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IRREDUNDANT COMPLETE COTOTAL DOMINATING NUMBER OF GRAPHS

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ABSTRACT **:The complete cototal domination set is said to be irredundant complete cototal dominating set if for each $u \in S$, $N_G\,[S - \{u\}] \neq N[S]$. The minimum cardinality taken over all an irredundant complete dominating set is called an irredundant complete cototal domination number and is denoted by $\gamma_{ircc}(G)$. Here a new domination parameter called an irredundant complete cototal dominating set was introduced and the study of bounds of $\gamma_{ircc}(G)$ was initiated.**

Keywords: **Irredundant complete cototal domination set, irredundant complete cototaldominating number**.

I. INTRODUCTION

Consider a simple graph G with non-isolated vertices. The total number of vertices are denoted by p and the total number of edges by q. The vertex is said to be pendent if its degree is one and the vertex adjacent to the pendent vertex is known as support vertex. Generally, we follow the terminologies of Harary to learn the basic concepts of graph theory. Domination theory was framed by Claude Berge during 1960's. A set S is called total dominating set if the induced subgraph < S > has no isolated vertices. The total dominating number γ_t is minimum cardinality of a total dominating set [4]. A dominating set is said to be a cototal dominating set if the inducedsubgraph < V-S > has no isolated vertices. The cototal domination number is the minimum cardinality of a cototal dominating set [5]. By introducing a parameter in cototal domination, the irredundant complete cototal dominating set is defined. In this paper we have defined the irredundant complete cototal dominating set and derived the properties and found the bounds of irredundant complete cototal domination number.

II.PRELIMINARIES

Definition 2.1 [2]

A set $S \subseteq V$ is dominating set if each vertex in V is dominated by some vertex in S. The domination number (G) is minimum cardinality of a dominating set.

Definition 2.2 [3]

A dominating set $S \subseteq V(G)$ is called as connected dominating set if induced subgraph $< S >$ is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set.

Definition 2.3 [4]

A dominating set $S \subseteq V(G)$ is called as total dominating set if induced subgraph $< S >$ has no isolated vertices. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set.

Definition 2.4 [5]

A dominating set $S \subseteq (G)$ is called as cototal dominating set if induced subgraph $< V - S >$ hasno isolated vertices. The cototal domination number $\gamma_{cl}(G)$ is the minimum cardinality of a cototal dominating set.

Definition 2.5 [7]

A total dominating set $S \subseteq (G)$ is called as complete cototal dominating set if induced subgraph $< V - S >$ has no isolated vertices. The complete cototal domination number $\gamma_{cc}(G)$ is the minimum cardinality of a complete cototal dominating set.

Definition 2.6 [12]

In the mathematical field of graph theory, the friendship graph (or Dutch windmill graph or *n*-fan) F_n is a planar undirected graph with $2n+1$ vertices and $3n$ edges.

Definition 2.7 [12]

A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset, and every possible edge that could connect vertices in different subsets is part of the graph.

III.MAIN RESULTS

Definition 3.1

A complete cototal dominating set S is called an irredundant complete cototal dominating set if for each $u \in S$, $N_G[S - \{u\}] \neq N_G[S]$. The irredundant complete cototal dominating number is the cardinality of the smallest irredundant complete cototal dominating set and is denoted by $\gamma_{ircc}(G)$.

Theorem 3.2
If a graph G is a Friendship graph F_n for each $n \geq 2$, then $\gamma_{ircc}(G) = 3$.

Proof:
Consider a Friendship graph F_n for each $n \geq 2$ having $2n + 1$ vertices $w_1, w_2, \ldots w_{2n}, v$ and $3n$ edges $w_1w_2, \ldots w_{2n-1}w_{2n}, w_iv$, $1 \leq i \leq 2n$. Middle vertex of F_n for each $n \geq 2$ is v. We take total dominating set as $\{v, u\}$, where u is any w_i such that irredundant complete cototal dominating set is $\{v, u\} \cup \{x\}$, x denotes the isolated vertex. Hence $\gamma_{ircc}(G) = 3$ for a Friendship graph $F_n, n \geq 2$.

Theorem 3.3
If a graph G is complete bipartite graph $k_{m,n}$, then $\gamma_{ircc}(G) = 2$.
Proof:
Consider a complete bipartite graph $k_{m,n}$ having $(m + n)$ vertices $x_1, x_2, \ldots , x_m, y_1, \ldots y_n$ and mn edges x_iy_j, $i = 1,2,\ldots m$, $j = 1,2, \ldots n$. We take total dominating set as $\{x_1, y_1\}$. Hence irredundant complete cototal dominating set is $\{x_1, y_1\}$. Therefore $\gamma_{ircc}(G) = 2$ for a complete bipartite graph $k_{m,}$.

Bounds for $\gamma_{ircc}(G)$.

Theorem 3.4

Let G be a graph with no isolated vertices, then $2 \leq \gamma_{ircc}(G) \leq p$. The proof of

the theorem follows from the definition.

Theorem 3.5
Let G be the graph without isolated vertices , then $\gamma_{ircc}(G) = p$ iff every edge of G is incident to a support vertex.

Proof:

If p=2,3 then the theorem holds. Let $p \geq 4$. Consider $S \subseteq V(G)$ as an irredundant complete cototal dominating set. Therefore all pendent vertices and support vertices belong to S. If every edge of G is incident to a support vertex then $S = V(G)$ which implies $\gamma_{ircc}(G) = p$. Suppose every edge of G is incident to a support vertex then there exist an edge uv such that both are not support vertex and $\deg(u) \geq 2$, $\deg(v) \geq 2$. If $S = V - \{u, v\}$, then S is dominating set and $< V-S >$ is connected. If $< S >$ has no isolated vertices then S is an irredundant complete cototal dominating set with cardinality $p-2$, which is a contradiction. Hence $\gamma_{ircc}(G) = p$.

Theorem 3.6
If G is a nontrivial tree, then $1+\Delta(G) \leq \gamma_{ircc}(G)$.
Proof:
Consider a nontrivial tree G with an irredundant complete cototal dominating set S. Then S has all pendent vertices and support vertices of G. Hence $\gamma_{ircc}(G) \geq 1 + \Delta(G)$

Theorem 3.7

For any nontrivial connected graph G, $\gamma_{ircc}(G) \leq 2q - p + 2$

Proof:

Consider a nontrivial connected graph G. Then $q \geq p - 1$. Also we know that $\gamma_{ircc}(G) \leq p$. Hence it can be rewritten as $\gamma_{ircc}(G) \leq 2(p - 1) - p + 2$, which implies $\gamma_{ircc}(G) \leq 2q - p + 2$.

Theorem 3.8

Let G be a tree then every edge of a tree is incident with a support vertex iff $\gamma_{ircc}(G) = 2q - p + 2$.

Proof:

Let G be tree and every edge of the tree is incident with a support vertex. We have to prove $\gamma_{ircc}(G) = 2q - p + 2$. Since every edge of G is incident to a support vertex, $\gamma_{ircc}(G) = p$. Also we know that $q = p - 1$ for any tree. Hence $\gamma_{ircc}(G) = p = 2(p - 1) - p + 2$ becomes $\gamma_{ircc}(G) = p = 2q - p + 2$.

Theorem 3.9

If G be a graph with $\Delta(G) = p - 1$, then $\gamma_{ircc}(G) \geq \gamma_c(G) + 1$.

Proof:

Let the maximum degree of a graph G be $p - 1$. Then $\gamma_c = 1$. Since $\gamma_{ircc}(G) \leq 2$ we get $\gamma_{ircc}(G) \geq \gamma_c(G) + 1$.

Theorem 3.10

Let G be any graph with non-isolated vertices, then $\frac{2p}{\Delta(G)+1} \leq \gamma_{ircc}(G)$.

Proof:

Let G be a graph without isolates and let the irredundant complete cototal dominating set of G be S. The number of edges in G is denoted by t, having only one vertex in S and all other vertices in $V - S$. Since deg $(v) \leq \Delta(G)$ $\forall\, v \in S$. Each vertex in S is adjacent to atleast one member of S, then $t \leq (\Delta(G) - 1)|S| = (\Delta(G) - 1)\gamma_{ircc}(G)$. And each vertex in $V - S$ is adjacent to atleast one vertex of $V - S$, then $t \leq 2|V - S| = 2[p - \gamma_{ircc}(G)]$. Hence $\Delta(G)\gamma_{ircc}(G) - \gamma_{ircc}(G) \geq 2(p - \gamma_{ircc}(G)]$. Therefore $\frac{2p}{\Delta(G)+1} \leq \gamma_{ircc}(G)$.

Theorem 3.11

For any connected graph G, $\gamma_{ircc}(G) = \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

Proof:

Let the irredundant complete cototal dominating set of G be S. Each vertex of S is a neighbor of at most $\Delta(G) - 1$ vertices of $V - S$ and neighbor of atleast one vertex in S. Therefore $|S|(\Delta(G) - 1) + |S| > n$. Hence $\gamma_{ircc}(G) = \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

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