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Generalized Alpha Closed Sets in Neutrosophic Bipolar Vague Topological Spaces

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A B S T R A C T

This paper is devoted to the study of neutrosophic bipolar vague topological spaces. In this paper neutrosophic bipolar vague generalized alpha closed sets and neutrosophic bipolar vague generalized open sets are introduced. Some of its properties are studied.

Keywords: Neutrosophic bipolar vague topology, neutrosophic bipolar vague generalized alpha closed sets, neutrosophic bipolar vague generalized alpha open sets

1. Introduction

Levine [6] studied the Generalized closed sets in general topology. Several investigations were conducted on the generalizations of the notion of the fuzzy set, after the introduction of the concept of fuzzy sets by Zadeh [15]. In the traditional fuzzy sets, the membership degree of component ranges over the interval [0, 1]. Few types of fuzzy set extensions in the fuzzy set theory are present, for example, intuitionistic fuzzy sets[1], interval-valued fuzzy sets[14], vague sets[12] etc. As a generalization of Zadeh's fuzzy set, the notion of vague set theory was first introduced by Gau W.L and Buehrer D.J [4]. In 1996, H.Bustince and P.Burillo indicated that vague sets are intuitionistic fuzzy sets [2].

In 1995, the definition of Smarandache's neutrosophic set, neutrosophic sets and neutrosophic logic have been useful in many real applications to handle improbability. Neutrosophy is a branch of philosophy which studies the source, nature and scope of neutralities, as well as their interactions with different ideational scales [13]. The neutrosophic set uses one single value to indicate the truth-membership grade, indeterminacy-membership degree and falsity membership grade of an element in the universe X. The conception of Neutrosophic Topological space was introduced by A.A.Salama and S.A.Alblowi [10].

Bipolar-valued fuzzy sets, which was introduced by Lee [7,8] is an extension of fuzzy sets whose membership degree range is extended from the interval [0, 1] to [-1, 1]. The membership degrees of the Bipolar valued fuzzy sets signify the degree of satisfaction to the property analogous to a fuzzy set and its counter-property in a bipolar valued fuzzy set, if the membership degree is 0 it means that the elements are unrelated to the corresponding property. Furthermore if the membership degree is on (0, 1] it indicates that the elements somewhat fulfil the property, and if the membership degree is on [-1,0) it indicates that elements somewhat satisfy the entire counter property. After that, Deli et al. [3] announced the concept of bipolar neutrosophic sets, as an extension lead of neutrosophic sets.

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Neutrosophic vague set is a combination of neutrosophic set and vague set which was well-defined by Shawkat Alkhazaleh [12]. Neutrosophic vague theory is a useful tool to practise incomplete, indeterminate and inconsistent information. Satham Hussain [11] introduced Neutrosophic bipolar vague sets. Mohana K and Princy R [9] have introduced Neutrosophic Bipolar Vague sets in topological spaces.
In this paper, a new class of sets known as neutrosophic bipolar vague generalized alpha closed sets and open sets are introduced in neutrosophic bipolar vague topological spaces. Also some of its characteristics have been analysed and compared.

2. Preliminaries

Definition 2.1: If A={< x, $[T_A^-, T_A^+]^+$, $[I_A^-, I_A^+]^+$, $[F_A^-, F_A^+]^+$, $[T_A^-, T_A^+]^-$, $[I_A^-, I_A^+]^-$, $[F_A^-, F_A^+]^-$>} and

B={< x, $[T_B^-, T_B^+]^+$, $[I_B^-, I_B^+]^+$, $[F_B^-, F_B^+]^+$, $[T_B^-, T_B^+]^-$, $[I_B^-, I_B^+]^-$, $[F_B^-, F_B^+]^-$>} where

$(T^+)^+ = 1-(F^-)^+$, $(F^+)^+ = 1-(T^-)^+$ and $(T^+)^- = -1-(F^-)^-$, $(F^+)^- = -1-(T^-)^- T^+$, I^+, F^+: X→[0,1] and T^-, I^-, F^- :X→[-1,0] are two neutrosophic bipolar vague sets then their union, intersection and complement are well-defined as follows:

1.A∪B= { $\max[T_A^-, T_B^-]^+$, $\max[T_A^+, T_B^+]^+$,
$\min[I_A^-, I_B^-]^+$, $\min[I_A^+, I_B^+]^+$,
$\min[F_A^-, F_B^-]^+$, $\min[F_A^+, F_B^+]^+$,
$\min[T_A^-, T_B^-]^-$, $\min[T_A^+, T_B^+]^-$,
$\max\ [I_A^-, I_B^-]^-$, $\max[I_A^+, I_B^+]^-$,
$\max[F_A^-, F_B^-]^-$, $\max[F_A^+, F_B^+]^-$}.

2.A∩B={ $\min[T_A^-, T_B^-]^+$, $\min[T_A^+, T_B^+]^+$,
$\max[I_A^-, I_B^-]^+$,$\max[I_A^+, I_B^+]^+$,
$\max[F_A^-, F_B^-]^+$, $\max[F_A^+, F_B^+]^+$,
$\max[T_A^-, T_B^-]^-$, $\max[T_A^+, T_B^+]^-$,
$\min\ [I_A^-, I_B^-]^-$, $\min[I_A^+, I_B^+]^-$,
$\min[F_A^-, F_B^-]^-$, $\min[F_A^+, F_B^+]^-$}.

3. $\bar{A}$={<$[F_A^-, F_A^+]^+$, $[1-I_A^-, 1-I_A^+]^+$, $[T_A^-, T_A^+]^+$, $[F_A^-, F_A^+]^-$, $[1-I_A^-, 1-I_A^+]^-$, $[T_A^-, T_A^+]^-$>}.

Definition 2.2: Suppose A and B be two neutrosophic bipolar vague sets defined over a universe of disclosure X. We say that A⊆B if and only if $[T_A^- \leq$ *TB−]+,[TA+≤TB+]+,[IA−≥IB−]+, [IA+≥IB+]+, [FA−≥FB−]+,[FA+≥FB+]+, [TA−≥TB−]−, [TA+≥TB+]−,* $[I_A^- \leq I_B^-]^-$,$[I_A^+ \leq I_B^+]^-$,$[F_A^- \leq F_B^-]^-$, $[F_A^+ \leq F_B^+]^-$.

Definition 2.3: A neutrosophic bipolar vague topology NBVT on a nonempty set X is a family NBV_τ of Neutrosophic bipolar vague set in X sustaining the following axioms:

1. 0, 1 ∈ NBV_τ.
2. $G_1 \cap G_2 \in NBV_\tau$, for any $G_1, G_2 \in NBV_\tau$.
3. $\cup G_i \in NBV_\tau$ for any arbitrary family { G_i: $G_i \in NBV_\tau$, i∈I}.

Under such case the pair (X, NBV_τ) is known as the neutrosophic bipolar vague topological space and any NBVS in NBV_τ is known as bipolar vague open set in X . The complement $\bar{A}$ of a neutrosophic bipolar vague open set (NBVOS) A in a neutrosophic bipolar vague topological space (X, NBV_τ) is referred as a neutrosophic bipolar vague closed (NBVCS) in X.

Definition 2.4: Let A_{NBV} be a NBVS of the universe U where $\forall u_i \in U$, $\hat{T}^P{}_{A_{NBV}}(x) = [1,1]$, $\hat{I}^P{}_{A_{NBV}}(x) = [0,0]$, $\hat{F}^P{}_{A_{NBV}}(x) = [0,0]$, $\hat{T}^N{}_{A_{NBV}}(x) = [-1,-1]$, $\hat{I}^N{}_{A_{NBV}}(x) = [0,0]$,

$$\hat{F}^N{}_{A_{NBV}}(x) = [0,0].$$

Then A_{NBV} is called unit NBVS (1_{NBV} in short), where $1 \leq i \leq n$.

Definition 2.5: Let A_{NBV} be a NBVS of the universe U where $\forall u_i \in U$, $\hat{T}^P{}_{A_{NBV}}(x) = [0,0]$, $\hat{I}^P{}_{A_{NBV}}(x) = [1,1]$, $\hat{F}^P{}_{A_{NBV}}(x) = [1,1]$, $\hat{T}^N{}_{A_{NBV}}(x) = [0,0]$, $\hat{I}^N{}_{A_{NBV}}(x) = [-1,-1]$,

$$\hat{F}^N{}_{A_{NBV}}(x) = [-1,-1].$$

Then A_{NBV} is called zero NBVS (0_{NBV} in short), where $1 \leq i \leq n$.

Definition 2.6: Suppose (X, NBV_τ) is a neutrosophic bipolar vague topological space and

A={< x, $[T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^-$>} be a NBVS inX .

Then the neutrosophic bipolar vague interior and neutrosophic bipolar vague closure of A are well-defined by,

NBVcl(A)=⋂{K:K is a NBVCS in X and A⊆K},

NBVint(A)=⋃{G:G is a NBVOS in X and G⊆A}.

3. Neutrosophic Bipolar Vague Topology

Definition 3.1A NBVS$A = \{< [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- >\}$ in a NBVTS (X, τ) is said to be a neutrosophic bipolar vague semi open set (NBVSCS in short) if $A \subseteq cl(int(A))$.

Every NBVOS in (X, τ) is a NBVSOS in X.

Definition 3.2A NBVS A of a NBVTS (X, τ) is a

i) Neutrosophic bipolar vague pre closed set (NBVPCS in short) if $cl(int(A)) \subseteq$ A.

ii) Neutrosophic bipolar vague pre open set (NBVPOS in short) if $A \subseteq int(cl(A))$.

Definition 3.3 A NBVS A of a NBVTS (X, τ) is a

i) Neutrosophic bipolar vagueα- open set (NBVαOS in short) if $A \subseteq int\left(cl(int(A))\right)$.

ii) Neutrosophic bipolar vagueα- closed set (NBVαCS in short) if $cl\left(int(cl(A))\right) \subseteq$ A.

The family of all NBVαCSs (resp. NBVαOSs) of a NBVTS (X, τ) is denoted by NBVαC(X) (resp. NBVαO(X)).

Definition 3.4A NBVS A of a NBVTS (X, τ) is a

i) Neutrosophic bipolar vagueγ- open set (NBV γOS in short) if $\subseteq int(cl(A)) \cup cl(int(A))$.

ii) Neutrosophic bipolar vagueγ- closed set (NBV γCS in short) if $cl(int(A)) \cap \ int(cl(A)) \subseteq A$.

Defintion 3.5 A NBVS A of a NBVTS (X, τ) is a neutrosophic bipolar vague semi pre open set (NBVSPOS in short) if there exists a NBVPOS B such that $B \subseteq A \subseteq cl(B)$.

Definition 3.6 A NBVS A of a NBVTS (X, τ) is a neutrosophic bipolar vague semi pre closed set (NBVSPCS in short) if there exists a NBVPCS B such that $int(B) \subseteq A \subseteq B$.

The family of all NBVSPCSs (resp. NBVSPOSs) of an NBVTS (X, τ) is denoted by NBVSPC(X) (resp. NBVSPO(X)).

Definition 3.7A NBVS A of a NBVTS (X, τ) is a

i) Neutrosophic bipolar vague regular open set (NBVROS in short) if $A = int(cl(A))$.

ii) Neutrosophic bipolar vague regular closed set (NBVRCS in short) if $A = cl(int(A))$.

Definition 3.8A NBVS A of a NBVTS (X, τ) is a neutrosophic bipolar vague generalized closed set (NBVGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NBVOS in X.

Note that every NBVCS in (X, τ) is a NBVGCS in X.

Definition 3.9A NBVS A of a NBVTS (X, τ) is a neutrosophic bipolar vague generalized open set (NBVGOS in short) ifA^c is a NBVGCS in X.

Definition 3.10 Let A be a NBVS of a NBVTS (X, τ) . Then the semi closure of A (scl(A) in short) is defined as scl(A) = $\cap \{K \mid K \text{ is a } NBVSCS \text{ in } X \text{ and } A \subseteq K\}$.

Definition 3.11 Let A be a NBVS of a NBVTS (X, τ) . Then the semi interior of A (sint(A) in short) is defined as sint(A) = $\cup \{K \mid K \text{ is a } NBVSOS \text{ in } X \text{ and } K \subseteq A\}$.

Definition 3.12A NBVS A of a NBVTS (X, τ) is a neutrosophic bipolar vague generalized semi closed set (NBVGSCS in short) if scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a NBVOS in X.

Note that every NBVCS in (X, τ) is a NBVGSCS in X.

Definition 3.13A NBVS A of a NBVTS (X,τ) is a neutrosophic bipolar vague generalized semi open set (NBVGSOS in short) if A^c is a NBVGSCS in X. The family of all NBVGSCSs (resp. NBVGSOSs) of a NBVTS (X,τ) is denoted by NBVGCS(X) (resp. NBVGSO(X).

Definition 3.14 Let A be a NBVS of a NBVTS (X,τ). Then

i) $scl(A) = A \cup int(cl(A))$.

ii) $sint(A) = A \cap cl(int(A))$.

If A is a NBVS of X then $scl(A^c) = (sint(A))^c$.

Definition 3.15 Let A be a NBVS of a NBVTS (X,τ). Then the pre closure of A (pcl(A) in short) is defined as pcl(A) $= \cap\{K \mid K \; is \; a \; NBVPCS \; in \; X \; and \; A \subseteq K\}$.

Definition 3.16 Let A be a NBVS of a NBVTS (X,τ). Then the pre interior of A (pint(A) in short) is defined as pint(A) $= \cup\{K \mid K \; is \; a \; NBVPOS \; in \; X \; and \; K \subseteq A\}$.

Definition 3.17A NBVS A of a NBVTS (X,τ) is a neutrosophic bipolar vague generalized pre closed set (NBVGPCS in short) if pcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a NBVOS in X.

Definition 3.18A NBVS A of a NBVTS (X,τ) is a neutrosophic bipolar vague generalized pre open set (NBVGPOS in short) if A^c is a NBVGPCS in X.

Definition 3.19 Let A be a NBVS of a NBVTS (X,τ). Then the α closure of A (α cl(A) in short) is defined as α cl(A) $= \cap\{K \mid K \; is \; a \; NBV\alpha CS \; in \; X \; and \; A \subseteq K\}$.

Definition 3.20 Let A be a NBVS of a NBVTS (X,τ). Then the α interior of A (α int(A) in short) is defined as α int(A) $= \cup\{K \mid K \; is \; a \; NBV\alpha OS \; in \; X \; and \; K \subseteq A\}$.

4. Neutrosophic Bipolar Vague Generalized Alpha Closed Sets

In this section we introduce neutrosophic bipolar vague generalized alpha closed sets and study some of their properties.

Definition 4.1 A NBVS A of a NBVTS (X,τ) is a neutrosophic bipolar vaguegeneralized alpha closed set (NBVGαCS in short) if αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a NBVαOS in (X,τ).
The family of all NBVGαCSs of a NBVTS (X,τ) is denoted by NBVG$\alpha C(X)$.

Example 4.2 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.2, 0.3], [0.6,0.6], [0.7,0.8], [-0.5, -0.4], [-0.6, -0.6], [-0.6, -0.5]\rangle$. Here the only α open sets are $0_\sim, 1_\sim \; and \; G$. Then the NBVS $A = \langle x, [0.1,0.2], [0.8,0.8], [0.8,0.9], [-0.4, -0.1], [-0.6, -0.6], [-0.9, -0.6]\rangle$is a NBVG$\alpha$CS in (X,τ).

Theorem 4.3 Every NBVCS in (X,τ) is a NBVGαCS, but not conversely.
Proof. Let $A \subseteq U$ and U is a NBVαOS in (X,τ). Since $\alpha cl(A) \subseteq cl(A)$ and A is a NBVCS, $\alpha cl(A) \subseteq cl(A) = A \subseteq U$. Therefore A is a NBVGαCS in X.

Example 4.4 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.2, 0.3], [0.6,0.6], [0.7,0.8], [-0.5, -0.4], [-0.6, -0.6], [-0.6, -0.5]\rangle$.
Let $A = \langle x, [0.1,0.2], [0.8,0.8], [0.8,0.9], [-0.4, -0.1], [-0.6, -0.6], [-0.9, -0.6]\rangle$ be any NBVS in X. Here $\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ for all NBVαOS G in X. A is a NBVαCS, but not a NBVCS in X, since $cl(A) = G^c \neq A$.

Theorem 4.5 Every NBVαCS is a NBV$G\alpha$CS but not conversely.
Proof. Let $A \subseteq U$ and U is a NBVαOS in (X,τ). By hypothesis $\alpha cl(A) = A$. Hence $\alpha cl(A) \subseteq U$. Therefore A is a NBV$G\alpha$CS in X.
Example 4.6 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where $G = \langle x, [0.4,0.8], [0.4,0.4], [0.2,0.6], [-0.4, -0.2], [-0.5, -0.5], [-0.8, -0.6]\rangle$. Let $A = \langle x, [0.5,0.9], [0.3,0.3], [0.1,0.5], [-0.5, -0.4], [-0.4, -0.4], [-0.6, -0.5]\rangle$be any NBVS in X. Clearly $\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ for all NBVαOS G in X. Therefore A is a NBVGαCS, but not a NBVαCS in X, since $cl\left(int\left(cl(A)\right)\right) = G^c \nsubseteq A$.

Theorem 4.7 Every NBVRCS is a NBVGαCS, but not conversely.

Proof. Let A be a NBVRCS in (X,τ). By definition $A = cl(int(A))$. This implies $cl(A) = cl(int(A))$. Therefore $cl(A) = A$. That is A is a NBVCS in X. By theorem 4.3, A is a NBVGαCS in X.

Example 4.8 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where $G = \langle x, [0.4,0.5], [0.5,0.5], [0.5,0.6], [-0.4,-0.3], [-0.6,-0.6], [-0.7,-0.6\rangle$. Let $A = \langle x, [0.4,0.5], [0.6,0.6], [0.5,0.6], [-0.6,-0.5], [-0.5,-0.4], [-0.5,-0.4]\rangle$be any NBVS in X. A is a NBVGαCS, but not NBVRCS in (X,τ) since $cl(int(A)) = G^c \neq A$.

Example 4.9 Every NBVGCS need not be NBVGαCS in X.

Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where

$G = \langle x, [0.3,0.4], [0.6,0.6], [0.6,0.7], [-0.5,-0.4], [-0.7,-0.7], [-0.6,-0.5]\rangle$.

Let $A = \langle x, [0.4,0.5], [0.6,0.6], [0.5,0.6], [-0.4,-0.3], [-0.6,-0.6], [-0.7,-0.6]\rangle$be any NBVS in X. Here A is a NBVGCS in X. Consider the NBVαOS $G_1 = \langle x, [0.4,0.5], [0.5,0.5], [0.5,0.6], [-0.4,-0.3], [-0.5,-0.5], [-0.7,-0.6]\rangle$. Here $A \subseteq G_1$ but $\alpha cl(A) \nsubseteq G_1$. Hence A is not a NBVGαCS in (X,τ).

Theorem 4.10 Every NBVGαCS is a NBVαGCS in X. But the converse is not true in general.

Proof. Let $A \subseteq U$ and U is a NBVαOS in (X,τ). Since every α open set is an open set, we have $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NBVOS in (X,τ). Hence A is a NBVαGCS in X.

Example 4.11 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where

$G = \langle x, [0.3,0.5], [0.7,0.7], [0.5,0.7], [-0.5,-0.3], [-0.7,-0.7], [-0.7,-0.5]\rangle$.

Let $A = \langle x, [0.4,0.6], [0.4,0.4], [0.4,0.6], [-0.4,-0.5], [-0.4,-0.4], [-0.5,-0.6]\rangle$ be any NBVS in X. Here A is a NBVαGCS in X. Consider the NBVαOS

$G_1 = \langle x, [0.5,0.7], [0.3,0.3], [0.3,0.5], [-0.6,-0.5], [-0.3,-0.3], [-0.5,-0.4]\rangle$. Here $A \subseteq G_1$ but $\alpha cl(A) \nsubseteq G_1$. Hence A is not a NBVGαCS in (X,τ).

Theorem 4.12 Every NBVGαCS is a NBVGSCS but its converse may not be true.

*Proof.*Let $A \subseteq U$ and U is a NBVαOS in (X,τ). By hypothesis, $\alpha cl(A) \subseteq U$, which implies $cl\left(int(cl(A))\right) \subseteq U$. That is, $int(cl(A)) \subseteq U$, which implies $A \cup int(cl(A)) \subseteq U$. Therefore $scl(A) \subseteq U, U$ is a NBVOS. Therefore A is a NBVGSCS in (X,τ).

Example 4.13 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.4,0.5], [0.6,0.6], [0.5,0.6], [-0.4,-0.3], [-0.5,-0.5], [-0.7,-0.6]\rangle$ be any NBVS in X.Here the only α open sets are $0_\sim, 1_\sim$ *and* G. Let $A =< x, [0.3,0.4], [0.6,0.6], [0.6,0.7], [-0.3,-0.2], [-0.6,-0.6], [-0.8,-0.7] >$be any NBVS in X. Then $scl(A) = G$. Clearly $scl(A) \subseteq G$, whenever $A \subseteq G$, for all NBVOS G in X. A is a NBVGSCS in (X,τ), but not NBVGαCS, since $\alpha cl(A) = G^c \nsubseteq G$.

Theorem 4.14 Every NBVGαCS is a NBVGPCS but its converse may not be true.

Proof. Let $A \subseteq U$ and U is a NBVαOS in (X,τ). By hypothesis, $\alpha cl(A) \subseteq U$, which implies $cl\left(int(cl(A))\right) \subseteq U$. That is, $cl(int(A)) \subseteq U$, which implies $A \cup cl(int(A)) \subseteq U$. Therefore $pcl(A) \subseteq U, U$ is a NBVOS. Therefore A is a NBVGPCS in (X,τ).

Example 4.15 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.3,0.5], [0.6,0.6], [0.5,0.7], [-0.5,-0.3], [-0.7,-0.7], [-0.7,-0.5]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ *and* G. Let

$A = \langle x, [0.2,0.3], [0.3,0.3], [0.7,0.8], [-0.4,-0.3], [-0.7,-0.7], [-0.7,-0.6]\rangle$ be any NBVS in X. $pcl(A) \subseteq G$. Therefore A is a NBVGPCS in (X,τ), but not a NBVGαCS, since $\alpha cl(A) = G^c \nsubseteq G$.

Remark 4.16 A NBVP closedness is independent of a NBVGα closedness.

Example 4.17 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.2,0.3], [0.6,0.6], [0.7,0.8], [-0.5,-0.4], [-0.6,-0.6], [-0.6,-0.5]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ *and* G. Let

$A = \langle x, [0.4,0.5], [0.5,0.5], [0.5,0.6], [-0.5,-0.5], [-0.6,-0.6], [-0.5,-0.5\rangle$ be a NBVGαCS(X) but not a NBVPCS(X), since $cl(int(A)) = G^c \nsubseteq G$.

Example 4.18 Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.3,0.4], [0.5,0.5], [0.6,0.7], [-0.4,-0.3], [-0.6,-0.6], [-0.7,-0.6]\rangle$. Let

$A = \langle x, [0.2,0.4], [0.6,0.6], [0.6,0.8], [-0.3,-0.2], [-0.7,-0.7], [-0.8,-0.7]\rangle$ be a NBVPCS(X) but not a NBVGαCS(X), since $\alpha cl(A) = G^c \nsubseteq G$.

Remark 4.19 A NBVS closedness is independent of a NBVGα closedness.

Example 4.20 Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G=\langle x,[0.4,0.5],[0.5,0.5],[0.5,0.6],[-0.4,-0.3],[-0.6,-0.6],[-0.7,-0.6]\rangle$.

Let $A=\langle x,[0.4,0.5],[0.6,0.6],[0.5,0.6],[-0.6,-0.5],[-0.5,-0.4],[-0.5,-0.4]\rangle$ be a NBVGαCS(X) but not a NBVGSCS(X), since $int(cl(A))=G\nsubseteq A$.

Example 4.21 Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G=\langle x,[0.4,0.5],[0.6,0.6],[0.5,0.6],[-0.4,-0.3],[-0.5,-0.5],[-0.7,-0.6]\rangle$.

Let $A=\langle x,[0.4,0.5],[0.6,0.6],[0.5,0.6],[-0.4,-0.3],[-0.5,-0.5],[-0.7,-0.6]\rangle$ be a NBVSCS(X) but not a NBVGαCS(X), since $\alpha cl(A)=G^c\nsubseteq G$.

Remark 4.22 The intersection of any two NBVGαCS is not a NBVGαCS in general as seen from the following example.

Example 4.23 Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where $G=\langle x,[0.2,0.3],[0.7,0.7],[0.7,0.8],[-0.5,-0.4],[-0.7,-0.7],[-0.6,-0.5]\rangle$. Then the NBVSs $A=\langle x,[0.1,0.2],[0.7,0.7],[0.8,0.9],[-0.5,-0.4],[-0.4,-0.5],[-0.6,-0.5]\rangle$,
$B=\langle x,[0.5,0.6],[0.3,0.3],[0.4,0.5],[-0.4,-0.3],[-0.7,-0.7],[-0.7,-0.6]\rangle$ are NBVGαCSsbut $A\cap B$ is not a NBVGαCS in X.

Theorem 4.24 If A is a NBVOS and a NBVGαCS in(X,τ), then A is a NBVαCS in X.

Proof. Let A be a NBVOS in X. Since $A\subseteq A$, by hypothesis $\alpha cl(A)\subseteq A$. But from the definition $A\subseteq \alpha cl(A)$. Therefore $\alpha cl(A)=A$. Hence A is a NBVαCS of X.

Theorem 4.25 Let (X,τ) be a NBVTS. Then $NBV\alpha O(X)=NBV\alpha C(X)$ if and only if every NBVS in (X,τ) is a NBVGαCS.

Proof. **Necessity:**

Suppose that $NBV\alpha O(X)=NBV\alpha C(X)$. Let $A\subseteq U$ and U is a NBVOS in X. This implies $\alpha cl(A)\subseteq \alpha cl(U)$ and U is a $NBV\alpha OS$ in X. Since by hypothesis U is a $NBV\alpha CS$ in $X,\alpha cl(U)=U$. This implies $\alpha cl(A)\subseteq U$. Therefore A is a $NBVG\alpha CS$ of X.

Sufficiency:

Suppose that every NBVS in (X,τ) is a $NBVG\alpha CS$. Let $U\in NBVO(X)$, then $U\in NBV\alpha O(X)$ and by hypothesis $\alpha cl(U)\subseteq U\subseteq \alpha cl(U)$. Therefore $U\in NBV\alpha C(X)$. Hence $NBV\alpha O(X)\subseteq NBV\alpha C(X)$. Let $A\in NBV\alpha C(X)$, then A^c is a $NBV\alpha OS$ in X. But $NBV\alpha O(X)\subseteq NBV\alpha C(X)$. Therefore $A\in NBV\alpha O(X)$. Hence $NBV\alpha C(X)\subseteq NBV\alpha O(X)$. Thus $NBV\alpha O(X)=NBV\alpha C(X)$.

5. Neutrosophic Bipolar Vague Generalized Alpha Open Sets

In this section we introduce neutrosophic bipolar vague generalized alpha open sets and studied some of its properties.

Definition 5.1. A NBVS $A$is said to be a neutrosophic bipolar vague generalized alpha open set $(NBVG\alpha OS\ in\ short)$ in (X,τ) if the compliment A^c is a $NBVG\alpha CS$ in X.

The family of all $NBVG\alpha OSs$ of a $NBVTS(X,\tau)$ is denoted by $NBVG\alpha O(X)$.

Theorem 5.2. For any$NBVTS(X,\tau)$, we have the following:

1. Every $NBVOS$ is a $NBVG\alpha OS$.
2. Every $NBV\alpha OS$ is a $NBVG\alpha OS$.
3. Every $NBVROS$ is a $NBVG\alpha OS$. But the converse is not true in general.

Proof. Straight forward.

Example 5.3. Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G=\langle x,[0.3,0.4],[0.6,0.6],[0.6,0.7],[-0.5,-0.4],[-0.7,-0.7],[-0.6,-0.5]\rangle$. Here the only α open sets are $0_\sim,1_\sim\ and\ G$. Let
$A=\langle x,[0.3,0.5],[0.3,0.3],[0.5,0.7],[-0.7,-0.6],[-0.6,-0.6],[-0.4,-0.3]\rangle$ be any NBVS in $X.A$ is a $NBVG\alpha OS$, but not a $NBVOS$ in X.

Example 5.4. Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G=\langle x,[0.3,0.4],[0.6,0.6],[0.6,0.7],[-0.5,-0.4],[-0.7,-0.7],[-0.6,-0.5]\rangle$. Here the only α open sets are $0_\sim,1_\sim\ and\ G$. Let
$A=\langle x,[0.3,0.5],[0.3,0.3],[0.5,0.7],[-0.7,-0.6],[-0.6,-0.6],[-0.4,-0.3]\rangle$ be any NBVS in X. A is a $NBVG\alpha OS$, but not a $NBV\alpha OS$ in X.

Example 5.5. Let $X=\{a\}$ and let $\tau=\{0_\sim,G,1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G=\langle x,[0.4,0.2],[0.5,0.6],[0.8,0.6],[-0.4,-0.2],[-0.7,-0.5],[-0.8,-0.6]\rangle$. Here the only α open sets are $0_\sim,1_\sim\ and\ G$. Let
$A=\langle x,[0.5,0.6],[0.5,0.5],[0.4,0.5],[-0.5,-0.4],[-0.6,-0.5],[-0.6,-0.5]\rangle$ be any SFS in X. A is a $SFG\alpha OS$, but not a $SFROS$ in X.

Theorem 5.6. For any $NBVTS(X,\tau)$, we have the following:

1. Every $NBVG\alpha OS$ is a $NBVGSOS$.
2. Every $NBVG\alpha OS$ is a $NBVSGOS$.
3. Every $NBVG\alpha OS$ is a $NBVGPOS$. But the converses are not true in general.

Proof. Straight Forward.

Example 5.7. Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.3,0.5], [0.6,0.6], [0.5,0.7], [-0.4,-0.3], [-0.6,-0.6], [-0.7,-0.6]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ $and\ G$.

Let $A = \langle x, [0.6,0.7], [0.4,0.4], [0.3,0.4], [-0.8,-0.7], [-0.3,-0.3], [-0.3,-0.2]\rangle$ be any NBVS in X. A is a $NBVGSOS$, but not a $NBVG\alpha OS$ in X.

Example 5.8. Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.3,0.5], [0.6,0.6], [0.5,0.7], [-0.4,-0.3], [-0.6,-0.6], [-0.7,-0.6]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ $and\ G$. Let

$A = \langle x, [0.6,0.7], [0.4,0.4], [0.3,0.4], [-0.8,-0.7], [-0.3,-0.3], [-0.3,-0.2]\rangle$ be any NBVS in X. A is a $NBVSGOS$, but not a $NBVG\alpha OS$ in X.

Example 5.9. Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.2,0.4], [0.5,0.5], [0.6,0.8], [-0.4,-0.2], [-0.6,-0.6], [-0.8,-0.6]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ $and\ G$. Let

$A = \langle x, [0.7,0.8], [0.5,0.5], [0.2,0.3], [-0.8,-0.7], [-0.2,-0.2], [-0.3,-0.2]\rangle$ be any NBVS in X. A is a $NBVGPOS$, but not a $NBVG\alpha OS$ in X.

Remark 5.10. The union of any two $NBVG\alpha OS$ is not a NBV$G\alpha OS$ in general as seen from the following example.

Example 5.11. Let $X = \{a\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a neutrosophic bipolar vague topology on X, where$G = \langle x, [0.4,0.5], [0.7,0.7], [0.5,0.6], [-0.5,-0.4], [-0.6,-0.5], [-0.6,-0.5]\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ $and\ G$.

Let $A = \langle x, [0.4,0.5], [0.2,0.2], [0.5,0.6], [-0.5,-0.4], [-0.3,-0.3], [-0.6,-0.5]\rangle$,

$B = \langle x, [0.5,0.6], [0.5,0.5], [0.4,0.5], [-0.6,-0.5], [-0.4,-0.4], [-0.5,-0.4]\rangle$ are NBVGαOSs but $A \cup B$ is not a NBVGαOS in X.

Theorem 5.12. A $NBVSA$ of a $NBVTS(X,\tau)$ is a NBVGαOS if and only if $G \subseteq \alpha int(A)$, whenever G is a NBVαCS(X) and $G \subseteq A$.

Proof. **Necessity:**

Assume that A is a NBVGαOS in X. Also let G be a NBVαCS in X such that $G \subseteq A$. Then G^c is a NBVαOS in X such that $A^c \subseteq G^c$. Since A^c is a NBVGαCS, $\alpha cl(A^c) \subseteq G^c$. But $\alpha cl(A^c) = \left(\alpha int(A)\right)^c$. Hence $\left(\alpha int(A)\right)^c \subseteq G^c$. This implies $G \subseteq \alpha int(A)$.

Sufficiency:

Assume that $G \subseteq \alpha int(A)$, whenever G is a NBVαCS and $G \subseteq A$. Then $\left(\alpha int(A)\right)^c \subseteq G^c$, whenever G^c is a NBVαOS and $\alpha cl(A^c) \subseteq G^c$. Therefore A^c is a NBVGαCS. This implies A is a NBVGαOS.

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