International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Effect of Wall Thickness on Deformation of Double Spine Mono-Symmetric Box Girder due to Torsional-Distortional Loads

[1]Nwokoye, Ogonna Samuel and [2]Nwabineli, Emmanuel Onochie

[1]Department of Civil Engineering, Federal Polytechnic Oko, Nigeria
ogonnanwokoye@gmail.com
[2]Department of Ceramic and Glass, Akanuibiam Federal Polytechnic, Unwana, Nigeria

ABSTRACT

In this work, the effect of wall thickness on the response of thin-walled double spine mono-symmetric box girder frame to torsional-distortional loads was studied using Vlasov's theory of thin walled structures. The differential equations for torsional-distortional analysis of a double spine mono-symmetric box girder section were obtained following Vlasov's theory. The evaluation of Vlasov's coefficient of the governing differential equations formed a major part of this work. This involved the valuation of the various strain mode diagrams for the box section and an assessment of the strain mode interaction to ascertain the relevant strain modes that affect torsional and distortional displacements Vlasov's coefficient for the governing equations of equilibrium were obtained using Morh's integral for displacement computations [diagram multiplication of the strain modes diagram].By substituting the coefficients into the governing equilibrium equation ,a set of coupled fourth order differential equations of equilibrium were obtained. The solution of these equations [by method of Fourier sine series] gave the torsional and distortional displacement of the 20m simply supported box girder section. The maximum torsional and distortional displacements of the box girder were 2.972mm and 8.773mm respectively. A Matlab computer program was developed for this work and used to verify the result manual analysis and the maximum values for torsional and distortional displacement were 2.975mm and 8.763mm respectively. The thickness of the box girder plates was varied from 0.1m, 0.125m, 0.15m, 0.175m, 0.2m, 0.225m and 0.25m and the result show a decrease in distortional displacement as the thickness of the plate increased

Keywords: Box girder, Distortion, Double spine, Mono-symmetric

I. INTRODUCTION

A box girder bridge superstructure consists of a deck slab, two or more vertical or inclined webs and a bottom slab which results in a single or a multi-cell rectangular or trapezoidal cross-section. The recent applicability of box girders have got wide acceptance in flyover bridge system design due to the following factors: (i) better in stability (ii) serviceability (iii) aesthetics and (iv) Its structural efficiency than T-beam or I-girder, Saxena and Maru (2013). A box girder bridge is a bridge in which the beams are in the shape of hollow box. Box girder can be made of prestressed concrete structural steel or a composite of steel and reinforced concrete. The box is usually rectangular or trapezoidal in cross section. According to Chidolue (2012) "The development of the curved beam theory by saint-venant (1843) and later, the thin-walled beam theory by vlasov's (1965) marked the birth of all research efforts published to date on the analysis and design of straight and curved box girder bridges" there are several methods available for the analysis of box girder bridges.

In each analysis method, the structure is simplified by means of assumptions in the geometry materials and relationship between its components. The accuracy of the structural analysis is dependent upon the method and its assumptions fortunately structural designers are careful enough not to ignore the effect of torsion on a structural member. The effect of warping and distortion on a structural component are however poorly evaluated or ignored in the analysis because of the rigorous analysis involved in their evaluation. There is therefore the need to develop a simple analytical model to enable designers put into consideration the primary, condition of cross sectional deformation in the analysis and design of box girder structures. Thus a better and more elaborate assessment of all the effect of loads on a thin walled box girder bridge structure can be achieved by a consideration of the effect of warping torsion and distortion of thin walled double spine mono symmetric box girder bridge

II. REVIEW OF PAST WORK

Before the advent of Vlasov's 'theory of thin-walled beams the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure. A modification of BEF analogy was developed by Hsu et al (1995) as a practical approach to the distortional analysis of steel box girders. The equivalent beam on elastic foundation (BEF) method as it is called is an enhancement of the BEF method. It is adoptable to the analysis of closed (or quasi-closed) box girders and provides a simplified procedure to account for deformation of the cross section, the effect of rigid or flexible interior diaphragms and continuity over the supports. Osadebe and Mbajiogu (2006) employed the variational principles of cross sectional deformation on the assumption of Vlasov's theory and developed a fourth order differential equation of distortional equilibrium for thin walled box girder structures. Their formulation took into considerations shear deformations which were reflected in the equation of equilibrium by second derivative term. Numerical analysis of a single cell box girder subjected to distortional loading enabled them to evaluate values of distortional displacement, distortional warping stresses and distortional shear which they compared with BEF analogy results and concluded that the effect of shear deformations can be substantial and should not be disregarded under distortional loading. Several investigators; Chidolue et al (2015), Osadebe and Chidolue (2012), Chidolue and Osadebe (2012), and Mbachu and Osadebe (2013), considered torsional, distortional and flexural stresses in thin-walled mono symmetric box girder structures involving single cell and multicell sections on the other hand, Xian and Xu (2014) Sarode and Vesmawala (2014), Ozgur (2017) and Rubeena (2010), considered horizontally curved reinforced concrete box girder bridges for torsional-distortional and flexural stresses while Qiao (2009), Aric and Granata (2016), considered horizontally curved prestressed concrete box girder structures. Arici et al (2015) considered horizontally curved steel box girder structures. Eze (2010) studied reinforced concrete box column based on the numerous literatures consulted in the literature survey the following observations and comments can be made.

1. Research work done on thin-walled box girder structures covers essentially single cell box girder structure and multi-cell box girder structure either straight or curved.
2. Literature on multiple spine box girders appears to be scarce. Thus, there appears to be a dearth of information on the torsional-distortional behavior of thin-walled double spine box girder bridge structure.

III. VLASOV'S STRESS - STRAIN RELATIONS

The longitudinal warping and transverse (distortional) displacements given by Vlasov (1958) are

$$u(x,s) = U(x)\,\varphi(s) \tag{1a}$$

$$v(x,s) = V(x)\,\Psi(s) \tag{1b}$$

Where U(x) and V(x) are unknown functions governing the displacements in the longitudinal and transverse directions respectively, and φ and Ψ are generalized warping and distortional strain modes respectively. These strain modes are known functions of the profile coordinates, and are chosen in advance for any type of cross section. The displacements may be represented in series form as;

$$u(x,s) = \sum_{i=1}^{m} U_i(x)\varphi_i(s) \tag{2a}$$

$$v(x,s) = \sum_{K=1}^{n} V_k(x)\,\Psi_k(s) \tag{2b}$$

Where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the space frame.

$\varphi_i(s)$ and $\Psi_k(s)$ are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements).

These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.

From the theory of elasticity the strain in the longitudinal and transverse directions are given by;

$$\frac{\partial u}{\partial x}(x,s) = \sum_{i=1}^{m} U_i'(x)\varphi_i(s) \text{ and} \qquad a \tag{3}$$

$$\frac{\partial v}{\partial x}(x,s) = \sum_{K=1}^{n} V_k'(x)\Psi_k(s) \qquad b$$

The expression for shear strain is $\gamma(x,s) = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x}$

Or $\boldsymbol{\gamma}(x,s) = \sum_{i=1}^{m} \varphi'_i(s)U_i(x) + \sum_{K=1}^{n} \Psi_k(s)V_k'(x)$ (4) Using the above displacement fields and basic stress-stain relationships of the theory of elasticity the expression for normal and shear stresses become:

$$\sigma(x,s) = E\frac{\partial u}{\partial x}(x,s) = E\sum_{i=1}^{m} \varphi_i(s)U_i'(x) \quad (5)$$

$$\tau(x,s) = G\gamma(x,s) = \left(G \sum_{i=1}^{m}\varphi_i'(s)U_i(x)+ \sum_{k=1}^{n}\Psi_k(s)V_k'(x)\right) \quad (6)$$

The (m+n) functions sought for, u_i (x) and v_k (x), are determined from (m+n) equations for the strip frame, obtained by equating to zero the work done by external and internal forces in (m+n) independent virtual displacements

Every virtual displacement is as a result of an infinitesimal variation experienced by one of the generalized strain coordinates which determine the position of all joints and bars of the frame. This application of the principle of virtual displacements is called the method of variations.

Transverse bending moment generated in the boxes structure due to distortion is given by;

$$M(x,s) = \sum_{k=1} M_k(s)V_k(x) \quad (7)$$

Where $M_k(s)$ = bending moment generated in the cross sectional frame of unit width due to a unit distortion, V(x) = 1

IV. POTENTIAL ENERGY FUNCTIONAL

The potential energy of a box structure under the action of a distortion load of intensity q is given by:

$$\prod = U + W_E \quad (8)$$

Where,

$\prod$ = the total potential energy of the box structure,

U = Strain energy

W_E = External potential or work done by the external loads.

From strength of material, the strain energy of a structure is given by

$$U=\frac{1}{2}\int_L\int_S \left(\sigma^2(x,s)/E + \tau^2(x,s)/G\right)t(s)\ dxds + M^2(x,s)\ EI_{(s)} \quad (9)$$

And work done by external load is given by;

W_E = qv(x,s)dxds

$$=\int_s\int_x q\Sigma V_h(x)\varphi_h(s)dsdx = \int_x \Sigma q_h V_h dx \quad (10)$$

Substituting expressions (9) and (10) into Eqn. (8) we obtain that,

$$\prod=\frac{1}{2}\left[\int_L\int_S \left\{\alpha^2(x,s)/E+ \tau^2(x,s)/G\ t(s)\right\}dxds + M^2(x,s)/EI_{(s)} - qv(x,s)\right] \quad (11)$$

Where,

$\sigma(x,s)$ = Normal stress

$\tau(x,s)$ = Shear stress

M (x,s) = Transverse distortion bending moment

q = Line load per unit area applied in the plane of the plate

$I_{(s)} = \frac{t^3(s)}{12(1-v^2)}$ = moment of inertia

E = Modulus of elasticity

G = Shear modulus

v = poisson ratio

t = thickness of plate

Substituting the expression for $\sigma(x,s)$ (eqn (5), $\tau(x,s)$ eqn. (6), M (x,s) eqn. (7) and $v(x,s)$ eqn. (1) into eqn (11) we obtain that:

$$\prod = E\Sigma\varphi_i(s)U'_i(x)^* \Sigma\varphi_i(s)U_j''(x)^*t(s)dsdx +$$

$$+G[\Sigma\varphi_i'(s)U_i(x)+\Sigma\Psi_k(s)V_k'(x)]^*[\Sigma\varphi_j'(s)U_j(x)+\Sigma\Psi_h(s)V_h'(x)]^*t(s)dsdx +$$

$$+1/El\ [\sum_{K=1}M_k(s)V_k{}^*\sum_{h=1}M_h(s)V_h(x)]dsdx \quad -\int_x \Sigma q_h V_h dx \qquad (12)$$

Simplifying further nothing that t(s)ds=dA we obtain;

$$\prod = \frac{1}{}E\sum_{1=1}^{m}U_i'(x)U_i'(x)^*\sum_{j=i}^{m}\varphi_i'(s)\varphi_j'(s)^*dAdx$$

$$+\frac{1}{2}G\sum_{i=1}^{m}U_i(x)U_j(x)\sum_{j=i}^{m}\varphi_i'(s)\varphi_j'(s)dAdx$$

$$+1/2G\sum_{J=i}^{n}U_j(x)V_k'(x)^*\sum_{K=1}^{n}\varphi_j'(s)\Psi_k(s)^*dAdx$$

$$+1/2G\sum_{h=i}^{n}U_j(x)U_h'(x)^*\sum_{i=1}^{m}\varphi_i'(s)\Psi_h(s)^*dAdx$$

$$+\frac{1}{2}G\sum_{h=1}^{n}\frac{V_k'(x)V_h'(x)}{El_{(s)}}{}^*\sum_{k=1}^{n}\Psi_k(s)\Psi_h(s)^*dAdx$$

$$+\frac{1}{2}\sum_{k=1}^{n}\frac{M_k(x)M_h(x)}{El_{(s)}}{}^*\sum_{k=1}^{n}V_k(x)V_h(x)^*dxds$$

$$-\Sigma q_h V_h dx \qquad (13)$$

Let,

$$a_{ij} = a_{ji} = \int\varphi_i(s)\varphi_j(s)dA \qquad (a)$$

$$b_{ij} = b_{ji} = \int\varphi_i'(s)\varphi_j'(s)dA \qquad (b)$$

$$c_{kj} = c_{jk} = \int\varphi_k'(s)\Psi_j(s)dA \qquad (c)$$

$$c_{ih} = c_{hi} = \int\varphi_i'(s)\Psi_k(s)dA \qquad (d)$$

$$r_{kh} = r_{hk} = \int\Psi_k(s)\Psi_h(s)dA; \qquad (e)$$

$$s_{kh} = s_{hk} = \frac{1}{E}\int\frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad (f)$$

$$q_h = \int q\Psi_h ds \qquad (g) \qquad (14)$$

Substituting eqns. (14) into eqn. (13) gives the potential energy functional:

$$\prod = \frac{1}{2}E\Sigma a_{ij}\, U_i'(x)U_i'(x)dx$$

$$+\frac{1}{2}G\left(\Sigma b_{ij}U_i(x)U_j(x)+\Sigma c_{kj}U_k(x)V_j'(x)\right)$$

$$+\frac{1}{2}G\left(\Sigma c_{ih}U_i(x)V_h'(x)+\Sigma r_{kh}V_k'(x)V_h'(x)\right)dx$$

$$+\frac{1}{2}E\Sigma s_{hk}\, V_k(x)V_h(x)dx \quad -\Sigma q_h V_h dx \qquad (15)$$

The governing equations of distortional equilibrium are obtained by minimizing the above functional eqn. (15), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique, eqns. (15) and (16).

$$\frac{\partial\Pi}{\partial u_j} - \frac{d}{dx}\left(\frac{\partial\Pi}{\partial u_j}\right) = 0 \qquad \text{(a) (16)}$$

$$\frac{\partial\Pi}{\partial V_h} - \frac{d}{dx}\left(\frac{\partial\Pi}{\partial V_h}\right) = 0 \qquad \text{(b)}$$

Carrying out the partial differentiation of eqn. (15) with respect to U_j and U_j' gives

$$\frac{\partial\Pi}{\partial U_j} = G[\Sigma b_{ij}U_i(x) + \Sigma c_{kj}V'_k(x)],$$

$$\frac{\partial\pi}{\partial U_j} = E\Sigma a_{ij}U'_i(x);\ \frac{d}{dx}\frac{\partial\Pi}{\partial U_j} = Ea_{ij}U''_i(x)$$

Therefore $$\frac{\partial\Pi}{\partial U_i} - \frac{d}{dx}\left(\frac{\partial\Pi}{\partial U'_j}\right) = 0$$

$$\Rightarrow G[\Sigma b_{ij}U_i(x) + \Sigma c_{kj}V'_k(x)] - E\Sigma a_{ij}U''_i(x) = 0$$

Or $$E\Sigma a_{ij}U''_i(x) - G\Sigma b_{ij}U_i(x) - G\Sigma c_{kj}V'_k(x) = 0$$

Diving through by G, and re-arranging we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k'(x) = 0 \qquad (17)$$

Where $k = \frac{E}{G} = 2(1+\nu)$

Performing similar operations with respect to V_h and V_h' we obtain the second equation as follows.

$$\frac{\partial\Pi}{\partial V_h} = E\Sigma s_{hk}V_k(x) - \Sigma q_h$$

$$\frac{\partial\Pi}{\partial V_h'} = G[\Sigma c_{ih}U_i(x) + \Sigma r_{kh}V_k'(x)]$$

$$\frac{d}{dx}\left(\frac{\partial\Pi}{\partial V_h'}\right) = G[\Sigma c_{ih}U_i'(x) + \Sigma r_{kh}V_k'(x)]$$

$$\therefore \frac{\partial\Pi}{\partial V_h} - \frac{d}{dx}\left(\frac{\partial\Pi}{\partial V_h'}\right) = -G[\Sigma c_{ih}U_i'(x) + \Sigma r_{kh}V_k'(x)] + E\Sigma s_{hk}V_k(x) - \Sigma q_h = 0$$

$$\Sigma c_{ih}U_i'(x) + \Sigma r_{kh}V_k''(x) - k\Sigma s_{hk}V_k(x) + \frac{1}{G}\Sigma q_h = 0 \qquad (18)$$

Equations (17) and (18) are vlasov's differential equations of distortional equilibrium for a box girder. The matrix form of eqns.(17 and 18) are:

$$k\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ & & \end{pmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ \end{Bmatrix} - \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ & & \end{pmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ \end{Bmatrix} -$$

$$a_{31} \; a_{32} \; a_{33} \qquad U_3'' \qquad b_{31} \; b_{32} \; b_{33} \qquad U_3$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{pmatrix} \begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \qquad (19a)$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix} \begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - k \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} +$$

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix} \begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} = \frac{1}{G} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \qquad (19b)$$

V. STRAIN MODES

From the energy formulation of the equilibrium it was noted that φ and Ψ represent generalized warping and distortional strain modes respectively and from eqns. (2a and 2b) $\varphi_i(s)$ and $\Psi_k(s)$ are elementary displacements) respectively. It was also noted that these displacements are chosen among all displacements possible and are called the generalized strain coordinates of a strip frame. Thus, Vlasov's coefficients of differential equations of equilibrium, eqn.(14), which involve a combination of these elementary displacements and their derivatives may be obtained by consideration of the box girder bridge cross section as a strip frame and then applying unit displacement one after the other at the nodal points of the frame in longitudinal direction, to determine the corresponding out of plane displacements at the joints in **n** possible transverse directions, the corresponding transverse (in-plane) displacements can also be obtained. The first order derivatives of these displacement functions may be obtained by numerical differentiation and used for computation of the coefficients with the aid of Morh's integral for displacement computations.

Consideration of the double spine mono-Symmetric strip frame in fig 1.shows that it has eight degrees of freedom in the longitudinal direction and seven in the transverse direction. From equation (2a and 2b), where in this case m = 8 and n = 7, it follows that we have fifty-six displacement quantities to compute and hence, fifty-six differential equations of distortional equilibrium will be required.The application of Vlasov's generalized strain modes as modified by Varbanov (1970) reduces the number of displacement quantities and hence the differential equations of equilibrium required to solve for them to seven, irrespective of the number of degrees of freedom possessed by the structure.

In the generalized strain modes, there are three strain fields in the longitudinal direction φ_1,φ_2, and φ_3. Thus, from eqn.

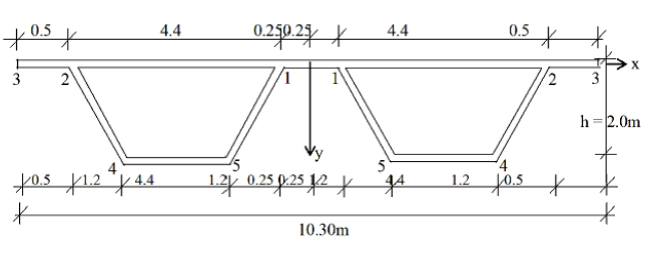


Fig.1 Double Spine Mono-Symmetric Box Girder Section

(2a) we have $\varphi(x,s) = \varphi_1(x)\ \varphi_1(s) + \varphi_2(x)\ \varphi_2\ (s) + \varphi_3(x)\ \varphi_3(s)$ Or

$$\varphi(x,s) = \sum_{i=1}^{3} \varphi_i(x)\ \varphi_i(s) \qquad (20a)$$

In the transverse direction four strain modes are also recognized Ψ_1,Ψ_2 and Ψ_3. Thus, we have $\Psi(x,s) = \Psi_1(x)\ \Psi_1(s) + \Psi_2(x)\ \Psi_2(s) + \Psi_3(x)\ \Psi_3(s) + \Psi_4(x)\ \Psi_4(s)$ Or

$$\Psi(x,s) = \sum_{k=1}^{4} \Psi_k(x)\ \Psi_k(s) \qquad (20b)$$

Where φ_1 = out of plane displacement parameter when the load is acting (vertically) normal to the top flange of the girder, i.e. bending is about horizontal axis.
φ_2 = out of plane displacement parameter when the load is acting tangential to the plane of the flanges i.e. bending is about vertical axis.
φ_3 = out of plane displacement parameter due to distortion of the cross section i.e; the warping function.
Ψ_1 = In-plane displacement parameter due to the load giving rise to φ_1
Ψ_2 = In-plane displacement parameters due to the load giving rise to φ_2
Ψ_3 = In-plane displacement parameter due to the distortion of the cross section i.e non uniform torsion.
Ψ_4 In-plane displacement functions due to pure rotation or Saint Venant torsion of the cross section.

IV. STRAIN MODE DIAGRAMS

Consider a simply supported girder loaded as shown in Fig.2a. if we assume the normal bean theory, i.e.; neutral axis remaining neutral before and after bending then the distortion of the cross section will be as shown in Fig.2 where, θ is the distortion angle (rotation of the vertical axis). The displacement φ_1 at any distance R, from the centroid is given by $\varphi_1 = R\theta$. If we assure a unit rotation of the vertical (z) axis then $\varphi_1 = R$, at any point on the cross section. Note that φ_1 can be positive or negative depending on the value of R, in the tension or compression zone of the girder. Thus, φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.

Similarly, if the load is acting in a horizontal (y-y) direction, normal to the x-z plane in Fig.2, then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2, displacement out of plane. The values of φ_2, are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian.

The warping function $\varphi_{3,}$ of the beam cross section is obtained as detailed in Fig.3a it has been explained that the warping function is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either x or y direction and without longitudinal extension. Ψ_1 and Ψ_2 are in-plane displacement of the cross section in x-z and x-y planes respectively while Ψ_3 is the distortion of the cross section. They can be obtained by numerical differentiation of $\varphi_{1,}$ $\varphi_{2,}$ and φ_3 diagrams respectively.

Ψ_4 is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, Ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. It is assumed to be positive if the member moves in the positive directions of the coordinate axis and negative otherwise.

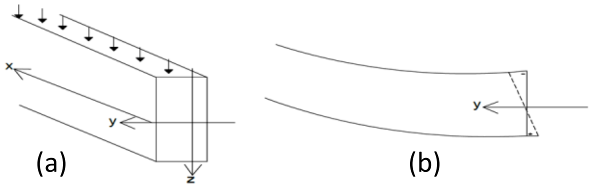
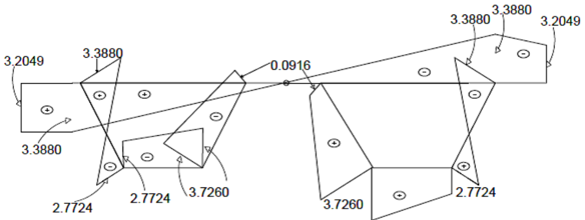
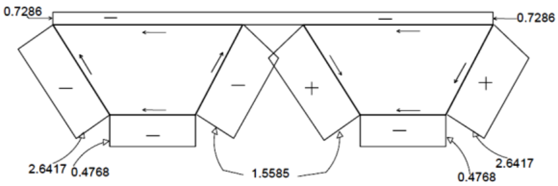


Fig.2 Simply Supported Girder Section and Cross Section Distortion

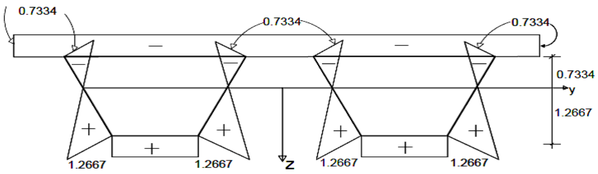


(a) Warping function ω_m (φ_3)

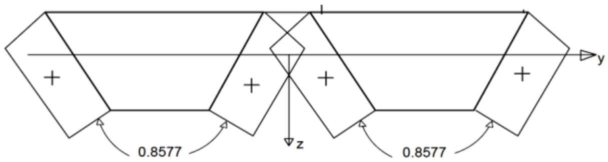
Fig.3 Generalized Strain Mode Diagram



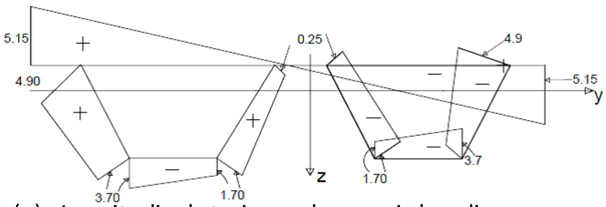
(b) Distortion diagram $\varphi'_3 = \Psi_3$



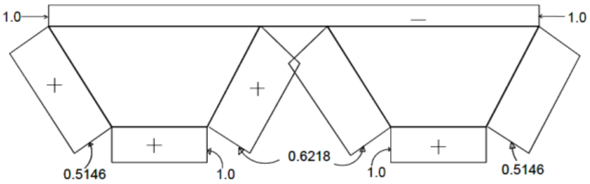
(c) Longitudinal strain mode diagram y-y axis bending φ_1



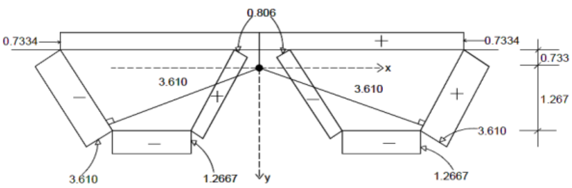
(d) Transverse strain mode in y direction Ψ_1



(e) Longitudinal strain mode z-z axis bending φ_2



(f) Transverse strain mode in z direction Ψ_2



(g) Rotation strain mode , Ψ_4

For monosymmetric section, the relevant Vlasov's coefficients for Torsional-distortional equilibrium are a_{33}, $b_{33} = r_{33}$, $r_{34} = r_{43}$, r_{44}

$a_{33} = \int \varphi_3(s)\, \varphi_3(s)\, dA \quad = 24.682$

$b_{33} = \int \varphi^1_3 \quad \varphi^1_3\, (s)\, dA \quad = 9.918$

$r_{34} = \int \Psi_{3}s\ \Psi_{4}s\, dA \quad = 7.107$

$r_{44} = \int \Psi_4 \Psi_4 \, dA$ $= 15.33$

Note,

$b_{33} = C_{33} = r_{33} = 9.918$

$r_{34} = r_{43} = 7.107$

The coefficient $S_{hk} = S_{kh} = \frac{I}{E}\int \frac{M_3(S) M_3(S)}{EIs}$ (21)

Where M_3 (s) is the distortional bending moment

VI. DETERMINATION OF DISTORTIONAL BENDING MOMENT FOR THE BOX GIRDER

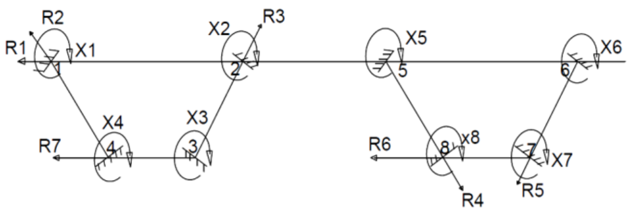


Fig. 4 Base System for Evaluation of Distortion Bending Moment

Fig.3a shows the base system for the evaluation of distortional bending moment for the double spine mono-symmetric box girder. The evaluation of the distortional bending moment involves the application of unit rotation $X_1$to X_8 at joint 1 to 8 respectively and applying unit transverse displacement of joints based on distortion diagram

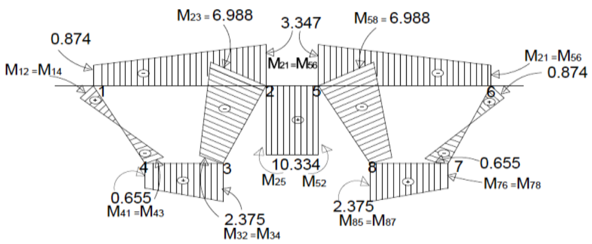


Fig.5 Bending Moment Diagram Due to Distortion of the Cross Section

Shk = shk = $\frac{1}{E}\int \frac{M_k(s) M_h(s)}{EIs}$ ds

Shk = Skh = $S_{33} = 2.891 \times 10^{-3} I_s$ only S_{33} has value

VII. FORMULATION OF DIFFERENTIAL EQUATIONS OF EQUILIBRIUM

The relevant coefficients for torsional-distortional equilibrium are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{43}, r_{44}, and s_{33}. Substituting these into the matrix notation equation (8) and (9) we obtain:

$$k\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & b_{33}\end{bmatrix}\begin{bmatrix}U_1''\\ U_2''\\ U_3''\end{bmatrix} - \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & b_{33}\end{bmatrix}\begin{bmatrix}U_1\\ U_2\\ U_3\end{bmatrix} - \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & b_{33}\end{bmatrix}\begin{bmatrix}V_1'\\ V_2'\\ V_3'\\ V_4'\end{bmatrix} = 0$$

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & C_{33}\\ 0 & 0 & C_{43}\end{bmatrix}\begin{bmatrix}U_1''\\ U_2''\\ U_3''\end{bmatrix} - K\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & S_{33} & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ V_3\\ V_4\end{bmatrix} +$$

$$\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & r_{33} & r_{34}\\ 0 & 0 & r_{43} & r_{44}\end{bmatrix} = -\frac{1}{G}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}$$

Multiplying out we obtain

$Ka_{33}\, U_3'' - b_{33}U_3 - C_{33}V_3' - C_{34}V_4' = 0$

$C_{33}\, U_3' - KS_{33}V_3 + r_{33}V_3' + r_{34}V_4' = -\frac{q_3}{G}$

$C_{43}\, U_3' - r4_3V_3' + r_{44}V_4' = -\frac{q_4}{G}$

Simplifying further we obtain

$\beta_1 V_4'' - \gamma_1 V_3 = K_1$ (22a)

$\propto_1 V_3'^{v} + \propto_2 V_4'^{v} - \beta_2 V_4'' = K_2$ (22b)

Where $\propto_1 = ka_{33}c_{43}$; $\propto_2 = ka_{33}\, r_{44}$;

$\beta_1 = r_{34}\, c_{43} - c_{33}\, r_{44}$

$\beta_2 = b_{33}\, r_{44} - c_{34}\, c_{43}$;

$\gamma_1 = c_{43}\, ks_{33}$ (23)

$K_1 = -C_{33}\,\frac{q_4}{G} - C_{43}\,\frac{q_3}{G}$; $K_2 = \left(\frac{b_{33}q_4}{G}\right)$ (24)

Torsional – Distortional Analysis of Mono-Symmetric Box Girder Structure

In this section the solutions of the differential equations of equilibrium are obtained for the double spine mono-symmetric box girder section shown in fig.1. Live loads are considered according to AASHTO-LRFD following the HL-93 loading. Uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A two span simply supported bridge deck structure, 20m per span, was considered. The obtained torsional loads are as follows

$q_3 = 1410.318$KN, $q_4 = 3732.202$ KN

Parameters for the governing equations (22a and 22b) are:

$\propto_1 = K\, a_{33}\, C_{43}$; $\propto_2 = Ka_{33}\, r_{44}$

$\beta_1 = r_{34}\, C_{43} - C_{33}\, r_{44}$; $\beta_2 = b_{33}\, r_{44} - C_{34}\, C_{43}$

$\gamma_1 = C_{43}\, K\, S_{33}$; $K_1 = C_{33}\frac{E4}{G} = C_{43}\,\frac{E3}{G}$

$K_2 = b_{33}\,\frac{q_4}{G}$; $S_{33} = 2.891 \times 10^{-2}\, I_s$

$k = 2\,(1 + V)$; $k = 2\,(1 + 0.25) = 2.5$; $\nu = 0.25$ for concrete

$E = 24 \times 10^9$ N/m^2; $G = 9.6 \times 10^9$ N/m^2

$\therefore \propto_1 = 2.5 \times 24.682 \times 7.107 = 438.537$

$\propto_2 = 2.5 \times 24.682 \times 15.153 = 935.016$

$\beta_1 = 7.107 \times 7.107 - 9.918 \times 15.153 = -99.778$

$\beta_2 = 9.918 \times 15.153 - 7.107 \times 7.107 = 99.778$

$\gamma_1 = 7.107 \times 2.5 \times 2.891 \times 10^{-2} = 0.5137$

$K_1 = 9.918 \times \frac{3732.202 \times 10^3}{9.6 \times 10^9} - 7.107 \times \frac{1410.318 \times 10^3}{9.6 \times 10^9} = 0.0028109$

$K_2 \;=\; \frac{(9.918\ x\ 3732.202\ x10\ )}{9.6\ x10^{9}} = 3.856x10^{-3}$

Substituting the coefficients $\propto_1, \propto_2, \beta_1, \beta_2, \gamma_1, K_1$ and K_2 We obtain equations (25) and (26) below

$438.537\ V_3^{IV} + 935.016\ V_4^{1V} - 99.778\ V_4^{II} = 3.856\ X\ 10^{-3}$ (25a)

$-99.778V_4^{II} - 0.5137\ V_3 = 2.811\ x\ 10^{-3}$ (25b)

Integrating by method of Trigonometric Series with accelerated convergence we obtain

$V_3\ (x) = 8.773\ x\ 10^{-3} \sin \frac{\pi x}{20}$ (26) $V_4\ (x) = 2.972\ x\ 10^{-3} \sin \frac{\pi x}{20}$

Table 1: Variation of torsional and distortional displacements along the length of the girder (20m simply supported)

Distance x from left support (m)	$\sin \frac{\pi x}{20}$	Distortional Displacement V_3 (x) x 10^{-3} m	Torsional Displacement V_4 (x) x10^{-3} m
0	0.000	0.000	0.000
2	0.309	2.711	0.918
4.	0.588	5.158	1.747
6.	0.809	7.097	2.404
8.	0.951	8.343	2.826
10	1.000	8.773	2.972
12	0.951	8.343	2.826
14.	0.809	7.097	2.404
16	0.588	5.158	1.747
18	0.309	2.711	0.918
20	0.000	0.000	0.000

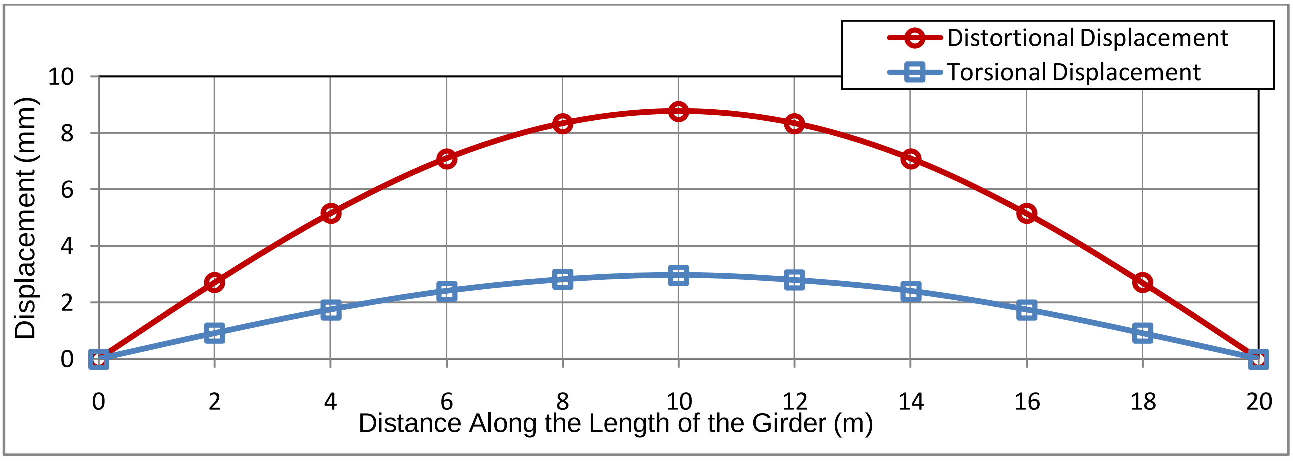


Fig. 6 Variation of torsional and distortional displacement along the length of the girder

Double Spine Mono-Symmetric Box Girder Section with Varied Thickness of the Plate.

In this section, the box girder plate thickness was changed from 200 mm for all plates to 100 mm, 125 mm, 150 mm, 175 mm, 225 mm and 250 mm for all plates and torsional-distortional analysis conducted for the box with the various thickness as stated above. The results of the analysis are shown for the warping function of the box girder, the Vlasov's coefficient, the distortional moment and torsional-distortional displacements.

```
Box girder analysis with plate thickness 0.1m
Please provide input configuration in metres.
Enter box girder height: 2
Enter box girder thickness: .1
Enter distance center to box: .25
Enter top flange span: 4.4
Enter cantilever span: .5
Enter outer web horizontal distance: 1.2
Enter bottom flange span: 2
Enter inner web horizontal distance: 1.2
Enter the longitudinal span: 20
=======================================================================
Analysis Summary
=======================================================================

Box Input Configurations
------------------------------------------------------------------------
Height of box:                  2m
Thickness of box:               0.1m
Center to right box span:       0.25m
Center to left box span:        0.25m
Right box top flange span:      4.4m
Left box top flange span:       4.4m
Right cantilever span:          0.5m
Left cantilever span:           0.5m
Right outer web horizontal range:   1.2m
Left outer web horizontal range:    1.2m
Right outer web span:           2.3324m
Left outer web span:            2.3324m
Right box bottom flange:        2m
Left box bottom flange:         2m
Right inner web horizontal range:   1.2m
Left inner web horizontal range:    1.2m
Right inner web span:           2.3324m
Left inner web span:            2.3324m

Section Properties
------------------------------------------------------------------------
Cross sectional area of profile:          2.363 m^2
First moment of area of section about y:     0 m^3
First moment of area of section about x:     0 m^3
First moment of sectional area about pole B: 0 m^3
Second moment of area of section about y:    19.7408 m^4
Second moment of area of section about x:    2.8439 m^4
Sectorial product of area:                -7.2262 m^4

The coordinates of the centroid
-------------------------------
x_bar_s: 0 m
y_bar_s: 0.73338 m
```

Warping Function Values for Double Spine Monosymmetric Box Girder

----

Location	xm	wB	0.36606x	wm
0	0	0	0	0
1	0.25	0	0.091514	0.091514
2	4.65	-5.0908	1.7022	-3.3886
3	5.15	-5.0908	1.8852	-3.2056
4	3.45	1.5106	1.2629	2.7735
5	1.45	3.1966	0.53078	3.7274
1	-0.25	0	-0.091514	-0.091514
2	-4.65	5.0908	-1.7022	3.3886
3	-5.15	5.0908	-1.8852	3.2056
4	-3.45	-1.5106	-1.2629	-2.7735
5	-1.45	-3.1966	-0.53078	-3.7274

Bending Moments Values Due To Distortion of the Section

M_12 = -0.87443
M_21 = -3.3511
M_23 = -6.9943
M_32 = -2.3759
M_25 = 9.9312
M_52 = 9.9312
M_34 = 2.3759
M_43 = 0.65498
M_41 = -0.65498
M_14 = 0.87443
M_56 = -3.3511
M_65 = -0.87443
M_67 = 0.87443
M_76 = -0.65498
M_78 = 0.65498
M_87 = 2.3759

Vlasov's Coefficients for Torsional/Distortional Analysis
of the Double Spine Monosymmetric Box Girder Section
a_33: 12.3587
b_33: 4.9614
c_33: 4.9614
r_33: 4.9614
r_34: 3.1595
r_44: 7.9922
s_33: 0.0016907

Variation Of Torsional And Distortional Displacements Along The Length
Of The Box Girder (L metres simply supported) Table.

X(m)	Sin(pix)/20	V3(m)	V4(m)
0	0	0	0
1	0.15643	0.0056496	0.00073454
2	0.30902	0.01116	0.001451
3	0.45399	0.016396	0.0021317
4	0.58779	0.021228	0.0027599
5	0.70711	0.025537	0.0033202
6	0.80902	0.029218	0.0037987
7	0.89101	0.032179	0.0041837
8	0.95106	0.034347	0.0044657
9	0.98769	0.03567	0.0046377

10	1	0.036115	0.0046955
11	0.98769	0.03567	0.0046377
12	0.95106	0.034347	0.0044657
13	0.89101	0.032179	0.0041837
14	0.80902	0.029218	0.0037987
15	0.70711	0.025537	0.0033202
16	0.58779	0.021228	0.0027599
17	0.45399	0.016396	0.0021317
18	0.30902	0.01116	0.001451
19	0.15643	0.0056496	0.00073454
20	1.2246e-16	4.4228e-18	5.7503e-19

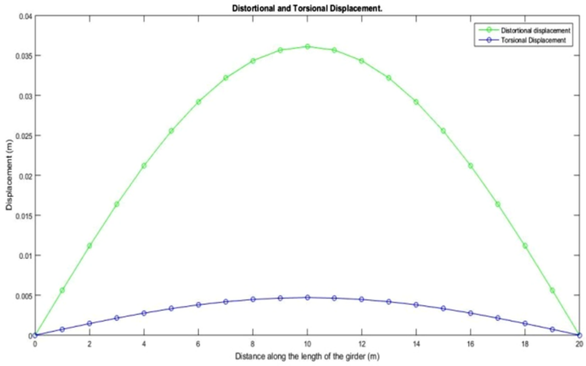


Fig 4

Box girder analysis with plate thickness 0.125m
Please provide input configuration in metres.
Enter box girder height: 2
Enter box girder thickness: .125
Enter distance center to box: .25
Enter top flange span: 4.4
Enter cantilever span: .5
Enter outer web horizontal distance: 1.2
Enter bottom flange span: 2
Enter inner web horizontal distance: 1.2
Enter the longitudinal span: 20

==

Analysis Summary

==

Box Input Configurations

Height of box: 2m
Thickness of box: 0.125m
Center to right box span: 0.25m
Center to left box span: 0.25m
Right box top flange span: 4.4m
Left box top flange span: 4.4m
Right cantilever span: 0.5m
Left cantilever span: 0.5m
Right outer web horizontal range: 1.2m
Left outer web horizontal range: 1.2m
Right outer web span: 2.3324m
Left outer web span: 2.3324m
Right box bottom flange: 2m
Left box bottom flange: 2m
Right inner web horizontal range: 1.2m
Left inner web horizontal range: 1.2m
Right inner web span: 2.3324m
Left inner web span: 2.3324m

Section Properties

Cross sectional area of profile: 2.9537 m^2
First moment of area of section about y: 0 m^3
First moment of area of section about x: 0 m^3
First moment of sectional area about pole B: 0 m^3
Second moment of area of section about y: 24.6759 m^4
Second moment of area of section about x: 3.5549 m^4
Sectorial product of area: -9.0267 m^4

The coordinates of the centroid

----

x_bar_s: 0 m
y_bar_s: 0.73338 m

Warping Function Values for Double Spine Monosymmetric Box Girder

----

Location	xm	wB	0.36581x	wm
0	0	0	0	0
1	0.25	0	0.091453	0.091453
2	4.65	-5.0899	1.701	-3.3889
3	5.15	-5.0899	1.8839	-3.206
4	3.45	1.512	1.262	2.774
5	1.45	3.1984	0.53043	3.7288
1	-0.25	0	-0.091453	-0.091453
2	-4.65	5.0899	-1.701	3.3889
3	-5.15	5.0899	-1.8839	3.206
4	-3.45	-1.512	-1.262	-2.774
5	-1.45	-3.1984	-0.53043	-3.7288

Bending Moments Values Due To Distortion of the Section

M_12 = -0.87514
M_21 = -3.3527
M_23 = -6.9966
M_32 = -2.3762
M_25 = 9.9349
M_52 = 9.9349
M_34 = 2.3762
M_43 = 0.65463
M_41 = -0.65463
M_14 = 0.87514
M_56 = -3.3527
M_65 = -0.87514
M_67 = 0.87514
M_76 = -0.65463
M_78 = 0.65463
M_87 = 2.3762

Vlasov's Coefficients for Torsional/Distortional Analysis
of the Double Spine Monosymmetric Box Girder Section
a_33: 15.4549
b_33: 6.2042
c_33: 6.2042
r_33: 6.2042
r_34: 3.95
r_44: 9.9903
s_33: 0.0041307

Variation Of Torsional And Distortional Displacements Along The Length Of The Box Girder (L metres simply supported) Table.

X(m)	Sin(pix)/20	V3(m)	V4(m)
0	0	0	0
1	0.15643	0.0039485	0.00062612
2	0.30902	0.0077998	0.0012368
3	0.45399	0.011459	0.0018171
4	0.58779	0.014836	0.0023526
5	0.70711	0.017848	0.0028301
6	0.80902	0.02042	0.003238
7	0.89101	0.02249	0.0035662
8	0.95106	0.024005	0.0038065
9	0.98769	0.02493	0.0039532
10	1	0.025241	0.0040024
11	0.98769	0.02493	0.0039532
12	0.95106	0.024005	0.0038065
13	0.89101	0.02249	0.0035662
14	0.80902	0.02042	0.003238
15	0.70711	0.017848	0.0028301
16	0.58779	0.014836	0.0023526
17	0.45399	0.011459	0.0018171
18	0.30902	0.0077998	0.0012368
19	0.15643	0.0039485	0.00062612
20	1.2246e-16	3.0911e-18	4.9016e-19

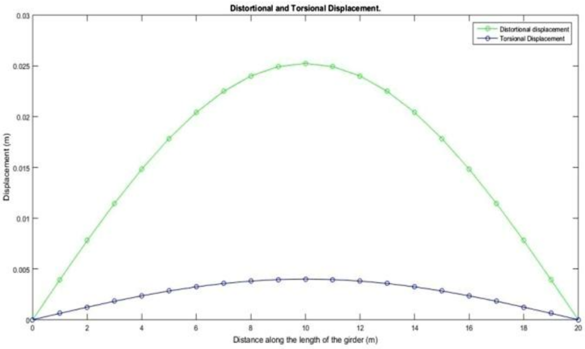


Fig5

Box girder analysis with plate thickness 0.150m
Please provide input configuration in metres.
Enter box girder height: 2
Enter box girder thickness: .15
Enter distance center to box: .25
Enter top flange span: 4.4
Enter cantilever span: .5
Enter outer web horizontal distance: 1.2
Enter bottom flange span: 2
Enter inner web horizontal distance: 1.2
Enter the longitudinal span: 20

==

Analysis Summary

==

Box Input Configurations

Height of box: 2m
Thickness of box: 0.15m
Center to right box span: 0.25m

Center to left box span: 0.25m
Right box top flange span: 4.4m
Left box top flange span: 4.4m
Right cantilever span: 0.5m
Left cantilever span: 0.5m
Right outer web horizontal range: 1.2m
Left outer web horizontal range: 1.2m
Right outer web span: 2.3324m
Left outer web span: 2.3324m
Right box bottom flange: 2m
Left box bottom flange: 2m
Right inner web horizontal range: 1.2m
Left inner web horizontal range: 1.2m
Right inner web span: 2.3324m
Left inner web span: 2.3324m

Section Properties

Cross sectional area of profile: 3.5444 m^2
First moment of area of section about y: 0 m^3
First moment of area of section about x: 0 m^3
First moment of sectional area about pole B: 0 m^3
Second moment of area of section about y: 29.6111 m^4
Second moment of area of section about x: 4.2659 m^4
Sectorial product of area: -10.8272 m^4

The coordinates of the centroid

----

x_bar_s: 0 m
y_bar_s: 0.73338 m

Warping Function Values for Double Spine Monosymmetric Box Girder

----

Location	xm	wB	0.36565x	wm
0	0	0	0	0
1	0.25	0	0.091412	0.091412
2	4.65	-5.0893	1.7003	-3.3891
3	5.15	-5.0893	1.8831	-3.2063
4	3.45	1.5129	1.2615	2.7744
5	1.45	3.1995	0.53019	3.7297
1	-0.25	0	-0.091412	-0.091412
2	-4.65	5.0893	-1.7003	3.3891
3	-5.15	5.0893	-1.8831	3.2063
4	-3.45	-1.5129	-1.2615	-2.7744
5	-1.45	-3.1995	-0.53019	-3.7297

Bending Moments Values Due To Distortion of the Section

M_12 = -0.87562
M_21 = -3.3538
M_23 = -6.998
M_32 = -2.3764
M_25 = 9.9373
M_52 = 9.9373
M_34 = 2.3764
M_43 = 0.65439
M_41 = -0.65439
M_14 = 0.87562

M_56 = -3.3538
M_65 = -0.87562
M_67 = 0.87562
M_76 = -0.65439
M_78 = 0.65439
M_87 = 2.3764

Vlasov's Coefficients for Torsional/Distortional Analysis
of the Double Spine Monosymmetric Box Girder Section
a_33: 18.5511
b_33: 7.447
c_33: 7.447
r_33: 7.447
r_34: 4.7406
r_44: 11.9884
s_33: 0.0085695

Variation Of Torsional And Distortional Displacements Along The Length
Of The Box Girder (L metres simply supported) Table.

X(m)	Sin(pix)/20	V3(m)	V4(m)
0	0	0	0
1	0.15643	0.0027686	0.00055696
2	0.30902	0.005469	0.0011002
3	0.45399	0.0080347	0.0016164
4	0.58779	0.010403	0.0020927
5	0.70711	0.012514	0.0025176
6	0.80902	0.014318	0.0028804
7	0.89101	0.015769	0.0031723
8	0.95106	0.016832	0.0033861
9	0.98769	0.01748	0.0035165
10	1	0.017698	0.0035604
11	0.98769	0.01748	0.0035165
12	0.95106	0.016832	0.0033861
13	0.89101	0.015769	0.0031723
14	0.80902	0.014318	0.0028804
15	0.70711	0.012514	0.0025176
16	0.58779	0.010403	0.0020927
17	0.45399	0.0080347	0.0016164
18	0.30902	0.005469	0.0011002
19	0.15643	0.0027686	0.00055696
20	1.2246e-16	2.1674e-18	4.3602e-19

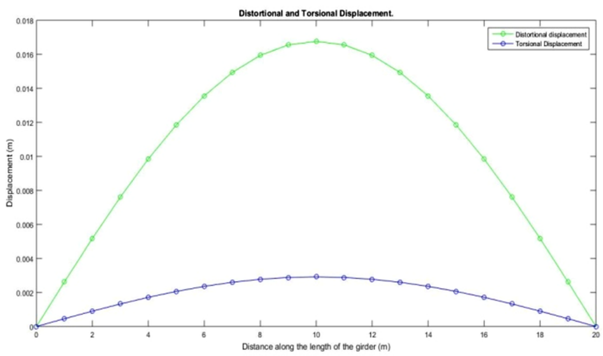


Fig 6

DISCUSSION OF RESULTS:

The governing differential equations of torsional-distortional equilibrium for the double spine mono-symmetric box girder structures are given by eqn. (25a and 25b).

The solution of the torsional-distortional equations of equilibrium for the double spine monosymmetric box girder studied is given by:

$$V_3 = 8.773\text{x}10^{-3} \sin \frac{\pi x}{L}$$

$$V_4 = 2.972\text{x}10^{-3} \sin \frac{\pi x}{L} \qquad (26)$$

Where L represents the span of the girder

The torsional and distortional deformations obtained by integration of eqn. (25) are given by eqn. (26). The results of the analysis are presented in table 1 with graphical presentation in fig.3. The maximum (mid-span) torsional displacement was 2.97mm while the mid-span distortional displacement was 8.77mm. Thus the maximum distortional deformation is about 3 times that of torsional deformation. This explains why torsional stresses may be neglected but not distortional stresses.

Fig 4 shows the torsional-distortional displacement of a double spine mono-symmetric box girder bridge section with thickness of all plates 0.1m. The maximum torsional-distortional displacement is 0.0046955m and 0.036115m. Fig 5 shows the same double spine mono-symmetric box girder bridge section with thickness of all plates 0.125m. The torsional-distortional displacement is 0.0040024m and 0.025241m respectively while Fig 5 shows the torsional-distortional displacement of double spine mono-symmetric box girder with thickness of plate equal 0.15m. The torsional-distortional displacement is 0.0035604m and 0.017698m respectively. Also Fig 6 shows the torsional-distortional displacement for double spine mono-symmetric box girder section with all plate thickness 0.175m. The torsional-distortional displacements are 0.0032372m and 0.012421m respectively. Torsional-distortional displacement of double spine mono-symmetric box girder bridge section decreases with increase in plate thickness.

CONCLUSION

- The distortional deformations were found to be about three times that of torsional deformation.
- The response of double spine mono-symmetric box girder structure to torsional and distortional loads is similar to that of single and multi cellular box girders obtained from earlier studies by other researchers; Chidolue and Osadebe 2012.
- Torsional-distortional displacement of double spine mono-symmetric box girder bridge section decreases with increase in plate thickness.
- The generalized forth order differential equations for torsional-distortional analysis of double spine mono-symmetric box girder structure and indeed, all mono-symmetric box girder structures are given by eqns.(22a) and (22b)

REFERENCES

American Association of State Highway and Transportation Officials (AASHTO), (1998)."Load and Resistance Factor Design, LRFD, bridge design specifications" Washington, D.C.

British standard (BS5400-2), (1978) “Steel, concrete and composite bridges"

Cagri Ozgur (2017) “Behavior of a horizontally curved and skiwed I-girder bridge”.School of Engineering Georgia Institute of Technology.

Chapman, J C., Dowling, P. J., Lim and Billington, (2012). The structural behavior of steel and concrete box girder bridges, international journal of physical sciences, vol.7 (6), pp 111-120

Chidolue, C.A., Amadou, A. and Ezeagu, C. A., 2015, Torsional-distortional performance of multi-cell trapezoidal box girder with all inclined web members, International Journal of Engineering Research and Application, Vol.5, Issue 2, pp.45-51

Ezeh J.C., (2010) “Buckling behavior of axially compressed multi cell doubly symmetric thin-walled column using Vlasov’s theory”.

Osadebe N.N. and Chidolue, C.A. (2012), Tortional-distortional response of thin-walled mono symmetric box girder structures, International Journal of Engineering Research and Application, vol.2, issue 3, pp 814 – 821

Osadebe, N.N. and Chidolue, C.A.,(2012), Response of double cell mono symmetric box girder structure to torsional-distortional deformations, International Journal of Engineering and Advanced Technology, vol. 1, issue 4, pp 285 – 292

Osadebe, N.N., and Mbajiogu M.S.W. (2006)" Distortional analysis of thin-walled box girders" Nigerian journal of technology, vol.25 no.2, p.36-45.

Zhang Yuan-hai and Li Qiao (2009), "Flexural – torsional analysis of curved box girders with the consideration of the shear lag effect". Journal of China, vol. 26 No.10

Sarode B. Ashish and G.R Vesmawala, (2014), "parametric study of horizontally curved box girders for torsional behavior and stability". International refereed journal of engineering and sciences (IRJES), vol. 3, issue 2, pp 50 -55

Varbernov, C.P.(1976), Theory of Elasticity, Techntka Press Sofia, 4th edition, p.254-271 Vlasov, V.Z.(1961). "Thin walled elastic beams", 2nd Ed., National Science Foundation, Washington, D.C.

Vlasov V.Z. (1958). "Thin walled space structures" Gosstrojizdat, Moscow.

Xiang Yiqiang and Xu Jianwu, (2014), "analysis of cracking behavior for reinforced concrete curved box girder". Istanbul bridge conference, Istanbul, turkey.

Arici M., Micheal Fabio Granate and Maria Olivia, (2015) "Influence of secondary torsion on curved steel girder bridges with box and I-girder cross sections". KSCE journal of civil engineering, vol. 19, issue 7, pp 2157-2171

Jan Rubeena, Bhat, Javed Ahmad, (2010), "Evaluation of load distribution factors for curved box girder bridges" International journal of earth sciences and engineering, vol. 3, pp 710 – 719

Mbachu V.C. and Osadebe N.N (2013), "Torsional response of continuous thin walled box girder structures". International journal of engineering research and applications, vol. 3, issue 6, pp 1117 – 1122