ITD}(x) = \begin{cases}
1-\frac{1}{6}\left(\frac{q_2 - x}{q_2 - q_1}\right), & q_1 \le x \le q_2 \\
\frac{5}{6}-\frac{1}{6}\left(\frac{q_3 - x}{q_3 - q_2}\right), & q_2 \le x \le q_3 \\
\frac{4}{6}-\frac{1}{6}\left(\frac{q_4 - x}{q_4 - q_3}\right), & q_3 \le x \le q_4 \\
\frac{3}{6}-\frac{1}{6}\left(\frac{q_5 - x}{q_5 - q_4}\right), & q_4 \le x \le q_5 \\
\frac{2}{6}-\frac{1}{6}\left(\frac{q_6 - x}{q_6 - q_5}\right), & q_5 \le x \le q_6 \\
\frac{1}{6}\left(\frac{q_7 - x}{q_7 - q_6}\right), & q_6 \le x \le q_7 \\
\frac{1}{6}+\frac{1}{6}\left(\frac{q_8 - x}{q_8 - q_7}\right), & q_7 \le x \le q_8 \\
\frac{2}{6}+\frac{1}{6}\left(\frac{q_9 - x}{q_9 - q_8}\right), & q_8 \le x \le q_9 \\
\frac{3}{6}+\frac{1}{6}\left(\frac{q_{10} - x}{q_{10} - q_9}\right), & q_9 \le x \le q_{10} \\
\frac{4}{6}+\frac{1}{6}\left(\frac{q_{11} - x}{q_{11} - q_{10}}\right), & q_{10} \le x \le q_{11} \\
\frac{5}{6}+\frac{1}{6}\left(\frac{q_{12} - x}{q_{12} - q_{11}}\right), & q_{11} \le x \le q_{12} \\
\frac{1}{6}\left(\frac{x - q_{13}}{q_{13} - q_{12}}\right), & q_{12} \le x \le q_{13} \\
1 \quad , & x \ge q_{13}
\end{cases}
$$

Figure showing Graph of Membership function and non-membership function.



$$
\mu\tilde{p}_{ITD}(x)=\begin{cases}
0, & x\le p_1\\
\frac{1}{6}\left(\frac{x-p_1}{p_2-p_1}\right), & p_1\le x\le p_2\\
\frac{1}{6}+\frac{1}{6}\left(\frac{x-p_2}{p_3-p_2}\right), & p_2\le x\le p_3\\
\frac{2}{6}+\frac{1}{6}\left(\frac{x-p_3}{p_4-p_3}\right), & p_3\le x\le p_4\\
\frac{3}{6}+\frac{1}{6}\left(\frac{x-p_4}{p_5-p_4}\right), & p_4\le x\le p_5\\
\frac{4}{6}+\frac{1}{6}\left(\frac{x-p_5}{p_6-p_5}\right), & p_5\le x\le p_6\\
\frac{5}{6}+\frac{1}{6}\left(\frac{x-p_6}{p_7-p_6}\right), & p_7\le x\le p_6\\
1-\frac{1}{6}\left(\frac{x-p_7}{p_8-p_7}\right), & p_7\le x\le p_8\\
\frac{5}{6}-\frac{1}{6}\left(\frac{x-p_8}{p_9-p_8}\right), & p_8\le x\le p_9\\
\frac{4}{6}-\frac{1}{6}\left(\frac{x-p_9}{p_{10}-p_9}\right), & p_9\le x\le p_{10}\\
\frac{3}{6}-\frac{1}{6}\left(\frac{x-p_{10}}{p_{11}-p_{10}}\right), & p_{10}\le x\le p_{11}\\
\frac{2}{6}-\frac{1}{6}\left(\frac{x-p_{11}}{p_{12}-p_{11}}\right), & p_{11}\le x\le p_{12}\\
\frac{1}{6}\left(\frac{p_{13}-x}{p_{13}-p_{12}}\right), & p_{12}\le x\le p_{13}\\
0\quad, & x\ge p_{13}
\end{cases}
$$

2.3. Some arithmetic operations on Intuitionistic Triskaidecagonal fuzzy number (ITFN)

Let two ITFN are

$$\tilde{p}_{ITD}=\{(p_1,p_2,p_3,\ p_4,\ p_5,\ p_6,\ p_7,\ p_8,\ p_9,\ p_{10},\ p_{11},\ p_{12},\ p_{13});\ (q_1,q_2,q_3,\ q_4,\ q_5,\ q_6,\ q_7,\ q_8,\ q_9,\ q_{10},\ q_{11},\ q_{12},\ q_{13})\}$$

$$\tilde{Q}_{ITD}=\{(r_1,r_2,r_3,\ r_4,\ r_5,\ r_6,\ r_7,\ r_8,\ r_9,\ r_{10},\ r_{11},\ r_{12},\ r_{13});\ (s_1,s_2,s_3,\ s_4,\ s_5,\ s_6,\ s_7,\ s_8,\ s_9,s_{10},\ s_{11},\ s_{12},s_{13})\}$$

then

$$\tilde{p}_{ITD}+\tilde{Q}_{ITD}=\{(p_1+r_1,\ \ p_2+r_2,\ p_3+r_3,\ p_4+r_4,\ p_5+r_5,\ p_6+r_6,\ p_7+r_7,\ p_8+r_8,\ p_9,+r_9,\ p_{10}+r_{10},\ p_{11}+r_{11}\ \ p_{12}+r_{12},\ p_{13}+r_{13});\ (q_1+s_1,\ q_2+s_2,\ q_3+s_3,\ q_4+s_4,\ q_5+s_5,\ q_6+s_6,\ q_7+s_7,\ q_8+s_8,q_9,+s_9,\ q_{10}+s_{10},\ q_{11}+s_{11},\ \ q_{12}+s_{12},\ q_{13}+s_{13})\}$$

$$\tilde{p}_{ITD} + \tilde{Q}_{ITD} = \{(p_1 - r_1,\ p_2 - r_2,\ p_3 - r_3,\ p_4 - r_4,\ p_5 - r_5,\ p_6 - r_6,\ p_7 - r_7,\ p_8 - r_8,\ p_9, - r_9,\ p_{10} - r_{10},\ p_{11} - r_{11}\ p_{12} - r_{12},\ p_{13} - r_{13});\ (q_1 - s_1,\ q_2 - s_2,\ q_3 - s_3,\ q_4 - s_4,\ q_5 - s_5,\ q_6 - s_6,\ q_7 - s_7,\ q_8 - s_8,\ q_9, - s_9,\ q_{10} - s_{10},\ q_{11} - s_{11}\ q_{12} - s_{12},\ q_{13} - s_{13})\}$$

2.4 Magnitude of Membership and Magnitude of Non-membership of a ITFN:

Let

$$\tilde{p}_{ITD} = \{(p_1, p_2, p_3,\ p_4,\ p_5,\ p_6,\ p_7,\ p_8,\ p_9,\ p_{10},\ p_{11},\ p_{12},\ p_{13});\ (q_1, q_2, q_3,\ q_4,\ q_5,\ q_6,\ q_7,\ q_8,\ q_9,\ q_{10},\ q_{11},\ q_{12},\ q_{13})\}\ \textit{is ITFN then}$$

Magnitude of Membership function

$$Mag\ \mu(\tilde{p}_{ITD}) = \frac{p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 + p_{10} + p_{11} + p_{12} + p_{13}}{13}$$

Magnitude of non – membership of ITFN :

$$Mag\ \omega(\tilde{p}_{ITD}) = \frac{q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 + q_{10} + q_{11} + q_{12} + q_{13}}{13}$$

2.5 New Ranking of ITFN:

$$R(\tilde{p}_{ITD}) = Max\ \lfloor Mag_{\mu}(\tilde{p}_{ITD}), Mag_{\omega}(\tilde{p}_{ITD})\rfloor$$

3. MATHEMATICAL FORMULATION OF INSTUITIONISTIC TRISKAIDECAGONAL FUZZY TRAVELLING SALESMAN PROBLEM:

Opimize $\tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij}\, x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, ...n$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, ...n$$

$$x_{ij} = 0\ or\ 1,\ i = 1, ...n,\ j = 1, ...n$$

Order should not be chosen more than one duration, that is $x_{ij} + x_{ji} \le 1\ \forall\ i, j\ s.t.\ x_{ij} \ge 0$

Where $\tilde{C}_{ij}$ is triskaidecagonal intuitionistic fuzzy number.

3.1 SOLUTION METHODOLOGY:

Method 1: The given intuitionistic triskaidecagonal fuzzy travelling Salesman Cost matrix is converted to three crisp travelling salesman problems and the cost of tour for each crisp problem is found by any popular methods. Finally the average of the three cost of tours is the optional solution of the given intuitionstic triskaidecagonal fuzzy travelling salesman problem.

Step-1: First Crisp problem is made by taking average of highest of the costs of the both truthfulness part and falseness part of the given intuitionistic triskaidecagonal fuzzy cost given in the cell. This is done for each cell. Thus formed travelling salesman problem is solved by any method.

Step -2: Second crisp problem is made by taking average lowest o the costs of the both truthfulness part and falseness part of the given intuitionistic triskaidecagonal fuzzy cost given in the cell. This is done for each cell and then the found TSP is solved by any method.

Step-3: Third crisp problem is made by taking average of the average of lowest and average of highest costs of the both truthfulness part and falseness part of intuitionistic triskaidecagonal fuzzy cost given in the cell found in step 1 and step 2. This is done for each cell and thus found TSP is solved by any method.

Step-4: The average of the tour costs in the TSPs found in Step-1, Step-2 and Step-3 is taken and this average is the final optional solution of the given Intuitionistic Triskaidecagonal Fuzzy Travelling Salesman Problem.

Method-2:

The given intuitionistic triskaidecagonal fuzzy travelling salesman problem is converted to crisp travelling salesman problems by using the new ranking function given in 2.5 in this paper. Then the TSP is solved. Hence optional tour cost is found.

At last the optional tour costs solved in method-1 and method-2 is compared.

4. NUMERICAL ILLUSTRATION:

a salesman wants to visit four cities C_1, C_2, C_3, C_4 the expense on the travelling from a city to another city are given in the form of Intuitionistic Triskaidecagonal Intuitionistic fuzzy numbers.

Home	Home	C_1	C_2	C_3	C_4
Home	C_1	∞	ITD_1	ITD_2	ITD_3
	C_2	ITD_1	∞	ITD_4	ITD_5
	C_3	ITD_2	ITD_4	∞	ITD_6
	C_4	ITD_3	ITD_5	ITD_6	∞

Where ITD_i are the Intuitionistic Triskaidecagonal fuzzy cost.

ITD_1= (46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70; 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73, 76)

ITD_2= (60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84; 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90)

ITD_3= (32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54; 56; 28, 29, 31, 33, 35, 37, 44, 47, 49, 51, 53, 55, 58)

ITD_4= (30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66; 26, 30, 33, 37, 41, 45, 48, 53, 57, 61, 65, 69, 74)

ITD_5= (52, 56, 60, 64, 66, 68, 70, 74, 78, 82, 86, 88, 92; 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 94, 98)

ITD_6= (74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98; 72, 74, 78, 79, 80, 82, 86, 88, 90, 92, 94, 96, 100)

Method-1:

Step-1: Average of highest

(70+76)/2=73,(84+90)/2=87,(56+58)/2=57,(66+74)/2=70,(92+98)/2=95,(98+100)/2=99

Crisp travelling of average of highest in the cell

Home	C_1	C_2	C_3	C_4
Home C_1	∞	73	87	57
C_2	73	∞	70	75
C_3	87	70	∞	99
C_4	57	95	99	∞

Modified Cost Matrix

Home	C_1	C_2	C_3	C_4
C_1	∞	16	30	0
Home C_2	3	∞	0	25
C_3	17	0	∞	29
C_4	0	38	42	∞

Path –I 1→ 4→ 2→ 3→ 1 Tour Cost 309 Units

Path –II 1→ 3→ 2→ 4→ 1 Tour Cost 309 Units

Step-2: Crisp TSP averages of the lowest in the Cell

	Home C_1	C_2	C_3	C_4
Home C_1	∞	43	57	30
C_2	43	∞	28	46
C_3	57	28	∞	73
C_4	30	46	73	∞

Modified Cost Matrix

	Home C_1	C_2	C_3	C_4
Home C_1	∞	13	27	0
C_2	15	∞	29	18
C_3	29	0	∞	45
C_4	0	16	43	∞

Path –I $1\to 4\to 2\to 3\to 1$ Tour Cost 161 Units

Path –II $1\to 3\to 2\to 4\to 1$ Tour Cost 161 Units

Step-3: Crisp TSP of averages of the averages of lowest and highest in the cell.

	Home C_1	C_2	C_3	C_4
Home C_1	∞	58	72	43.5
C_2	58	∞	49	72.5
C_3	72	49	∞	86
C_4	43.5	72.5	86	∞

Modified Cost Matrix

Path –I ,$1\to 4\to 2\to 3\to 1$, Tour Cost 235 Units

Path –II, $1\to 3\to 2\to 4\to 1$, Tour Cost 235 Units

Averages of tour costs found in Step -1,

Step-2, Step-3 will be $\dfrac{309+161+235}{3} = 235 \;\; units$

Method 2:

$$Mag_{\mu}(ITD_1) = 58, Mag_{\omega}(ITD_1) = 54.69, R(ITD_1) = 58$$

$$Mag_{\mu}\ (ITD_2) = 72,\quad Mag_{\omega}(ITD_2) = 72,\ \ R\ (ITD_2) = 72$$

$$Mag_{\mu}\ (ITD_3) = 44,\quad Mag_{\omega}(ITD_3) = 42.30,\ \ R\ (ITD_3) = 44$$

$$Mag_{\mu}\ (ITD_4) = 48,\quad Mag_{\omega}\ (ITD_4) = 46.846,\ \ R\ (ITD_4) = 48$$

$$Mag_{\mu}\ (ITD_5) = 72,\quad Mag_{\omega}\ (ITD_5) = 69.769,\ \ R\ (ITD_5) = 72$$

$$Mag_{\mu}\ (ITD_6) = 86,\quad Mag_{\omega}\ (ITD_6) = 85.469,\ \ R\ (ITD_6) = 86$$

So Crisp TSP

	Home C_1	C_2	C_3	C_4
Home C_1	∞	58	72	44
C_2	58	∞	48	72
C_3	72	48	∞	86
C_4	44	72	86	∞

By the same method applied in previous Optimal Tour is Crisp TSP

Tour –I 1→ 4→ 2→ 3→ 1, Tour Cost 236 Units

Tour –II 1→ 3→ 2→ 4→ 1, Tour Cost 236 Units

So we can compare the tour costs of the two method which are 235 units and 236 units, almost same**.**

5. CONCLUSION:

So the TSP with ITDN cost is solved in the paper by two methods, method 1 is done without use of ranking function and method 2 is done with the use of ranking function. The tour cost is almost same in the methods so method 1 is an alternative when ranking function is not used. The methods are used for both symmetric TSP and Asymmetric TSP.

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