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Tuning of a PD-PI Controller to Control Overdamped Second-Order-Like Processes

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A B S T R A C T

This paper presents an effective tuning approach for PD-PI controllers used to control overdamped second-order-like processes having natural frequency up to 1 rad/s and damping ratio from 1 to 1.5. The MATLAB program is used through its optimization toolbox to tune the controller using the ITAE error-based criterion without any functional constraints. The tuning technique used is compared with another technique and the effectiveness of the controller is investigated through comparison with using the minimum ITAE standard forms technique. The effectiveness of the used optimization technique is outlined considering the possibility of kick elimination associated with PID controllers and providing a robust controller.

Keywords:

PD-PI controller, controller tuning, overdamped second-order process, control system performance.

1. Introduction

This is work belongs to the research work published by the author to present a second generation for the conventional PID controllers. This generation comprises controllers and compensators all aiming at getting rid of the kick associated with the used of the conventional PID controllers.

Jain and Nigam (2008) investigated the optimization of PD-PI controller using Swarm Intelligence. They used the performance indices: ITAE, IAE, MSE and settling time. They used the inverted pendulum dynamic system as a case study. The mean square error (MSE) was superior [1]. Hassaan (2014) investigated the tuning of a PD-PI controller to control first order delayed processes. He used the integral of square error (ISE) as an objective function in an optimization technique based on using the MATLAB optimization toolbox. He compared his results with those of Ziegler-Nichols and Tavakoli tuning techniques [2].

Hassaan (2014) investigated the tuning of a PD-PI control used with a highly oscillating second order process having 85.4 % maximum percentage overshoot and 6 seconds settling time. He used the ISE error function as an objective function and could produce a step time response having a step-shape dynamics with zero overshoot and settling time [3]. Hassaan (2014) investigated the disturbance rejection associated with a highly oscillating second order process. He examined the use of ITAE, ISE, IAE, ITSE and ISTSE objective functions to tune the PD-PI controller for this purpose. He showed that the ITAE, ITSE and ISTSE error functions gave almost the same results and could minimize the maximum step disturbance time response to only 0.095 [4].

Panda, Patidar and Kolhe (2016) used cascaded PD-PI and conventional PID controllers to control two area interconneced power system. They tuned the two controllers using the teaching-learning based optimization technique. They concluded that the PD-PI controller performed better than the PID controller in many aspects [5]. Sain (2016) studied the design of PID, I-PD and PD-PI controllers for the control of the nonlinear ball and beam dynamic system. She linearized the nonlinear system around the equilibrium point. She tuned the PID controller using an ITAE performance index and shoed that the PD-PI controller was the best of the three controllers [6].

Singer, Hassaan and Elgamil (2020) investigated the use of a PD-PI controller to control a third order process having 57 % maximum percentage overshoot and 75 seconds settling time. They showed that the tuned PD-PI controller could reduce the maximum percentage overshoot to only 2 % and the settling time to 13.6 seconds and was superior when compared with other controllers [7]. Annamraju, Bhuky and Nandiraju (2021) applied an adaptive fuzzy-based fractional order cascade PD-PI controller to control the frequency in a standalone microgrid. They proposed a controller with the merits of cascaded PD-PI controller, fractional calculus and fuzzy logic approach providing better robustness when compared with other PID-based structures [8].

2. Process

The controlled process is second-order-like process having the transfer function, $G_p(s)$:

$$G_p(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \qquad (1)$$

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Where:

ω_n = process natural frequency

ζ = process damping ratio (> 1)

3. The PD-PI controller

The structure of a PD-PI controller for the control of a linear process is set in the forward path with the process as shown in Fig.1 [1]. This structure is for set-point tracking or disturbance rejection.

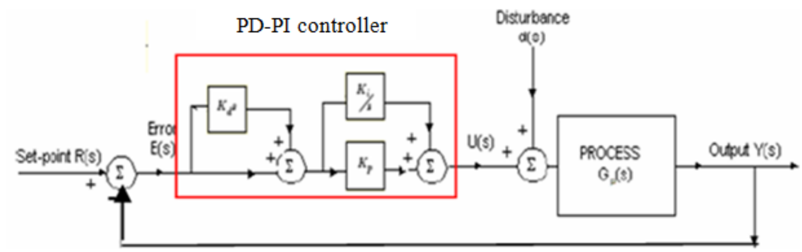

Fig.1 PD-PI controlled process [1].

The PD-PI controller has a transfer function $G_c(s)$ given by:

$$G_c(s) = (1 + K_d s)\,[K_{pc} + (K_i/s)] \qquad (2)$$

Where:

- K_d is the derivative gain
- K_{pc} is the proportional gain
- K_i is the integral gain

4. Control system transfer function

The closed loop transfer function of the control system is obtained using the block diagram of Fig.1 with unit feedback elements and Eqs.1and 2 and given by:

$$M(s) = (b_0 s^2+b_1 s+b_2) /(a_0 s^3+a_1 s^2+a_2 s+a_3) \qquad (3)$$

Where:

$b_0 = K_{pc}K_d\omega_n^2$

$b_1 = \omega_n^2(K_{pc} + K_iK_d)$

$b_2 = \omega_n^2K_i$

$a_0 = 1$

$a_1= 2\zeta\omega_n + K_{pc}K_d\omega_n^2$

$a_2 = \omega_n^2 (1 + K_{pc} + K_iK_d)$

$a_3 = \omega_n^2K_i$

5. Controller tuning and system step time response

The controller parameters are tuned as follows:

- The control and optimization toolboxes of MATLAB are used to assign the three parameters of the controller [9].
- The integral of the time multiplied by the absolute error of the control system (ITAE) is chosen as an objective function for the optimization process.
- The optimization command '*fminunc*' is used to minimise the objective function without using any functional constraints [9].
- The step response of the closed-loop control system is plotted using the command '*step*' of MATLAB [10].
- The controller is tuned using the above approach for an overdamped second process with assigned damping ratio and natural frequency.
- The time-based specifications of the closed- loop control system are extracted using the MATLAB command '*stepinfo*' [11].
- Process parameters covered in the present analysis:
  - Damping ratio: $1 \le \zeta \le 1.5$
  - Natural frequency: $0.1 \le \omega_n \le 1.0$ rad/s
- The tuned parameters of the controller are given in Table 1 through Table 4 depending on the process parameters values.

Table 1- Tuned controller parameters for 0.1 and 0.2 rad/s natural frequency

ω_n	0.1			0.2		
ζ	K_{pc}	K_d	K_i	K_{pc}	K_d	K_i
1	1076.4	1980.2	606.4	820.6	949.9	602.1
1.1	1075.9	1979.3	606.4	820.4	949.8	602.3
1.2	1075.5	1978.3	606.4	820.4	949.8	602.3
1.3	1075.1	1977.4	606.4	820.4	949.7	602.3
1.4	1074.7	1976.5	606.4	820.3	949.6	602.4
1.5	1074.3	1976.0	606.5	820.2	949.5	602.5

Table 2- Tuned controller parameters for 0.3 and 0.4 rad/s natural frequency

ω_n	0.3			0.4		
ζ	K_{pc}	K_d	K_i	K_{pc}	K_d	K_i
1	743.8	664.2	600.7	698.9	487.4	600.3
1.1	743.8	663.9	600.7	698.8	487.2	600.3
1.2	743.7	663.6	600.7	698.8	487.0	600.3
1.3	743.6	663.2	600.7	698.7	486.8	600.3
1.4	743.5	662.9	600.7	698.6	486.6	600.3
1.5	743.3	662.6	600.7	698.5	486.4	600.3

Table 3- Tuned controller parameters for 0.5 and 0.6 rad/s natural frequency

ω_n	0.5			0.6		
ζ	K_{pc}	K_d	K_i	K_{pc}	K_d	K_i
1	691.7	443.2	600.2	657.2	314.3	600.0
1.1	691.6	443.1	600.2	657.2	314.3	600.1
1.2	691.6	443.1	600.2	657.2	314.2	600.1
1.3	691.6	443.0	600.2	657.2	314.2	600.1
1.4	691.6	443.0	600.2	657.2	314.1	600.1
1.5	691.6	443.0	600.2	657.2	314.1	600.1

Table 4- Tuned controller parameters for 0.8 and 1.0 rad/s natural frequency

ω_n	0.8			1.0		
ζ	K_{pc}	K_d	K_i	K_{pc}	K_d	K_i
1	651.5	293.4	600.0	632.0	214.3	600.0
1.1	651.5	293.4	600.0	632.0	214.3	600.0
1.2	651.5	293.4	600.0	632.0	214.3	600.0
1.3	651.5	293.3	600.0	632.0	214.2	600.0
1.4	651.5	293.3	600.0	632.0	214.2	600.0
1.5	651.5	293.2	600.0	632.0	214.2	600.0

- It is obvious from Tables 1 through 4 that the damping ratio of the overdamped process has very little effect on the tuned PD-PI controller parameters. Therefore, it is worthwhile to present the tuned controller parameters in one table as function only of the process natural frequency as shown in Table 5 with the standard deviation around the mean for each controller gain except its integral gain.

- Table 5 reveals important comments:
  - The values between brackets give the standard deviation about the mean of each PDPI controller parameter.
  - The mean derivative gain has zero standard deviation.
  - The mean integral gain has values very close to each other with a mean value of 601.25 with 2.22 standard deviation.

- As an application of the present tuning technique of the PD-PI controller when used with overdamped second-order-like processes we consider a second order process with 0.5 rad/s natural frequency and a 1.3 damping ratio.
- The tuned controller parameters in this case are: $K_{pc} = 691.6$, $K_d = 443.1$ and $K_i = 601.25$.
- The unit step response of the control system incorporating the tuned PD-PI controller and the overdamped second-order-like process is shown in Fig.2 using the tuned parameters of the PD-PI controller as given in Table 5.

Table 5- Mean tuned controller parameters against process natural frequency

ω_n (rad/s)	Mean controller parameters $K_{pc,mean}$	$K_{d,mean}$	$K_{i,mean}$
0.1	1075.4 (0.834)	1978.0 (1.630)	606.4
0.2	820.4 (0.138)	949.7 (0.170)	602.3
0.3	743.6 (0.212)	663.4 (0.590)	600.7
0.4	698.7 (0.132)	486.9 (0.370)	600.3
0.5	691.6 (0.031)	443.1 (0.090)	600.2
0.6	657.2 (0.031)	314.2 (0.070)	600.1
0.8	651.5 (0.031)	293.3 (0.080)	600.0
1.0	632.0 (0.017)	214.2 (0.050)	600.0

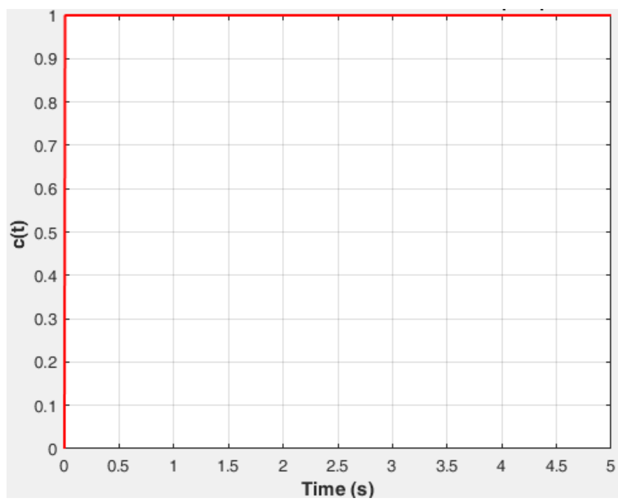

Fig.2 Unit step time response of the PD-PI controlled overdamped second order process.

- Using the tuning technique applied in this paper, the specifications of the closed control system incorporating the PD-PI controller are as follows:

- Maximum percentage overshoot: 0 %
- Settling time: 56 µs
- Gain margin: ∞
- Phase margin: 90^o

6. Comparison with minimum ITAE standard forms tuning

- The parameters of the PD-PI controller is tuned using the minimum ITAE standard forms of Graham and Lathrop [12]. The resulting controller parameters are:

  $K_{pc} = 485.3$

  $K_i = 1$

  $K_d = 0.0138$

- The unit step time response of the closed loop control system incorporating the PD-PI controller and the process with $\zeta = 1.3$ and $\omega_n = 0.5$ rad/s is shown in Fig.3. using both tuning techniques.

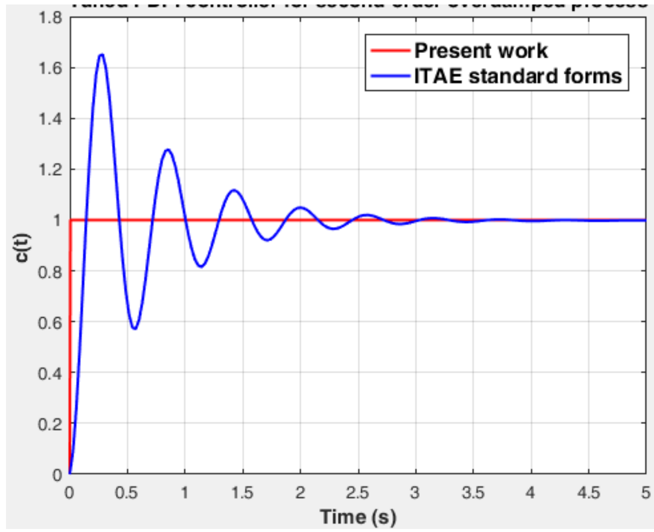

Fig.3 Unit step time response using two tuning techniques.

- The characteristics of the control system using the two tuning techniques are compared in Table 6.

Table 6- Characteristics comparison using two tuning techniques

Tuning technique	Present tuning	ITAE standard forms
OS_{max} (%)	0.0	65.1
T_s (s)	5.6×10^{-5}	2.4
GM (dB)	∞	∞
PM (degrees)	90	15.4

7. Conclusions

- The PD-PI controller was tuned for use with second-order-like overdamped processes damping ratio between 1 and 1.5 (overdamped second order processes) and natural frequency from 0.1 to 1 rad/s.
- It was tuned using the MATLAB optimization toolbox.
- The ITAE error criterion was used as an objective function for the optimization problem of the controller tuning.
- The tuning technique used was compared with another one using the minimum ITAE standard forms.
- The time response kick was almost eliminated using the proposed tuning technique.
- The tuned PD-PI controller was superior compared with the used tuning technique producing a very fast step time response with zero maximum percentage overshoot with kick elimination.
- With infinite Gain Margin and 90 degrees Phase Margin, the tuning technique presented in this paper produced robust controller when used with the overdamped second order-like process.

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DEDICATION

I have the honour to dedicate this research work to Dr. Saad Kassem, Professor of System Dynamics in the Department of Mechanical Design and Production of the Faculty of Engineering, Cairo University. This is because of his continuous support of having laboratory equipments to facilitate running live experiments for the MDP students. Beside his encouragements for valuable innovations within the Dynamics Group of the MDP Department.

BIOGRAPHY

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- Emeritus Professor of System Dynamics and Automatic Control.
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