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Observations on the Surd Equation

$$\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+4)y}}+\sqrt{x+\sqrt{(m^2+4)y}}\ (m\neq 0)$$

S.Vidhyalakshmi[1], M.A.Gopalan[2]

[1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,Trichy-620 002,Tamil Nadu, India.
Email: vidhyasigc@gmail.com

[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.
Email: mayilgopalan@gmail.com

Abstract:

In this paper, non-zero integer solutions to the surd equation with three unknowns given by $\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+4)y}}+\sqrt{x-\sqrt{(m^2+4)y}}$ are obtained.

Keywords: surd equation, transcendental equation ,integer solutions

Introduction:

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations [1,2]. In [3-18], the integral solutions of transcendental equations involving surds are analyzed for their respective integer solutions.

This communication analyses a transcendental equation with three unknowns given by $\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+4)y}}+\sqrt{x-\sqrt{(m^2+4)y}}$. Infinitely many non-zero integer triples (x,y,z) satisfying the above equation are obtained.

Notations:

$$t_{3,n}=\frac{n(n+1)}{2}$$

$$P_n^5=\frac{n^2(n+1)}{2}$$

$$CP_n^3 = \frac{n(n^2+1)}{2}$$

$$CP_n^5 = \frac{n(5n^2+1)}{6}$$

Method of analysis:

The surd equation to be solved is

$$\sqrt{2z-4} = \sqrt{x+\sqrt{(m^2+4)y}} + \sqrt{x+\sqrt{(m^2+4)y}}\ (m \neq 0) \quad (1)$$

On squaring both sides of (1),it simplifies to

$$z = 2 + x + \sqrt{x^2-(m^2+4)y^2} \quad (2)$$

To start with ,observe that the square root on the R.H.S. of (2) is removed

by choosing

$$x = k(m^2+2) \quad , y = km \quad k \geq 0 \quad (3)$$

and from (2)

$$z = k(m^2+4)+2 \quad (4)$$

A few numerical solutions are presented in Table:1 below

Table:1 Numerical solutions

K	m	x	y	z
1	1	3	1	7
2	2	12	4	18
3	2	18	6	26
4	5	108	20	118

Observations :

1. When k and m are both odd, the values of x ,y ,z are odd
2. $m(z-x-2) = 2y$
3. $m(2z-x-4) = (m^2+6)y$
4. $2(z-2) = (m^2+4)(x-my)$

5. $(z-x-1)^2 = 1+8t_{3,k}$

6. $y+z = 2+4k+2k\,t_{3,m}$

7. $6k(x+2y-k)$ is a nasty number

8. $y(m-4)+m(z-2) = 2k\,P_m^5$

9. $y(x-k) = 2k^2 CP_m^3$

10. $y(5x-10k) = 6k^2 CP_m^5$

However , there are other choices of m ,x ,y for eliminating the square-root on the R.H.S. of (2).

The corresponding values of m ,x ,y along with z are exhibited in Table:2 below:

Table: 2 Choices of m ,x ,y ,z

m	x	y	z
m	$(m^2+5)k$	$2k$	$2((m^2+4)k+1)$
$2k+1$	$(2k^2+2k+3)s$	s	$2+(4k^2+4k+5)s$
m	$(m^2+4)p^2+q^2$	$2pq$	$2((m^2+4)p^2+1)$
m	$m^{k+2}+2m^k$	m^{k+1}	$m^{k+2}+4m^k+2$

Conclusion:

In this paper, we have presented integer solutions to the surd equation

$\sqrt{2z-4} = \sqrt{x+\sqrt{(m^2+4)y}} + \sqrt{x+\sqrt{(m^2+4)y}}\ (m \neq 0)$

. To conclude one may attempt to find integer solutions to other choices of surd equations .

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