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Cordial Labeling and Anti- magic labeling for the Family of Fish graph

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A B S T R A C T

In this paper we find the total graph, middle graph, splitting graph and shadow graph of Fish graph and its cordial labeling and anti- magic labeling. And we prove that those total graph, middle graph, splitting graph and shadow graph of Fish Graph accept Cordial Labeling and anti- magic labeling.

Keywords: Fish Graph, Cordial Labeling, anti- magic labeling, Middle Graph, Shadow Graph, Splitting Graph, Total Graph

1. Introduction

Graph labeling have been traced back to the 19th century when the famous British mathematician Arthur Cayley proved that there are n^{n-2} distinct labeled trees of order n[3] .A graph labeling is an assignment of integers to the vertices or edge or both, subject to certain conditions have been motivated by practical problems, labeled graphs serve useful mathematical models for a broad range applications. A simple graph X is an ordered pair of sets X=(V,E).Elements of V are called vertices of X and elements of E are called edges of X. By a graph here we mean a finite undirected graph without loops and multiple edges [4]. In this paper we find the total graph, middle graph, splitting graph and shadow graph of Fish Graph and the cordial labeling and anti- magic labeling of those graphs.

2. Definitions

Definition 2.1:

The middle graph M (G) [9] of a graph G is the graph whose vertex set is V(G)UE(G) and in which two vertices are adjacent if and only if either

1. They are adjacent edges of G
2. One is a vertex of G and other is an edge incident on it.

Definition 2.2:

The total graph T (G) [12] of a graph G is the graph whose vertex set is V (G) U E(G) and two vertices are adjacent whenever they are either adjacent or incident in G.

The total graph T(G) of a graph G is the graph whose vertices correspond to the vertices and edges of G, and whose two vertices are joint if and only if

1. The corresponding vertices are adjacent, edges are adjacent
2. Vertices and edges are incident in G.

Definition 2.3:

The splitting graph S'(G) [5] of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that N (v) =N (v') where N(v) and N (v') are the neighborhood sets of v and v' respectively. Properties of Splitting graph,

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1. deg v in S'(G)=2 deg v in G and deg v' in S'(G)= deg v in G.
2. If G has n triangles, then S'(G) has a 4n triangles.

Definition 2.4:

The Shadow Graph $D_2(G)$ [11] of a connected graph G is constructed by taking two copies of G, say G' and G". Join each vertex u' in G' to the neighbors of the corresponding vertex u" in G".

Definition 2.5:

Let G = (V, E) be a graph. A mapping f: V(G)→ {0, 1} is called a binary vertex labeling of G and f (v) is called the label of the vertex v of G under f.

For an edge e=uv, the induced edge labeling f*: E(G)→{0,1} is given by

f*(e)=|f(u)-f(v)|. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_{f^*}(0)$, $e_{f^*}(0)$ be the number of edges having labels 0 and 1 respectively under f*.

Definition 2.6:

A binary vertex labeling of a graph G is called a Cordial labeling [7] if| $v_f(0) - v_f(1)$ |≤1 and | $e_{f^*}(0) - e_{f^*}(1)$ |≤ 1. A graph G is Cordial if it admits cordial labeling.

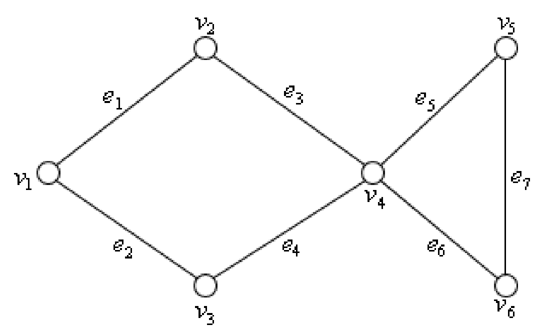
Definition 2.7:

An anti-magic labeling [6] of a finite simple undirected graph with q edges is a bijection from the set of edges to the set of integers { 1 , 2 , … , q } such that the vertex sums are pairwise distinct, where the vertex sum at vertex u is the sum of labels of all edges incident to such vertex.

A graph with q edges is called anti-magic, if its edges can be labeled with 1, 2, 3…... q such that the sums of the labels of the edges incident to each vertex are distinct.

Definition 2.8:

The Fish graph [2] is a special type of graph. It has the 6 vertices and 7 edges.



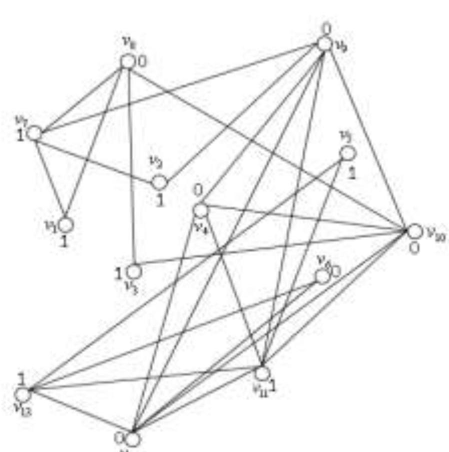
Fish graph

3. Cordial Labeling for the family of Fish graph

Theorem 3.1:

The Middle graph of a Fish graph is a Cordial graph.

Proof:



The number of vertices of the graph in Middle graph of a Fish graph is 13 and the number of edges of this graph is 25. Suppose the vertices are v_i , where i=1, 2... 13.

$$\text{Let } f(v_i)=\begin{cases} 1 \text{ if i is prime and 1} \\ 0 \text{ if i is not prime} \end{cases}$$

$$v_f(1)=(v_1,v_2,v_3,v_5,v_7,v_{11},v_{13})$$
$$v_f(0)=(v_4,v_6,v_8,v_9,v_{10},v_{12})$$
$$|v_f(1)|=7, |v_f(0)|=6$$
$$\therefore |v_f(1)-v_f(0)|=1\le 1$$

Hence the induced edge labeling are

$$\text{Let } f^*(v_iv_j)=\begin{cases} 1 \text{ if one of i and j is prime and 1 and other is not} \\ 0 \text{ if both of i and j is either prime and 1 or other is not} \end{cases}$$

$$e_{f^*}(1)=\begin{cases} v_1v_8, v_2v_9, v_3v_8, v_3v_{10}, v_4v_{11}, v_6v_{13}, \\ v_7v_8, v_7v_9, v_9v_{11}, v_{10}v_{11}, v_{11}v_{12}, v_{12}v_{13} \end{cases}$$

$$e_{f^*}(0)=\begin{cases} v_1v_7, v_2v_7, v_4v_9, v_4v_{10}, v_4v_{12}, v_5v_{11}, v_5v_{13}, \\ v_6v_{12}, v_8v_{10}, v_9v_{10}, v_9v_{12}, v_{10}v_{12}, v_{11}v_{13} \end{cases}$$

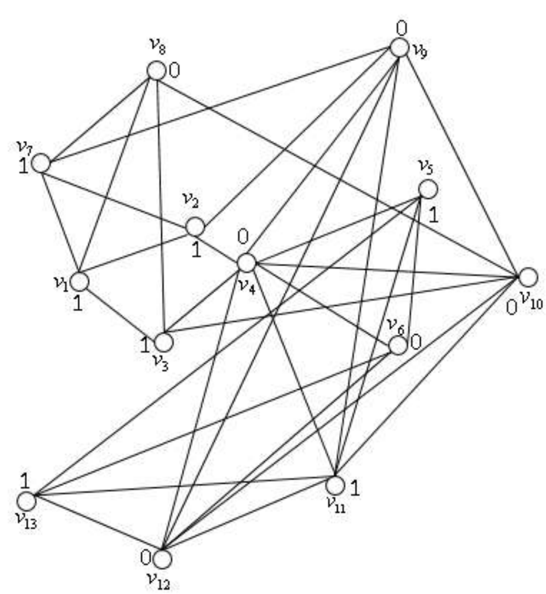
$$|e_{f^*}(1)|=12, |e_{f^*}(0)|=13$$
$$\therefore |e_{f^*}(1)\text{-}e_{f^*}(0)|=1\le 1$$

Hence the middle graph of a Fish graph is a cordial graph.

Theorem 3.2:

The Total graph of a Fish graph is a Cordial graph.

Proof:



The number of vertices of the total graph of Fish graph are 13 and the number of edges of this graph are 32. Suppose the vertices are v_i , where i=1, 2... 13.

$$\text{Let } f(v_i)=\begin{cases} 1 \text{ if i is prime and 1} \\ 0 \text{ if i is not prime} \end{cases}$$

$$v_f(1)=(v_1,v_2,v_3,v_5,v_7,v_{11},v_{13})$$
$$v_f(0)=(v_4,v_6,v_8,v_9,v_{10},v_{12})$$

$$\left|v_f(1)\right| = 7, \left|v_f(0)\right| = 6$$

$$\therefore \left|v_f(1) - v_f(0)\right| = 1 \le 1$$

Hence the induced edge labeling are

$$\text{Let } f^*(v_i v_j) = \begin{cases} 1 \text{ if one of i and j is prime and 1 and other is not} \\ 0 \text{ if both of i and j are either prime and 1 or not} \end{cases}$$

$$e_{f^*}(1) = \begin{cases} v_1v_8, v_2v_4, v_2v_9, v_3v_4, v_3v_8, v_3v_{10}, v_4v_5, v_4v_{11}, \\ v_5v_6, v_6v_{13}, v_7v_8, v_7v_9, v_9v_{11}, v_{10}v_{11}, v_{11}v_{12}, v_{12}v_{13} \end{cases}$$

$$e_{f^*}(0) = \begin{cases} v_1v_2, v_1v_3, v_1v_7, v_2v_7, v_4v_9, v_4v_6, v_4v_{10}, v_4v_{12}, \\ v_5v_{11}, v_5v_{13}, v_6v_{12}, v_8v_{10}, v_9v_{10}, v_9v_{12}, v_{10}v_{12}, v_{11}v_{13} \end{cases}$$

$$\left|e_{f^*}(1)\right| = 16, \left|e_{f^*}(0)\right| = 16$$

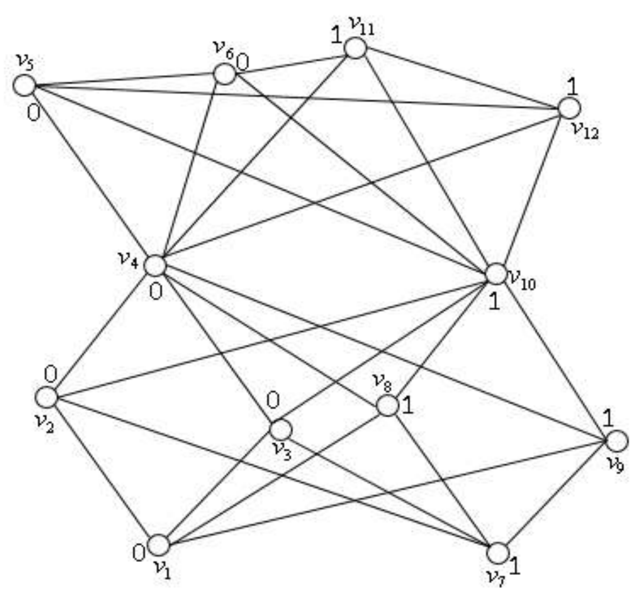
$$\therefore \left|e_{f^*}(1) - e_{f^*}(0)\right| = 0 \le 1$$

Hence the total graph of a Fish graph is a cordial graph.

Theorem 3.3:

The Shadow graph of a Fish graph is a cordial graph.

Proof:



The number of vertices of the Shadow graph 0f Fish graph are 12 and the number of edges of this graph are 28.

Suppose the vertices are v_i , where i=1, 2... 12.

$$\text{Let } f(v_i) = \begin{cases} 1 \text{ if } i \le 6 \\ 0 \text{ if } i > 6 \end{cases}$$

$$v_f(0) = (v_1, v_2, v_3, v_4, v_5, v_6)$$

$$v_f(1) = (v_7, v_8, v_9, v_{10}, v_{11}, v_{12})$$

$$\left|v_f(1)\right| = 6, \left|v_f(0)\right| = 6$$

$$\therefore \left|v_f(1) - v_f(0)\right| = 0 \le 1$$

Hence the induced edge labeling are

$$\text{Let } f^*(v_i v_j) = \begin{cases} 0 \text{ if both i and j are either } \le 6 \text{ or } > 6 \\ 1 \text{ if one of i and j is } \le 6 \text{and other is} > 6 \end{cases}$$

$$e_{f^*}(1) = \begin{cases} v_1v_8, v_1v_9, v_2v_7, v_2v_{10}, v_3v_4, v_3v_{10}, v_4v_8, \\ v_4v_9, v_4v_{11}, v_4v_{12}, v_5v_{10}, v_5v_{12}, v_6v_{10}, v_6v_{11} \end{cases}$$

$$e_{f^*}(0) = \begin{cases} v_1v_2, v_1v_3, v_2v_4, v_3v_7, v_4v_5, v_4v_6, v_5v_6, v_7v_8, \\ v_7v_9, v_8v_{10}, v_9v_{10}, v_{10}v_{11}, v_{10}v_{12}, v_{11}v_{12} \end{cases}$$

$$\left|e_{f^*}(1)\right| = 14, \left|e_{f^*}(0)\right| = 14$$

$$\therefore \left|e_{f^*}(1)\text{-}e_{f^*}(0)\right| = 0 \leq 1$$

Hence the Shadow graph of a Fish graph is a cordial graph.

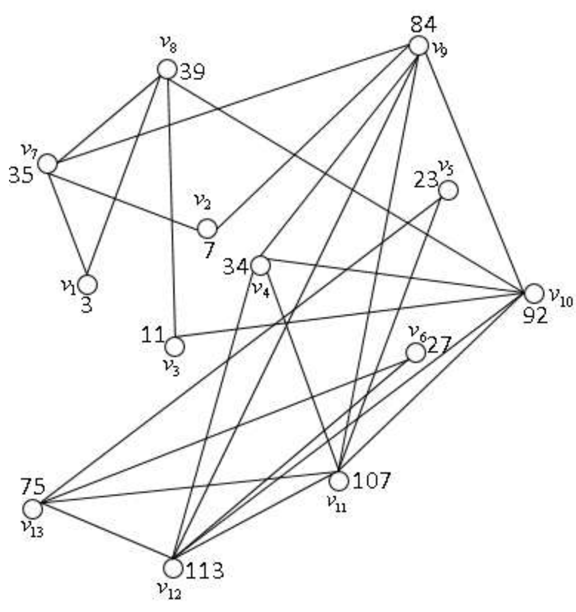
4. Anti-magic Labeling for Family of Fish graph

Theorem 4.1:

The middle graph of a Fish graph is anti-magic.

Proof:

The number of vertices of the middle graph of a Fish graph 8. Let them be v_i , i= 1 to 13. The number of edges of the graph is 13. Select vertex v_1 then find the vertices connected to v_1. Suppose k vertices are connected to v_1 then the edges corresponding it, are labeled as $v_1v_i = e_j$ =j where j=1to k and priority is given to smaller i where i's are among 1 to 13 vertices. Then go for the next vertex v_2 if k' vertices are connected to v_2 label the edges connected to it as $v_2v_i = e_j$ =j where j=k+1 to k+k' and priority is given to smaller i where i's are among 1 to 13 vertices. We end the processes if all edges are labeled.



The number of vertices: 13
The number of edges: 25. They are

$$v_1v_7 = e_1, v_1v_8 = e_2, v_2v_7 = e_3, v_2v_9 = e_4, v_3v_8 = e_5,$$
$$v_3v_{10} = e_6, v_4v_9 = e_7, v_4v_{10} = e_8, v_4v_{11} = e_9, v_4v_{12} = e_{10},$$
$$v_5v_{11} = e_{11}, v_5v_{13} = e_{12}, v_6v_{12} = e_{13}, v_6v_{13} = e_{14}, v_7v_8 = e_{15},$$
$$v_7v_9 = e_{16}, v_8v_{10} = e_{17}, v_9v_{10} = e_{18}, v_9v_{11} = e_{19}, v_9v_{12} = e_{20},$$
$$v_{10}v_{11} = e_{21}, v_{10}v_{12} = e_{22}, v_{11}v_{12} = e_{23}, v_{11}v_{13} = e_{24}, v_{12}v_{13} = e_{25}$$

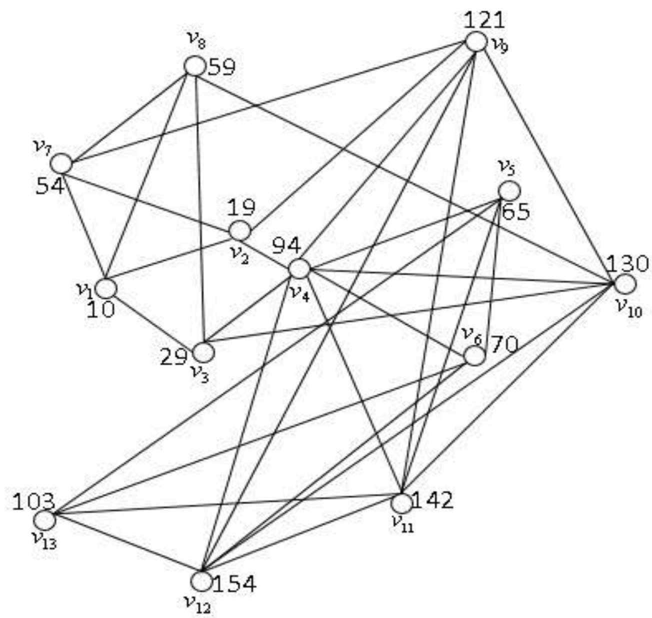
Hence the sum of the labels of the edges incident to the vertices v_i for i=1 to 13 are 3, 7, 11, 34, 23, 27, 35, 39, 84, 22, 23, 24 and 25 respectively. Thus the sums of the labels of the edges incident to each vertex are distinct. Hence the middle graph of a Fish graph is anti-magic.

Theorem 4.2:

The Total graph of Fish graph is anti-magic.

Proof:

The number of vertices of the Total graph of Fish graph is 13. Let them be v_i for i=1 to 13. The number of edges of the graph is 32.



The number of vertices: 13

The number of edges: 32. They are

$$v_1v_2 = e_1, v_1v_3 = e_2, v_1v_7 = e_3, v_1v_8 = e_4, v_2v_4 = e_5, v_2v_7 = e_6, v_2v_9 = e_7,$$
$$v_3v_4 = e_8, v_3v_8 = e_9, v_3v_{10} = e_{10}, v_4v_5 = e_{11}, v_4v_6 = e_{12}, v_4v_9 = e_{13}, v_4v_{10} = e_{14},$$
$$v_4v_{11} = e_{15}, v_4v_{12} = e_{16}, v_5v_6 = e_{17}, v_5v_{11} = e_{18}, v_5v_{13} = e_{19}, v_6v_{12} = e_{20},$$
$$v_6v_{13} = e_{21}, v_7v_8 = e_{22}, v_7v_9 = e_{23}, v_8v_{10} = e_{24}, v_9v_{10} = e_{25}, v_9v_{11} = e_{26},$$
$$v_9v_{12} = e_{27}, v_{10}v_{11} = e_{28}, v_{10}v_{12} = e_{29}, v_{11}v_{12} = e_{30}, v_{11}v_{13} = e_{31}, v_{12}v_{13} = e_{32}$$

Let the edges e_i be labeled by i for i=1 to 32.

Hence the sum of the labels of the edges incident to the vertices v_i for i=1 to 13 are 10, 19, 29, 94, 65, 70, 54, 59, 121,130, 148, 154 and 103 respectively.

Thus the sums of the labels of the edges incident to each vertex are distinct. Hence the total graph of a Fish graph is anti-magic.

Theorem 4.3:

The splitting graph of a Fish graph is anti-magic.

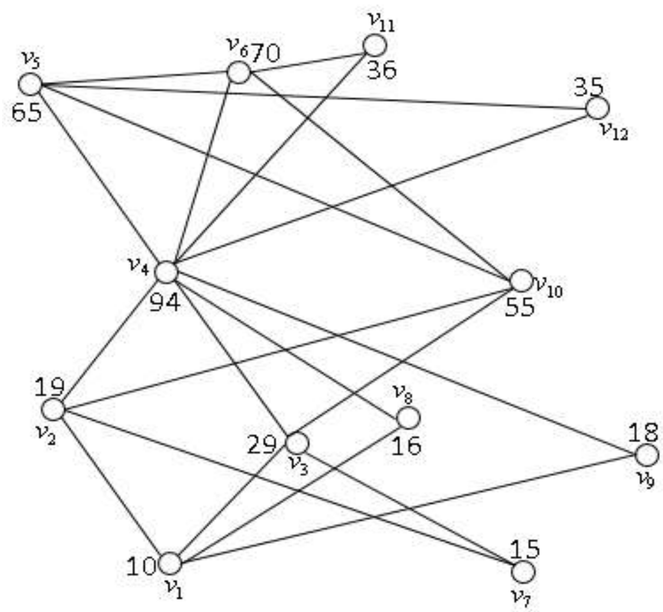
Proof:

The number of vertices of the Total graph of Fish graph is 8. Let them be v_i for i=1 to 12. The number of edges of the graph is 21.

The number of vertices: 12

The number of edges: 21. They are

$$v_1v_2 = e_1, v_1v_3 = e_2, v_1v_8 = e_3, v_1v_9 = e_4, v_2v_4 = e_5, v_2v_7 = e_6, v_2v_{10} = e_7,$$
$$v_3v_4 = e_8, v_3v_7 = e_9, v_3v_{10} = e_{10}, v_4v_5 = e_{11}, v_4v_6 = e_{12}, v_4v_8 = e_{13}, v_4v_9 = e_{14},$$
$$v_4v_{11} = e_{15}, v_4v_{12} = e_{16}, v_5v_6 = e_{17}, v_5v_{10} = e_{18}, v_5v_{12} = e_{19}, v_6v_{10} = e_{20}, v_6v_{11} = e_{21}$$



Let the edges e_i be labeled by i for i=1 to 12.

Hence the sum of the labels of the edges incident to the vertices v_i for i=1 to 12 are 10, 19, 29, 94, 65, 70, 15, 16, 18, 55, 36 and 35 respectively.

Thus the sums of the labels of the edges incident to each vertex are distinct. Hence the splitting graph of a Fish graph is anti-magic.

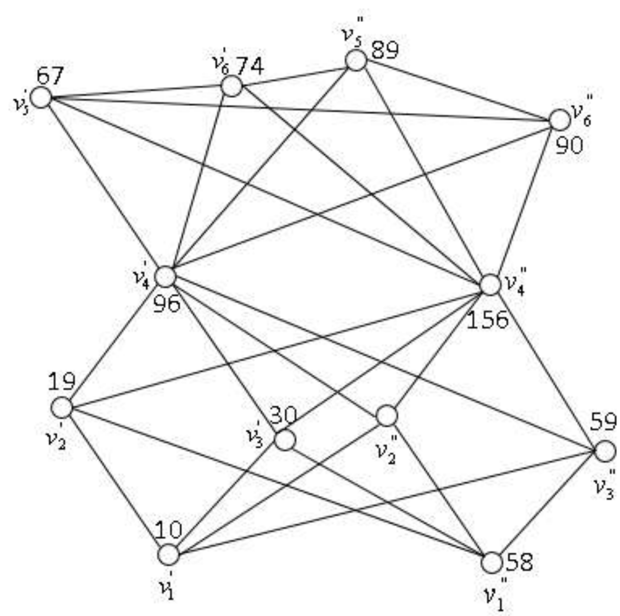
Theorem 4.4:

The shadow graph of a Fish graph is anti-magic.

Proof:

The number of vertices of the shadow graph of a Fish graph is 12.

Let them be $v_i{}'$ for i=1 to 6 and $v_i{}''$ for i=1 to 6. The number of edges of the graph is 28.



The number of vertices: 12

The number of edges: 28. They are

$$v_1'v_2' = e_1, v_1'v_2'' = e_2, v_1'v_3' = e_3, v_1'v_3'' = e_4, v_2'v_1'' = e_5, v_2'v_4' = e_6, v_2'v_4'' = e_7,$$
$$v_3'v_1'' = e_8, v_3'v_4' = e_9, v_3'v_4'' == e_{10}, v_4'v_2'' = e_{11}, v_4'v_3'' = e_{12}, v_4'v_5' = e_{13}, v_4'v_5'' = e_{14},$$
$$v_4'v_6' = e_{15}, v_4'v_6'' = e_{16}, v_5'v_4'' = e_{17}, v_5'v_6' = e_{18}, v_5'v_6'' = e_{19}, v_6'v_4'' = e_{20}, v_6'v_5'' = e_{21},$$
$$v_1''v_2'' = e_{22}, v_1''v_3'' = e_{23}, v_2''v_4'' = e_{24}, v_3''v_4'' = e_{25}, v_4''v_5'' = e_{26}, v_4''v_6'' = e_{27}, v_5''v_6'' = e_{28}$$

Let the edges e_i be labeled by i for i=1 to 28.

Hence the sum of the labels of the edges incident to the vertices v_i' for i=1 to 6 and v_i'' for i=1 to 6. are 10, 19, 30 96, 67, 74, 58, 59, 64, 156, 89 and 90respectively.

Thus the sums of the labels of the edges incident to each vertex are distinct. Hence the shadow graph of a Fish graph is anti-magic

5. Conclusion

In this paper we found the family of a Fish graph. Also we found that not all the graphs are accepting cordial labeling. As an example we proved that the Splitting graph of a Fish graph is not cordial graph. And also we found the graphs which are anti-magic graphs. It is very interesting and challenging as well to find these labeling for the family of a Fish graph which admit these labeling.

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