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Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) Modelling of Exchange Rate Return of United State Dollar to Nigeria Naira from January1981 to December 2015

Wiri, Leneenadogo[1] , Chims Benjamin. E[2] and Richard lgbudu. C[3]

Rivers State University, Nigeria

ABSTRACT

This study applied Generalized Autoregressive conditional heteroscedasticity (GARCH) in modelling United State Dollar (USD) to Nigeria Naira (NN) from January 1981 to December 2015. The time plot of the original series showed the present of trend and logarithm transformation of the exchange rate return series make it stationary. The return was estimated using both the conditional mean and conditional variance. The study applied both symmetric and asymmetric (GARCH) model that capture most of the feature of a financial time series data, such as volatility clustering and leverage effect in modelling the return series of USD/NN. However, six models were estimated for the conditional variance and asymmetric power autoregressive conditional heteroscedasticity (APARCH (1,1)) was adopted as the best model for USD/NN exchange return series and for the conditional mean of exchange return of United State Dollar to Nigeria Naira (USD/NN) follow an ARMA (1,1). Finally, the most adequate model for estimating volatility of the exchange rates is the asymmetric APARCH model,

Keywords: Exchange rate, conditional means, conditional variance, GARCH, Volatility clustering.

1.0 INTRODUCTION

The United State Dollar Exchange rate to Nigeria Naira is a key factor that influences Nigeria Economics activities. In the past years the foreign exchange market has becomes the most volatile in financial market in the world. The modelling of exchange rate volatility has important implication in many area in economic and financial market.(Cyprian, o. o et al.)

In finance, researchers always put comforts in modelling Failure which could lead to disaster and possible disappointment in the financial market. A crucial part of financial market is to measure the potential losses of a group of assets, and in order to measure these possible losses, estimates must be made for future volatilities. Volatility is one of the most important concept in finances, its measured variances of variable asset return. Volatility is often used as a basic measure of the total risk of financial assets. Policy maker are interested in measuring volatility process to learn about market expectation and uncertainty about exchange rate in foreign market (Christ book). A number of models have been developed to investigate volatility across different countries. The most common model to estimate exchange rate volatility is the GARCH model developed independently by Bollerslev (1986), Taylor (1986) and Engle (1986). The purpose of the ARCH model is to estimate the conditional variances of a time series data. Engle described the conditional variance by a simple quadratic function of its past lagged terms. Bollerslev in (1986) extended the basic ARCH mode and describes the conditional variances by its own lagged value and the square of the lagged terms of the shock. The GARCHmodel provide a good techniques for analysing financial time series and estimating conditional variance. (Cyprian, o. o al et.)

In financial markets, it is a conventional fact that a downward movement is always followed by greater volatility, which can cause a negative shock. In GARCH model, negative shocks increase volatility in financial markets than positive shocks. Volatility is higher after negative shocks than after positive shocks of the same size, this process is call leverage effect. Thisfeature was first suggested by Black (1976) for stock returns. He attributed asymmetry feature to leverage effects. In this context, negative shocks increase volatility in asset markets more than positive shocks. In foreign market, a shock, which increases the volatility of the market, increases the risk of holding the currency. Longmore and Robinson, (2004).

The aim of this paper is to model Nigeria Naira exchange rate volatility to United State Dollar (NN/USD). In modelling volatility there are two distinct models to consider, the conditional mean and the conditional variance which apply different method of univariatemodels formonthly exchange rate return series from January 1981 to December 2015. The time series models in this paper are ARMA(1,1) ARMA (1,2) ARMA(2,2), ARMA(2,1),ARCH(1,1), GARCH(1,1) , EGARCH(1,1), APARCH (1,1) and TS-GARCH(1,1)

2.0 METHODOLOGY

2.1 Conditional Mean Model

2.1.1 ARMA (p,q)

The exchange of Nigeria Naira (NN) to United State Dollar (USD) movement has the component of an AR process and MA process. The autoregressive moving average is use to model conditional mean in a series. The AR model includes lagged terms of the past values of series and that the MA model includes lagged terms of error term. By including both lagged terms, we arrive at ARMA model. Therefore ARMA (p,q), where p is the order of autoregressive term and q the order of the moving-average term, these can generally be represented as

$$y_t = \sum_{i=1}^{p} \alpha_i y_{t-1} + \varepsilon_t + \sum_{j-1}^{q} \beta_j \varepsilon_{t-1} \quad (1)$$

A series $\{y_t\}$ is said to follow an autoregressive moving average model *of orders p and q*, designated ARMA (p, q), where β_j and α_i are constants such that the model is stationary as well as invertible and $\{\varepsilon_t\}$ is a white noise process.

2.2.0 Volatility Models

The techniques to model volatilities is divide in to two main type, symmetric and asymmetric models. In symmetric model the conditionalvariance only depend on the magnitude and not the sign of asset, while in asymmetric model the negative and positive shock of the size have different effect of the future volatility. Chris Brook, (2008).

2.2.1 Symmetric Models

2.2.2 ARCH (1)

The autoregressiveconditional heteroscedastic (ARCH) model is used to model conditional variance of a series. This model is often used to describe the increase and descries in variation. Suppose we are modelling the variance of a series y_t the ARCH (1) model for the variance for y_t is condition on Y_{t-1} at time t.
Mathematically ARCH model is represented as follow

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \quad (2)$$

We impose the constraints that $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ to avoid negative variance

2.2.3. GARCH (1, 1) Model

The generalized autoregressive conditional heteroscedastic model used value of the past squared observation and past variance to model the variance at time t. the model allows the conditional variance to dependent upon previous lags itself. The models Measures the extent to which a volatility shock today feed through into the next period's volatility. Measures the rate at which this effect lies over time. An example of GARCH(1, 1) model is below

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (3)$$

This is a GARCH (1,1) model.σ_t^2Is known as the conditional variancesince it is a one-period ahead estimate for the variance.

2.2.4 GARCH- M (1,1)

In financial market, high risk is expected to produce high return. In these type of condition one may consider the GARCH IN MEAN model. The model allow the condition mean of a sequence to depend on it conditional variance.
The model is as follow

$$\gamma_t = \mu + \lambda\sigma_t^2 + y_t \quad (4)$$

$$y_t = \sigma_t \varepsilon_t$$

$$\varepsilon_t \sim (0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_t \sigma_{t-1}^2 \quad (5)$$

Where λ *and* μ are constant, if λ is positive the return is also positive related to volatility

2.3.0. Asymmetric Models

Since bad news (negative shocks) tends to have a large impact on volatility thangood news (positive shocks), hence there is need to talk about the ASYMMETRIC GARCH modeland we restricted our analysis to the more popular models of asymmetric GARCH, such asEGARCH, TS-GARCH, APARCH.

2.3.1. EGARCH Model:

The exponential GARCH (EGARCH) model.The model has several advantages over the pure GARCH specification. First, since the log (σ_i^2) is modelled, then even if the parameters are negative, (σ_i^2), will be positive. There is thus no need to artificially impose non-negativity constraints on the model parameters. Second, asymmetries are allowed for under the EGARCH formulation, since if the relationship between volatility and returns is negative, γ, will be negativethe modelhas the following representation:

$$\text{Log}h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \frac{|\varepsilon_{t-1}| + \gamma_i \varepsilon_{i-1}}{h_{t-1}^{\frac{1}{2}}} + \sum_{j=1}^{q} \beta_j \log h_{t-j} \quad (6)$$

Where, $\gamma$is leverage effect co-efficient. (If$\gamma > 0$ it indicates the presence of leverage effect).
Note that when $\varepsilon$is positive there is good news, when $\varepsilon$is negative there is bad news

2.3.2. TsGARCH Model:

Another GARCH method that is capable of modelling leverage effects is the Threshold GARCH (TGARCH) model, which has the following form:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \gamma_1 \varepsilon_{t-1}^2 s_{t-1} + \sum_{j=i}^{q} \beta_j h_{t-1} \quad (7)$$

Where $s_{t-1} = \begin{cases} 1 \ if \ \varepsilon_{i-1} < 0 \\ 0 \ if \ \varepsilon_{t-1} \geq 0 \end{cases}$

γ isleverage effects coefficient. (if$\gamma > 0$ it indicates the present of leverage effect). That is depending on whether ε is above or below the threshold value of zero,
ε_t^2has different effects on conditional variance h_t when ε_{t-i} is positive.

2.3.3 APARCH (1,1)

Asymmetric power ARCH (APARCH) model, this is ableto accommodate asymmetric effect and power transformation of the variance. Its specificationfor the conditional variance is as follow

$$\sigma_t^2 = \alpha_0 z_t + \sum_{i=1}^{q} \alpha_i(|u_t| - \gamma_t u_t) + \sum_{j=1}^{p} \beta_j \sigma_{t-1}^2 \quad (8)$$

Where $\sigma_t = \sqrt{h_t}$, the parameter γ (assumed positive and ranging between 1 and 2)

3.0 METHODS OF ESTIMATION OF GARCH MODELS

3.1 Maximum Likelihood Function

The method used for estimating GARCH model is the maximum likelihood estimator. The method is used to find the most likely value of the parameters given the actual series. The following step are involved in estimating GARCH model

(i) Specify the mean and variance equation, example (AR(1) and GARCH(1,1) model)

$$y_t = \mu + \theta y_{t-1} + \mu_t \mu \sim (0, \sigma_t^2) \quad (9)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (10)$$

(ii) Estimate the likelihood function to maximise the normality assumption of disturbance terms.

$$\log L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{\mu_t^2}{\sigma_t^2}. \quad (11)$$

3.2 Unit Root Test

In order to determine the integration of the variance we apply the augmented Dickey Fuller test wish base on the equation below

$$\Delta y_t = \varphi + \beta_t + \alpha_1 y_{t-1} + \sum_{i=1}^{k} d_i \Delta y_{t-1} + \mu_t \tag{12}$$

Where μ_t is a white noise process, the equation is use to test null hypothesis of unit root against the alternative hypothesis

3.3 Data Transformation

The monthly United State Dollar exchange rate Nigeria Naira from January 1980 to December 2015 are used. This make a total of 420 observation and are transformed for the needs of fitting the model to a logarithm returns. Let the series (e_t) denoted exchange rate of naira to US Dollar. The logarithm returns series γ_t is

$$\gamma_t = log^{(\frac{e_t}{e_{t-1}})} = log^{(\frac{return\ series_t}{return\ series_{t-1}})} \tag{13}$$

Where e_t is the Nigeria naira exchange rate to US Dollar at time t and e_{t-1} represent the exchange at time t-1.

4.0 Empirical Result

4.1 Data Properties:

The time plot of original series of exchange rate without transformation is showed in figure (1). To remove trend we take the first difference (d) of the logarithms (I) transformation of the data and the series are transformed in to financial time series showed in figure (2).

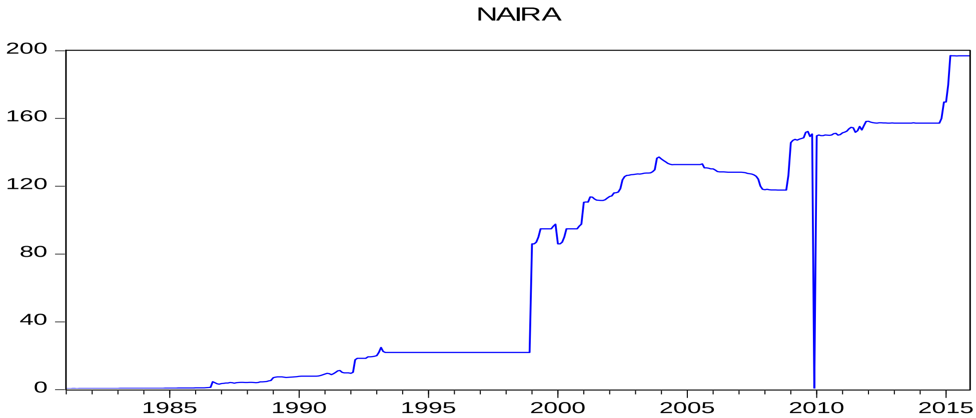

Figure 1 Time Plot of Differences Series

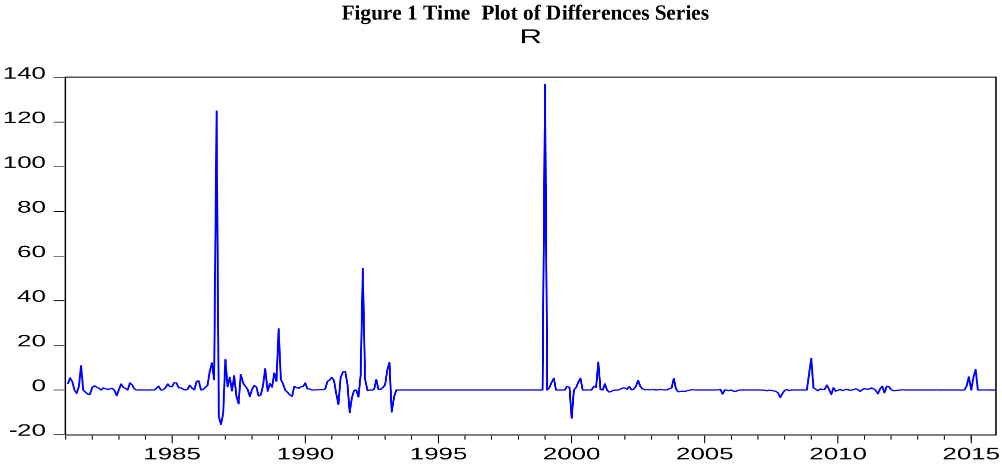

Figure 2 Time Plot of Original Series

Table (1.0) Value of Unit Root Test of NN/USD

Critical value	**ADF test statistics (-20.30455)**
1%	-3.445776
5%	-2.86235
10%	-2.570401

The value of USD/NN exchange rate after the logarithm transformation and first differences is stationary, since the value of ADF test statistics is less than the critical at (1%,5% and 10%) as showed in table (1)

Table (1.1) Summary Statistics for the returns of NN/USD exchange rates

observation	**Mean**	**median**	**max**	**min**	**Std dev**	**skewness**	**kurtosis**	**JarqueBera**	**probability**
417	0.01411	0.000	1.369	-0.154	0.0986	11.314	146.52	368529.1	0.0000

The mean of the monthly exchange rate return of NN/USD is positive and the standard deviation of the return is more volatile. The maximum return of the series is 1.369 which is higher return in financial market. The value of the kurtosis is above 3 the kurtosis for a normal distribution which showed the present of fat tail (leptokurtic) which is one of the characteristic of financial time series. The Jarque-Bera test showed that the series is not normal, then the null hypothesis of normality is rejected.

Table (1.2) Residuals Square Autocorrelation Function

Lag	**Acf**	**Pacf**	**Q-S**	**Prob**
13	-0.006	-0.006	0.1272	1.000
14	-0.007	-0.007	0.1469	1.000
15	-0.006	-0.007	0.1643	1.000

The correlogram of the square residual revealed the present of ARCH effect as the as the value of the ACF, PACF and Q-Statistics decline gradually and the lag of the ACF and PACF all lies within thesame level non lag cut across the level.

Estimating the Conditional Mean

The ARMA(p,q) is used to model the conditional mean and dynamic error in the series. The autoregressive function and moving average function are used to determine the order of ARMA(p,q) models. The following ARMA model estimated with AIC value showed below.

Table (1.3) Selection ARMA (p, q) models

models	**AIC**	**SC**
ARMA(2,1)	7.3667	7.395733
ARMA(1,1)	**7.364366**	**7.393328**
ARMA(2,1)	7.364391	7.393352
ARMA(2,2)	7.655257	7.68421

Four models were estimated with Akaike information criterion (AIC) and schwarz criterion are shown in table 1.3. The best model is models that minimise information criterion and schwarz criterion (ARMA (1,1) with AIC (7.364366) SC (7.393328) showed table 1.3 the return mean equation follow ARMA(1,1) process.

Heteroskedasticity Test: ARCH

F-statistic	0.005307	Prob. F(5,408)	1.0000
Obs*R-squared	0.026923	Prob. Chi-Square(5)	1.0000

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 08/09/18 Time: 13:33
Sample (adjusted): 1981M07 2015M12
Included observations: 414 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.029009	0.786977	1.307547	0.1918
WGT_RESID^2(-1)	-0.004035	0.049507	-0.081504	0.9351
WGT_RESID^2(-2)	-0.003839	0.049507	-0.077548	0.9382
WGT_RESID^2(-3)	-0.003449	0.049507	-0.069674	0.9445
WGT_RESID^2(-4)	-0.002476	0.049507	-0.050011	0.9601
WGT_RESID^2(-5)	-0.004111	0.049507	-0.083039	0.9339

R-squared	0.000065	Mean dependent var	1.010895
Adjusted R-squared	-0.012189	S.D. dependent var	15.75160
S.E. of regression	15.84731	Akaike info criterion	8.378264
Sum squared resid	102464.0	Schwarz criterion	8.436610
Log likelihood	-1728.301	Hannan-Quinn criter.	8.401338
F-statistic	0.005307	Durbin-Watson stat	2.000031
Prob(F-statistic)	0.999994		

Figure (1.4) Test For ARCH Effect

The table above showed the present ARCH since both the *F*-version and the *LM*-statistic are statistical significant, suggesting the presence of ARCH in the United State Dollar exchange to Nigeria Naira.

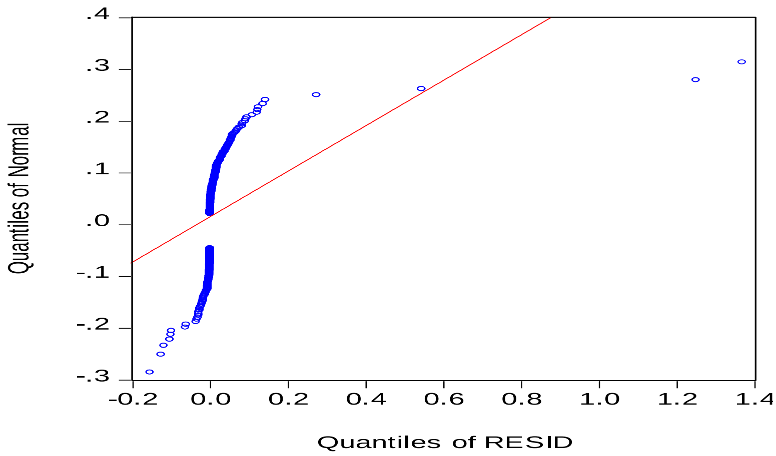

Figure (3) Quantile –Qauntile plot of standardized redual fitted from APARCH(1,1)

Model for USD/NN

The normal quantile–quantile plots of standardized residuals of United State Dollar exchange to Nigeria Naira show strong departures from normality.

Table (1.5) Parameter Estimation for ARCH-GARCH Models (USD/NN) Return with Volatility

Models/ Parameter.	ARCH (1,1)	GARCH (1,1)	GARCH-M (1,1)	APARCH (1,1)	TS GARCH (1,1)	EGARCH (1,1)
C	0.005237 (0.000)	1.616274 (0.2609)	0.17066 (0.1654)	**-7.14291 (0.000)**	1.213591 (0.0279)	2.51468 (0.00)
α_0	3.747975 (0.000)	0.000295 (0.000)	0.000291 (0.000)	**0.001738 (0.000)**	0.003744 (0.000)	-0.85826 (0.000)
α_1	0.14298 (0.0042)	-0.00527 (0.000)	-0.005227 (0.000)	**-0.093657 (0.000)**	0.095061 (0.000)	-0.36922 (0.000)
β		0.97644 (0.000)	0.976824 (0.000)	**0.404784 (0.000)**	-0.58899 (0.000)	0.610457 (0.000)
γ				**1.05156 (0.000)**	0.58899 (0.000)	0.780561 (0.000)
λ				**0.526657 (0.000)**		
$\alpha+\beta$		0.97117	0.971592	**0.30813**	-0.49134	0.24125
$\log L$	376.5976	402.7901	402.8836	**466.3154**	385.8132	403.418
AIC	-1.783	-1.90353	-1.903979	**-2.19720**	-1.81777	-1.90176
SC	-1.7543	-1.8648	-1.86543	**-2.139388**	-1.76954	-1.85357
Obs	419	419	419	**419**	419	914

This section seek to deduced the key result derived from estimate of all the GARCH(1,1) models. The parameter of the exchange rate return series is negative and positive, thereby satisfying the necessary and sufficient condition for asymmetric and symmetric GARCH models. The GARCH models for the return series all satisfy the covariance stationary condition that $\alpha + \beta < 1$. Comparing the Log Likelihood and information criterion in Table 1.4 within the conditional distribution, the model with conditional distribution of maximum Log-Likelihood and minimum information criterion, statistically estimate the better fitted model.

The APARCH model is found to be the best model, because it has the higher log-likelihood estimate and minimum AIC and SC information criterion (LogL=466.3154, AIC=-2.19720, SC= -2.139388). the coefficient on both the lagged squared residual and lagged conditional variance term in the conditional variance equation are highly statistically significant and the sum of the coefficient on the lagged squared error and lagged conditional variance is close to unity (**0.30813).** This a typical example of GARCH models for financial asset return series. This implies that shocks to the condition variance will be highly persistent.

However, Good news and Bad news have difference effect on the conditional variance: Good news has an impact on α while Bad news has an impact on($\alpha + \beta$). In APARCH model, the Good news has an impact on -0.93657 while Bad news has an impact on 0.30813. The value of λ is positive and statistically significant. This implies that, increased risk, given by an increase in the conditional variance which lead to a rise in the mean return of exchange rate.

The coefficient of the mean equation has a negative sign, but not statistically significant. This indicate that the exchange rate return, there is no feedback from the conditional variance to the conditional mean.

Conclusion

This paper examined the monthly exchange rate return series of United State Dollar to Nigeria Naira (USD/NN). The data span from January 1981 to December 2015. The study applied univariate specification of generalized autoregressive conditional heteroscedasticity (GARCH) models, including both symmetric and asymmetric model that capture the feature of a financial series, such as volatility clustering and leverage effect.However, six models were estimated from the GARCH family and the asymmetric power autoregressive conditional heteroscedasticity (APARCH (1,1)) was adopted as the best model for USD/NN exchange return series. Simultaneously four model was estimated for the conditional mean with the conditional variance and the model for conditional mean of exchange return of United State Dollar to Nigeria Naira is ARMA (1,1).

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