



## Implementation of Trajectory Tracking for Caterpillar Vehicles using A Back-Stepping Controller

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### ABSTRACT

This paper introduces trajectory tracking control of Caterpillar Vehicles (CVs) using a back-stepping control method. To design the proposed controller, the followings are done. Firstly, a system modeling of the Caterpillar Vehicle is presented. Secondly, a tracking error vector is defined. Thirdly, a back-stepping controller is designed based on the kinematic modeling and makes the tracking error vector go to zero. Fourthly, by choosing a suitable Lyapunov function candidate, the system stability is guaranteed and a control law can be obtained. Finally, in order to verify the effectiveness of the proposed controller, the simulation and experimental results are shown. The simulation and experimental results are shown for the Caterpillar Vehicle to track a desired trajectory and reaches the goal point well.

**Keywords:** Object Tracking; Caterpillar Vehicle; Back-stepping Controller; Laser Lidar Sensor

### 1. INTRODUCTION

Recently, autonomous robots have been used instead of human beings. The mobile robots are able to accomplish many tasks in dangerous places where humans cannot enter. These tasks may take place in unsafe environments such as nuclear waste facilities, and sites with harmful gases or high temperature. Therefore, mobile robots with a caterpillar type of wheels were chosen for these above tasks. Trajectory tracking control problem is one of the key techniques in mobile robot research. There are several types of methods that used for tracking control of the mobile robot in [1-5]. However, these previous papers have applied for mobile robots with conventional wheel. There were not applied for mobile robot with a caterpillar type of wheels yet. Therefore, this paper proposes a trajectory tracking for the Caterpillar Vehicles using the back-stepping control method. In order to verify the effectiveness of the proposed methods, the simulation and experimental results are shown.

### SYSTEM MODELING

In this paper, the Caterpillar Vehicle uses a differential drive wheeled configuration. The schematic modeling of the system is shown in Fig. 1.

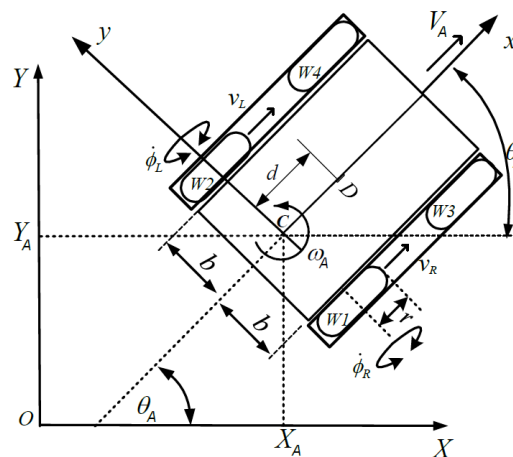


Fig. 1 Schematic diagram of the CV

In Fig. 1,  $OXY$  is a global coordinate frame.  $Cxy$  is the moving coordinate frame attached on the Caterpillar Vehicle (CV) platform. This CV has two driving standard wheels  $W1$  and  $W2$  with radius  $r$ , and  $W3$  and  $W4$  are passive wheels.  $(X_A, Y_A)$  is the CV position vector in global coordinate  $OXY$  with orientation angle  $\theta_A$ . The CV moves with linear velocity  $V_A$  and angular velocity  $\omega_A$ .  $v_R$  and  $v_L$  are the right and left wheel linear velocities, respectively.  $\dot{\phi}_R$  and  $\dot{\phi}_L$  are the right and left wheel angular velocities, respectively.  $b$  is a distance between wheels and  $Cx$  axis of the CV.  $D$  is a mass center of the CV.  $d$  is the distance between  $C$  and  $D$ .

The kinematic equation of the Caterpillar Vehicle as shown in Fig. 1 can be expressed as follows:

$$\dot{\mathbf{q}} = \mathbf{H}(\mathbf{q})\mathbf{z} \quad (1)$$

$$\mathbf{q} = \begin{bmatrix} X_A \\ Y_A \\ \theta_A \end{bmatrix}, \quad \mathbf{H}(\mathbf{q}) = \begin{bmatrix} \cos\theta_A & 0 \\ \sin\theta_A & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} V_A \\ \omega_A \end{bmatrix} \quad (2)$$

where  $\mathbf{q}$  is the posture vector of the Caterpillar Vehicle in global coordinate  $OXY$ ,  $\mathbf{H}(\mathbf{q})$  is full rank matrix,  $\mathbf{z}$  is the velocity vector of the Caterpillar Vehicle. Relation of the linear velocity  $V_A$  and angular velocity  $\omega_A$  with the right and left wheel angular velocities ( $\dot{\phi}_R, \dot{\phi}_L$ ) can be expressed as:

$$\mathbf{z} = \mathbf{T}\dot{\Phi} \quad \text{for } \mathbf{T} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & \frac{-r}{2b} \end{bmatrix}, \quad \dot{\Phi} = \begin{bmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{bmatrix} \quad (3)$$

By substituting Eq. (3) into Eq. (1), the kinematic equation of the Caterpillar Vehicle can be expressed as follows:

$$\begin{bmatrix} \dot{X}_A \\ \dot{Y}_A \\ \dot{\theta}_A \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta_A & \frac{r}{2}\cos\theta_A \\ \frac{r}{2}\sin\theta_A & \frac{r}{2}\sin\theta_A \\ \frac{r}{2b} & \frac{-r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{bmatrix} \quad (4)$$

$$\mathbf{q}_r = [X_r \quad Y_r \quad \theta_r]^T$$

## TRACKING CONTROLLER DESIGN

In this section, a tracking controller based on a back-stepping control method is designed for the CV to track the reference input

$$\mathbf{q}_r = [X_r \quad Y_r \quad \theta_r]^T \quad \text{with reference linear velocity } V_r \quad \text{and angular velocity } \omega_r.$$

An output error vector is defined as the difference between the output posture vector  $\mathbf{q}$  and reference input vector  $\mathbf{q}_r$  in coordinate  $Cxy$ :

$$\mathbf{e} = \mathbf{R}(\mathbf{q}_r - \mathbf{q}) \quad (5)$$

$$\text{with } \mathbf{e} = [e_1 \quad e_2 \quad e_3]^T,$$

$$\mathbf{R} = \begin{bmatrix} \cos\theta_A & \sin\theta_A & 0 \\ -\sin\theta_A & \cos\theta_A & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where  $\mathbf{R}$  is the orthogonal rotational matrix,  $\mathbf{q}_r = [X_r \quad Y_r \quad \theta_r]^T$  is a reference input vector, and  $\mathbf{q} = [X_A \quad Y_A \quad \theta_A]^T$  is the posture vector of

the CV. By using Eqs. (1) and (2), the first order time derivative of Eq. (5) can be obtained:

$$\dot{\mathbf{e}} = \begin{cases} \dot{e}_1 = V_r \cos e_3 - V_A + e_2 \omega_A \\ \dot{e}_2 = V_r \sin e_3 - e_1 \omega_A \\ \dot{e}_3 = \omega_r - \omega_A \end{cases} \quad (6)$$

A candidate Lyapunov function is chosen to analyze stability and determine control laws as follows:

$$V_0 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{(1 - \cos e_3)}{k_2} > 0 \quad (7)$$

where  $k_2$  is positive constants.

The first order time derivative of  $V_0$  can be obtained as follows:

$$\dot{V}_0 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + \dot{e}_3 \frac{\sin e_3}{k_2} \quad (8)$$

By using Eq. (6), the derivative of  $V_0$  can be obtained as follows:

$$\begin{aligned} \dot{V}_0 &= e_1 (-V_A + V_r \cos e_3) \\ &+ \frac{1}{k_2} (\sin e_3) (\omega_r - \omega_A + k_2 e_2 V_r) \end{aligned} \quad (9)$$

To satisfy condition  $\dot{V}_0 \leq 0$ , a desired control laws vector  $\mathbf{u}$  is chosen as follows:

$$\mathbf{u} \equiv \mathbf{z} = \begin{bmatrix} V_A \\ \omega_A \end{bmatrix} = \begin{bmatrix} V_r \cos e_3 + k_1 e_1 \\ \omega_r + k_2 V_r e_2 + k_3 \sin e_3 \end{bmatrix} \quad (10)$$

where  $k_1$ ,  $k_2$ , and  $k_3$  are positive constants. By substituting Eq. (10) into Eq. (9), the derivative of  $V_0$  can be obtained as follows:

$$\dot{V}_0 = -k_1 e_1^2 - \frac{k_3}{k_2} \sin e_3 \leq 0 \quad (11)$$

According to Lyapunov stability theory, if  $V_0 > 0$  and  $\dot{V}_0 \leq 0$ , the system is stable. And by Barbalat's Lemma, the error  $\mathbf{e}$  converge to zero as  $t \rightarrow \infty$ . The configuration of the tracking controller can be described as shown in Fig. 2.

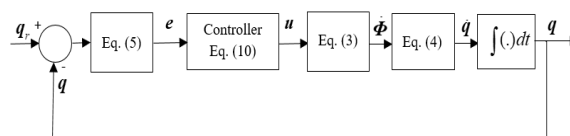


Fig. 2 Configuration of the tracking controller

## SIMULATION AND EXPERIMENTAL RESULTS

A Caterpillar Vehicle used in this experiment is shown in Fig. 3. Table 1 shows parameters of the CV. Table 2 shows their specifications. Table 3 shows the initial values and parameter values for the proposed tracking controller.

Table 1. Parameters values of CV

Parameters	Descriptions	Values
$2b$	Distance of wheels	0.12m
$r$	Radius of wheel	0.04m
$mw$	Weight of wheel	1kg
$mc$	Weight of CV	20kg

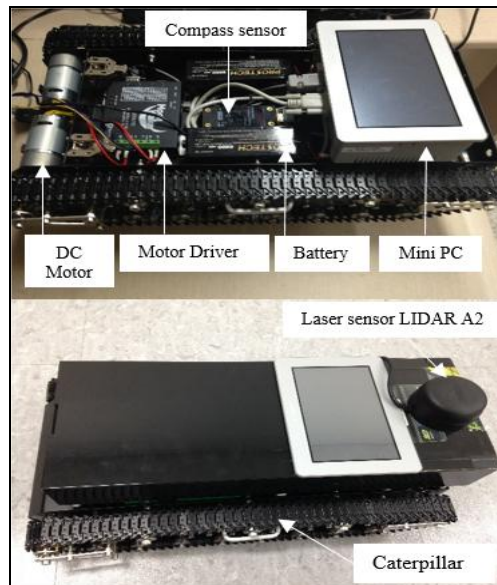


Fig. 3 Structure of the CV platform

Table 2. Specifications of the Equipment

No.	Equipment	Specifications
1	Mini PC (CPCV5)	CPU 1.83GHz, DDR 4GB, HDD 128GB, 2x GE (Realtek), 3x USB2.0, 1xUSB3.0, COM1,2: RS232C
2	Motor Driver (MDC24D200D)	DC Motor Operating Voltage: 8-30VDC Number of Channels: 2, Max Amps: 10A
3	Compass Sensor (HMR3000)	Resolution(Heading): $0.1^{\circ}$ , Interface: RS-232 Power Supply: 5V
4	Laser Sensor (LidarA2)	Distance Range: 0.15m-6m, Angular Range: $0-360^{\circ}$ , Scan Rate: 10Hz, Resolution: $0.9^{\circ}$ , Laser Wavelength: 785nm
5	Motor with encoders	BLDC motor Gear ratio: 131:1 Run speed(12V): 80rpm, 8400 counts per revolution.
6	Battery (PROTECH)	3 SIP, 7200mAh, 11.1V

Table 3. Parameter values of the proposed controller

Paras.	Values	Paras.	Values
$V_r$	0.1m/s	$XA(0)$	0m
$\omega_r$	0.1rad/s	$YA(0)$	0m
$k_1$	12	$\theta A(0)$	0rad
$k_2$	12	$VA(0)$	0m/s
$k_3$	21.5	$\omega A(0)$	0rad/s

The simulation and experimental trajectory tracking results are shown in Fig. 4. Fig. 4 shows the desired trajectory reference with a black continuous line from Start A(X=0mm, Y=0mm) to Goal D(X=3760mm, Y=0mm), the result trajectory by simulation with a magenta dashed line, and the result trajectory by the experiment with a blue center line.

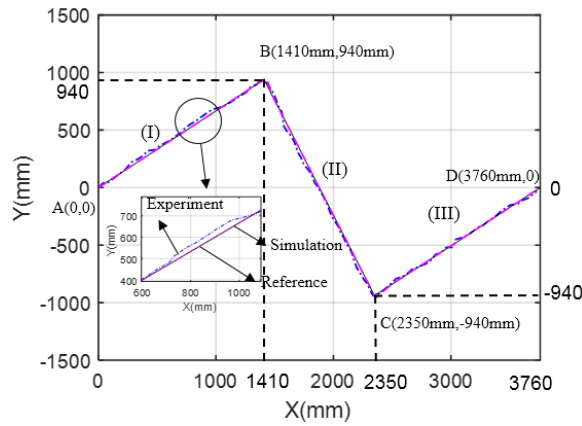


Fig. 4 Trajectory tracking results

The tracking errors of the experiment and the simulation are shown in Figs. 5-7. Fig. 5 shows tracking error  $e_x = (X_A - X_r)$  on X axis. The tracking error  $e_x$  of the experiment converges to zero after 5 seconds and  $e_\theta$  is bounded from -14mm to +13mm around zero in straight line, but  $e_x$  has big change in sharp edge and it is bounded from -20mm to +22mm around zero.

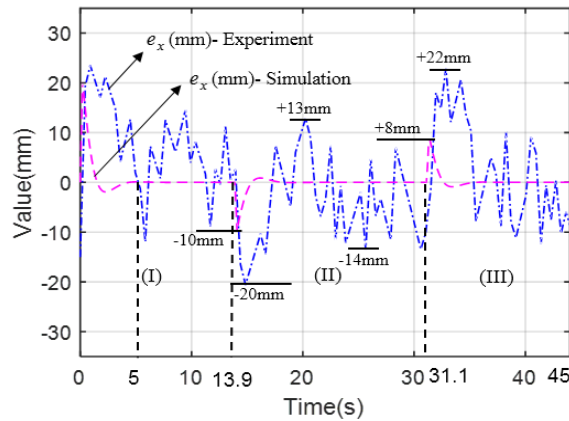


Fig. 5 Tracking errors  $e_x$  on X axis

Fig. 6 shows tracking error  $e_y = (Y_A - Y_r)$  on Y axis. The tracking error  $e_y$  of the experiment converges to zero after 4.5 seconds and it is bounded from -19mm to +18mm around zero in straight line, but  $e_y$  has big change in sharp edge and it is bounded from -25mm to +26mm around zero.

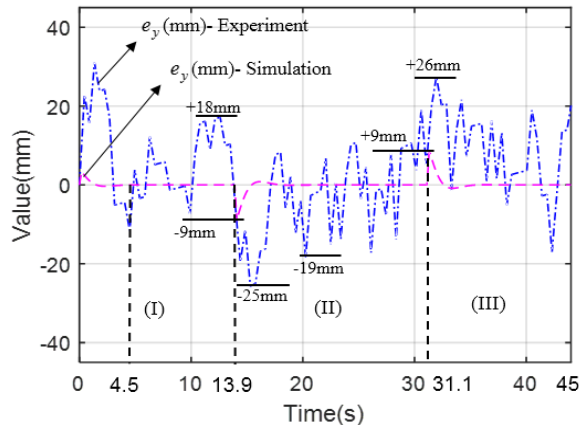


Fig. 6 Tracking errors  $e_y$  on Y axis

Fig. 7 shows heading error. The tracking error  $e_\theta = (\theta_A - \theta_r)$  of the both simulation and experiment are changed at  $t1 = 2s$ ,  $t2 = 13.9s$ , and  $t3 = 31.1s$  due to the reference direction of the CV. The heading error  $e_\theta$  of the experiment converges to zero and it is bounded from  $-0.18\text{rad}$  to  $+0.05\text{rad}$  around zero after 2 seconds in straight line, but  $e_\theta$  has big change in sharp edge and it is bounded from  $-0.90\text{rad}$  to  $+0.80\text{rad}$  around zero. Therefore, these simulation and experiment results show the proposed method have good tracking performance for the trajectory reference.

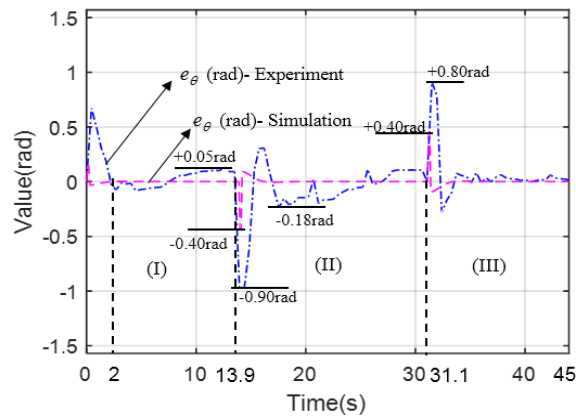


Fig. 7 Heading error results  $e_\theta$

## CONCLUSIONS

A tracking controller design for Caterpillar Vehicle to track desired trajectory reference using a back-stepping control method was proposed. In order to verify the effectiveness of the proposed methods, the simulation and experimental results were shown. The experiment results showed the proposed method had good tracking performance for the desired sharp trajectory reference. The tracking error in the sharp edges are big. However, the tracking error on X axis was bounded from  $-14\text{mm}$  to  $+13\text{mm}$  around zero, tracking error on Y axis was bounded from  $-19\text{mm}$  to  $+18\text{mm}$  around zero, and the heading error was bounded from  $-0.18\text{rad}$  to  $+0.05\text{rad}$  around zero in the straight line.

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