International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Pythagorean Fuzzy Ideals in Subtraction Algebras

R. Jenitha Vinnarasi[a*], *A. Stanis Arul Mary*[b]

[a] *PG Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore,India(TN),vinnarasijenitha@gmail.com*
[b] *Assistant Professor, Department of Mathematics,Nirmala College for Women, Coimbatore,India(TN),stanisarulmary@gmail.com*

ABSTRACT

In this paper, the notions of a Pythagorean Fuzzy subalgebra and a Pythagorean Fuzzy ideal of a subtraction algebra are introduced and its characterizations of above are investigated. Further as a special case, we proved that the homomorphic preimage of a Pythagorean Fuzzy subalgebra of a subtraction algebra is a Pythagorean Fuzzy subalgebra, and the onto homomorphic image of a Pythagorean Fuzzy subalgebra of a subtraction algebra is a Pythagorean Fuzzy subalgebra.

Keywords: Subtraction Algebra, (Pythagorean Fuzzy) subalgebra, (Pythagorean Fuzzy) ideal.

1. Introduction

B. M. Schein considered systems of the form (Φ; ∘, \), where Φ is a set of functions closed under the composition "∘" of functions (and hence (Φ; ∘) is a function semigroup) and the set theoretic subtraction " \ " (and hence (Φ; \) is a subtraction algebra). B. Zelinkadiscussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y. B. Jun et al. introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. S. S. Ahn and Y. H. Kim introduced the notions of an intersectional soft subalgebra and an intersectional soft ideal of a subtraction algebra and investigated some related properties of them.

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the Fuzzy set. As a generalization of Fuzzy sets, Atanassov introduced the degree of non-membership/falsehood (f) in 1986 and defined the intuitionistic Fuzzy set. Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the Neutrosophic set on three components (t, i, f) = (truth, indeterminacy, falsehood). Jun et. al introduced the notions of a Neutrosophic N – subalgebras and a (closed) Neutrosophic N- ideal in a BCK/BCI - algebras and investigated some related properties.

In this paper, we introduce the notions of a Pythagorean Fuzzy subalgebra and a Pythagorean Fuzzy ideal of a subtraction algebra. Characterizations of a Pythagorean Fuzzy subalgebra and a Pythagorean Fuzzy ideal are investigated. We show that the homomorphicpreimage of a Pythagorean Fuzzy subalgebra of a subtraction algebra is a Pythagorean Fuzzy subalgebra, and the onto homomorphic image of a Pythagorean Fuzzy subalgebra of a subtraction algebra is a Pythagorean Fuzzy subalgebra.

2. Preliminaries

By a subtraction algebra we mean an algebra $(X, -, 0)$ with a single binary operation " $-$ " that satisfies the following conditions: for any $x, y, z \in X$,

(S_1) $x - (y - x) = x$,

(S_2) $x - (x - y) = y - (y - x)$,

(S_3) $(x - y) - z = (x - z) - y$.

The subtraction determines an order relation on X: $a \le b$ if and only if $a - b = 0$, where $0 = a - a$ is an element that does not depend on the choice of $a \in X$. The ordered set $(X; \le)$ is a semi-Boolean algebras, that is, it is a semilattice with zero 0 in which every interval $[0,a]$ is a Boolean algebra with respect to the induced order. Hence $a \wedge b = a - (a - b)$; the complement of an element $b \in [0, a]$ is $a - b$; and if $b, c \in [0,a]$, then

$$b \vee c = (b^* \wedge c^*)^* = a - ((a - b) \wedge (a - c))$$

* *Corresponding author.*
E-mail address: vinnarasijenitha@gmail.com

$= a - ((a - b) - ((a - b) - (a - c)))$.

In a subtraction algebra, the following are true:

(a_1) $(x - y) - y = x - y$,
(a_2) $x - 0 = x$ and $0 - x = 0$,
(a_3) $(x - y) - x = 0$,
(a_4) $x - (x - y) \leq y$,

A non-empty subset A of a subtraction algebra X is called a subalgebra of X if $x - y \in A$ for any $x, y \in A$. A non-empty subset I of a subtraction algebra X is called an ideal of X if
(I_1) $0 \in I$,
(I_2) $(\forall x, y \in X)(x - y, y \in I$ imply $x \in I)$.
A mapping f: $X \to Y$ of subtraction algebras is called a homomorphism if $f(x - y) = f(x) - f(y)$ for all $x, y \in X$.

Definition 2.1
Let X be a space of points (objects) with generic elements in X denoted by x. A simple valued Pythagorean Fuzzy set Ain X is characterized by a truth-membership function $M_A(x)$, and a falsity -membership function $N_A(x)$. Then a simple valued Pythagorean Fuzzy set A can be denoted by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\},$$

where$M_A(x)$, $N_A(x) \in [0,1]$ for each point x in X. Therefore the sum of $M_A(x)$,$N_A(x)$ satisfies the condition $0 \leq M_A^2(x) + N_A^2(x) \leq 2$.

For convenience, "simple valued Pythagorean Fuzzy set" is abbreviated to "Pythagorean Fuzzy set" later.

Definition 2.2
Let A be a Pythagorean Fuzzy set in a subtraction algebra X and $\alpha, \beta \in [0,1]$ with $0 \leq \alpha+\beta \leq 2$ and an (α,β) - level set of X denoted by $A^{(\alpha,\beta)}$ is defined as

$$A^{(\alpha,\beta)} = \{x \in X \mid M_A(x) \geq \alpha, N_A(x) \leq \beta\}.$$

For any family $\{a_i | i \in \Lambda\}$, we define

$$\bigvee\{a_i|i \in \Lambda\} := \begin{cases} \max\{a_i|i \in \Lambda\} & \text{if } \Lambda \text{ is finite} \\ \sup\{a_i|i \in \Lambda\} & \text{otherwise} \end{cases}$$

and

$$\bigwedge\{a_i|i \in \Lambda\} := \begin{cases} \min\{a_i|i \in \Lambda\} & \text{if } \Lambda \text{ is finite} \\ \inf\{a_i|i \in \Lambda\} & \text{otherwise} \end{cases}$$

3 Pythagorean Fuzzy ideals

In this section, let X be a subtraction algebra unless otherwise specified.

Definition 3.1
A Pythagorean Fuzzy set Ain X is called a Pythagorean Fuzzy subalgebraof X if it satisfies:
(3.1) $(\forall x,y \in X)(M_A(x-y) \geq \min\{M_A(x), M_A(y)\}, N_A(x-y) \leq \max\{N_A(x), N_A(y)\})$.

Proposition 3.2
Every Pythagorean Fuzzy subalgebra of X satisfies the following conditions:

(3.2) $(\forall x \in X)(M_A(0) \geq M_A(x), N_A(0) \leq N_A(x))$.

Proof: Straightforward.

Example 3.3
Let X = {0, 1, 2, 3} be a subtraction algebra with the following table:

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	1
3	3	2	1	0

Define a Pythagorean Fuzzy set Ain X as follows:

$$M_A : X \to [0,1], x \to \begin{cases} 0.54 & \text{if } x \in \{0,3\} \\ 0.13 & \text{if } x \in \{1,2\} \end{cases}$$

and

$$N_A : X \to [0,1], x \to \begin{cases} 0.11 & \text{if } x \in \{0,3\} \\ 0.53 & \text{if } x \in \{1,2\} \end{cases}$$

It is easy to check that A is a Pythagorean Fuzzy subalgebra of X.

Theorem 3.4
Let A be a Pythagorean Fuzzy set in X and let $\alpha, \beta \in [0,1]$ with $0 \leq \alpha+\beta \leq 2$. Then A is a Pythagorean Fuzzy subalgebra of X if and only if all of (α, β) - level set $A^{(\alpha,\beta)}$ are subalgebras of X when $A^{(\alpha,\beta)} \neq \emptyset$.
Proof:
Assume that A is a Pythagorean Fuzzy subalgebra of X. Let $\alpha,\beta \in [0,1]$ be such that $0 \leq \alpha+\beta \leq 2$ and $A^{(\alpha,\beta)} \neq \emptyset$. Let $x, y \in A^{(\alpha,\beta)}$. Then $M_A(x) \geq \alpha$, $M_A(y) \geq \alpha$ and $N_A(x) \leq \beta$, $N_A(y) \leq \beta$. Using (3.1), we have $M_A(x - y) \geq \min\{M_A(x),M_A(y)\} \geq \alpha$ and $N_A(x - y) \leq \max\{N_A(x), N_A(y)\} \leq \beta$. Hence $x - y \in A^{(\alpha,\beta)}$. Therefore $A^{(\alpha,\beta)}$ is a subalgebra of X.

Conversely, all of (α,β)-level set $A^{(\alpha,\beta)}$ are subalgebras of X when $A^{(\alpha,\beta)} \neq \emptyset$. Assume that there exist $a_m, b_m \in X$ and $a_n, b_n \in X$ such that $M_A(a_m - b_m) < \min\{M_A(a_m), M_A(b_m)\}$ and $N_A(a_n - b_n) > \max\{N_A(a_n), N_A(b_n)\}$. Then $M_A(a_m - b_m) < \alpha_1 \leq \min\{M_A(a_m), M_A(b_m)\}$ and $N_A(a_n - b_n) > \beta_1 \geq \max\{N_A(a_n), N_A(b_n)\}$ for some $\alpha_1 \in (0,1]$ and $\beta_1 \in [0,1)$. Hence $a_m, b_m \in A^{(\alpha_1,\beta_1)}$, and $a_n, b_n \in A^{(\alpha_1,\beta_1)}$.But $a_m - b_m \notin A^{(\alpha_1,\beta_1)}$and $a_n - b_n \notin A^{(\alpha_1,\beta_1)}$, which is a contradiction. Hence $M_A(x - y) \geq \min\{M_A(x), M_A(y)\}$, and $N_A(x - y) \leq \max\{N_A(x), N_A(y)\}$ for any x, y, z ∈ X. Therefore A is a Pythagorean Fuzzy subalgebra of X. Since [0,1] is a completely distributive lattice with respect to the usual ordering, we have the following theorem.

Theorem 3.5
If $\{A_i | i \in \mathbb{N}\}$ is a family of Pythagorean Fuzzy subalgebras of X, then $(\{A_i | i \in \mathbb{N}\}, \subseteq)$ forms a complete distributive lattice.

Theorem 3.6
Let A be a Pythagorean Fuzzy subalgebra of X. If there exists a sequence $\{a_n\}$ in X such that $\lim_{n\to\infty} M_A(a_n) = 1$ and $\lim_{n\to\infty} N_A(a_n) = 0$, then $M_A(0) = 1$ and $N_A(0) = 0$.
Proof:
By Proposition 3.2, we have $M_A(0) \geq M_A(x)$ and $N_A(0) \leq N_A(x)$ for all $x \in X$. Hence we have $M_A(0) \geq M_A(a_n)$ and $N_A(0) \leq N_A(a_n)$ for every positive integer n. Therefore $1 = \lim_{n\to\infty} M_A(a_n) \leq M_A(0) \leq 1$ and $0 \leq N_A(0) \leq \lim_{n\to\infty} N_A(a_n) = 0$. Thus we have $M_A(0) = 1$ and $N_A(0) = 0$.

Proposition 3.7
If every Pythagorean Fuzzy subalgebraA of X satisfies the condition
(3.3) $(\forall x, y \in X)\ (M_A(x - y) \geq M_A(y)$ and $N_A(x - y) \leq N_A(y))$,
thenM_A and N_A are constant functions.
Proof:
It follows from (3.3) that $M_A(x) = M_A(x - 0) \geq M_A(0)$ and $N_A(x) = N_A(x - 0) \leq N_A(0)$ for any $x \in X$. By Proposition 3.2, we have $M_A(x) = M_A(0)$ and $N_A(x) = N_A(0)$ for any $x \in X$. Hence M_A and N_A are constant functions.

Theorem 3.8
Every subalgebra of X can be represented as an (α, β)-level set of a Pythagorean Fuzzy subalgebraA of X.
Proof:
Let S be a subalgebra of X and let A be a Pythagorean Fuzzy subalgebra of X. Define a Pythagorean Fuzzy set A in X as follows:

$$M_A : X \to [0,1], x \to \begin{cases} \alpha_1, & if\ x \in S \\ \alpha_2, & otherwise, \end{cases}$$

$$N_A : X \to [0,1], x \to \begin{cases} \beta_1, & if\ x \in S \\ \beta_2, & otherwise, \end{cases}$$

where $\alpha_1,\alpha_2 \in (0,1]$ and $\beta_1,\beta_2 \in [0,1)$ with $\alpha_1 > \alpha_2$, $\beta_1 < \beta_2$ and $0 \leq \alpha_1 + \beta_1 \leq 2$, $0 \leq \alpha_2 + \beta_2 \leq 2$. Obviously, $S = A^{(\alpha_1,\beta_1)}$. We prove that A is a Pythagorean Fuzzy subalgebra of X. Let $x, y \in X$. If $x, y \in S$, then $x - y \in S$ because S is a subalgebra of X. Hence $M_A(x) = M_A(y) = M_A(x - y) = \alpha_1$, $N_A(x) = N_A(y) = N_A(x - y) = \beta_1$ and so $M_A(x - y) \geq \min\{M_A(x),M_A(y)\}$, $N_A(x - y) \leq \max\{N_A(x),N_A(y)\}$. If $x \in S$ and $y \notin S$, then $M_A(x) = \alpha_1$, $M_A(y) = \alpha_2$, $N_A(x) = \beta_1$, $N_A(y) = \beta_2$ and so $M_A(x - y) \geq \min\{M_A(x), M_A(y)\} = \alpha_2$, $N_A(x - y) \leq \max\{N_A(x), N_A(y)\} = \beta_2$. Obviously, if $x \notin S$ and $y \notin S$, then $M_A(x - y) \geq \min\{M_A(x), M_A(y)\} = \alpha_2$, $N_A(x - y) \leq \max\{N_A(x), N_A(y)\} = \beta_2$. Therefore A is a Pythagorean Fuzzy subalgebra of X.

Theorem 3.9
Let A be a Pythagorean Fuzzy set of X and let $\alpha, \beta \in [0,1]$ with $0 \leq \alpha + \beta \leq 2$. Define a Pythagorean Fuzzy set A^*in X as follows:

$$M_{A^*} : X \to [0,1], x \to \begin{cases} M_A(x), & \text{if } x \in A^{(\alpha,\beta)} \\ 0, & \text{otherwise} \end{cases}$$

and

$$N_{A^*} : X \to [0,1], x \to \begin{cases} N_A(x), & \text{if } x \in A^{(\alpha,\beta)} \\ 1, & \text{otherwise} \end{cases}$$

If A is a Pythagorean Fuzzy subalgebra of X, then so is A^*.
Proof:
Let A be a Pythagorean Fuzzy subalgebra of X. By Theorem 3.4, all of (α, β) - level set $A^{(\alpha,\beta)}$ are subalgebras of X. If $x, y \in A^{(\alpha,\beta)}$, then $x - y \in A^{(\alpha,\beta)}$. Hence we have $M_{A^*}(x - y) = M_A(x - y) \geq \min\{M_A(x), M_A(y)\} = \min\{M_{A^*}(x), M_{A^*}(y)\}$ and $N_{A^*}(x - y) = N_A(x - y) \leq \max\{N_A(x), N_A(y)\} = \max\{N_{A^*}(x), N_{A^*}(y)\}$ for any $x, y \in X$. If $x \notin A^{(\alpha,\beta)}$ or $y \notin A^{(\alpha,\beta)}$, then $M_{A^*}(x) = 0$, $N_{A^*}(x) = 1$ or $M_{A^*}(y) = 0$, $N_{A^*}(y) = 1$. Therefore we get $M_{A^*}(x - y) \geq \min\{M_{A^*}(x),M_{A^*}(y)\} = 0$ and $N_{A^*}(x - y) \leq \max\{N_{A^*}(x),N_{A^*}(y)\} = 1$ for any $x, y \in X$. Thus $A^*$is a Pythagorean Fuzzy subalgebra of X.

Definition 3.10
A Pythagorean Fuzzy set A in X is called a Pythagorean Fuzzy ideal of X if it satisfies (3.2) and
(3.4) $(\forall x,y \in X)(M_A(x) \geq \min\{M_A(x - y),M_A(y)\}$ and $N_A(x) \leq \max\{N_A(x - y),N_A(y)\})$.

Proposition 3.11

Every Pythagorean Fuzzy ideal of X is a Pythagorean Fuzzy subalgebra of X.

Proof:

Let A be a Pythagorean Fuzzy ideal of X. Put x = x − y and y = x in (3.4). Then we have $M_A(x-y) \geq \min\{M_A((x-y)-x), M_A(x)\}$ and $N_A(x-y) \leq \max\{N_A((x-y)-x), N_A(x)\}$. It follows from ($a_3$) and (3.2) that $M_A(x-y) \geq \min\{M_A((x-x)-y), M_A(x)\} = \min\{M_A(0), M_A(x)\} \geq \min\{M_A(x), M_A(y)\}$ and $N_A(x-y) \leq \max\{N_A((x-y)-x), N_A(x)\} = \max\{N_A(0), N_A(x)\} \leq \max\{N_A(x), N_A(y)\}$, for any x, y ∈ X. Thus A is a Pythagorean Fuzzy subalgebra of X.

The converse of Proposition 3.11 may not be true in general (see Example 3.12.)

Example 3.12

(a) Let X = {0,a,b,c} be a subtraction algebra with the following table:

−	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Define a Pythagorean Fuzzy set Ain X as follows:

$$M_A : X \rightarrow [0,1], x \rightarrow \begin{cases} 0.72 & \text{if } x \in \{0,a\} \\ 0.11 & \text{if } x \in \{b,c\} \end{cases}$$

and

$$N_A : X \rightarrow [0,1], x \rightarrow \begin{cases} 0.13 & \text{if } x \in \{0,a\} \\ 0.71 & \text{if } x \in \{b,c\} \end{cases}$$

It is easy to check that A is a Pythagorean Fuzzy ideal of X.

(b) Let X = {0, 1, 2, 3} be a subtraction algebra as in Example 3.3. Define a Pythagorean Fuzzy set B in X as follows:

$$M_B : X \rightarrow [0,1], x \rightarrow \begin{cases} 0.53 & \text{if } x = 0 \\ 0.22 & \text{if } x \in \{1,2\} \\ 0.13 & \text{if } x = 3 \end{cases}$$

and

$$N_B : X \rightarrow [0,1], x \rightarrow \begin{cases} 0.11 & \text{if } x = 0 \\ 0.25 & \text{if } x \in \{1,2\} \\ 0.46 & \text{if } x = 3 \end{cases}$$

It is easy to check that B is a Pythagorean Fuzzy subalgebra of X. But it is not a Pythagorean Fuzzy ideal of X, since $M_B(3) = 0.13 \ngeq \min\{M_B(3\text{-}1), M_B(1)\} = \max\{M_B(2), M_B(1)\} = 0.22$.

Theorem 3.13

Let A be a Pythagorean Fuzzy set in X and let α, β ∈ [0,1] with 0 ≤ α+β ≤ 2. Then A is a Pythagorean Fuzzy ideal of X if and only if all of (α, β)-level set $A^{(\alpha,\beta)}$ are ideals of X when $A^{(\alpha,\beta)} \neq \emptyset$.

Proof:

Assume that A is a Pythagorean Fuzzy ideal of X. Let α,β∈ [0,1] be such that 0 ≤ α + β ≤ 2 and $A^{(\alpha,\beta)} \neq \emptyset$. Let x, y ∈ X be such that $x-y, y \in A^{(\alpha,\beta)}$. Then $M_A(x-y) \geq \alpha$, $M_A(y) \geq \alpha$ and $N_A(x-y) \leq \beta$, $N_A(y) \leq \beta$. By Definition 3.10, we have $M_A(0) \geq M_A(x) \geq \min\{M_A(x-y), M_A(y)\} \geq \alpha$ and $N_A(0) \leq N_A(x) \leq \max\{N_A(x-y), N_A(y)\} \leq \beta$. Hence $0, x \in A^{(\alpha,\beta)}$. Therefore $A^{(\alpha,\beta)}$ is an ideal of X.

Conversely, suppose that there exist a, b ∈ X such that $M_A(0) < M_A(a)$ and $N_A(0) > N_A(b)$. Then there exist $a_m \in (0,1]$ and $b_m \in [0,1)$ such that $M_A(0) < a_m \leq M_A(a)$ and $N_A(0) > b_m \geq N_A(b)$. Hence $0 \notin A^{(a_m,b_m)}$, which is a contradiction. Therefore $M_A(0) \geq M_A(x)$ and $N_A(0) \leq N_A(x)$ for all x ∈ X. Assume that there exist $a_m,b_m,a_n,b_n \in X$ such that $M_A(a_m) < \min\{M_A(a_m-b_m), M_A(b_m)\}$ and $N_A(a_n) > \max\{N_A(a_n-b_n), N_A(b_n)\}$. Then there exist $s_m \in (0,1]$ and $s_n \in [0,1)$ such that $M_A(a_m) < s_m \leq \min\{M_A(a_m-b_m), M_A(b_m)\}$ and $N_A(a_n) > s_n \geq \max\{N_A(a_n-b_n), N_A(b_n)\}$. Hence $a_m-b_m, a_n-b_n \in A^{(s_m,s_n)}$ and $b_m, b_n \in A^{(s_m,s_n)}$. But $a_m \notin A^{(s_m,s_n)}$ and $a_n \notin A^{(s_m,s_n)}$.This is a contradiction. Therefore $M_A(x) \geq \min\{M_A(x-y), M_A(y)\}$ and $N_A(x) \leq \max\{N_A(x-y), N_A(y)\}$, for any x,y∈ X. Therefore A is a Pythagorean Fuzzy ideal of X.

Proposition 3.14

Every Pythagorean Fuzzy ideal A of X satisfies the following properties:

(i) $(\forall x, y \in X)(x \leq y \Rightarrow M_A(x) \geq M_A(y), N_A(x) \leq N_A(y))$,

(ii) $(\forall x, y, z \in X)(x - y \leq z \Rightarrow M_A(x) \geq \min\{M_A(y), M_A(z)\}, N_A(x) \leq \max\{N_A(y), N_A(z)\})$.

Proof.

(i) Let x,y∈ X be such that x ≤ y. Then x − y = 0. Using (3.4) and (3.2), we have $M_A(x) \geq \min\{M_A(x-y), M_A(y)\} = \min\{M_A(0), M_A(y)\} = M_A(y)$ and $N_A(x) \leq \max\{N_A(x-y), N_A(y)\} = \max\{N_A(0), N_A(y)\} = N_A(y)$.

(ii) Let x, y, z ∈ X be such that x −y ≤ z. By (3.4) and (3.2), we get $M_A(x-y) \geq \min\{M_A((x-y)-z), M_A(z)\} = \min\{M_A(0), M_A(z)\} = M_A(z)$, $N_A(x-y) \leq \max\{N_A((x-y)-z), N_A(z)\} = \max\{N_A(0), N_A(z)\} = N_A(z)$. Hence $M_A(x) \geq \min\{M_A(x-y), M_A(y)\} \geq \min\{M_A(y), M_A(z)\}$, and $N_A(x) \leq \max\{N_A(x-y), N_A(y)\} \leq \max\{N_A(y), N_A(z)\}$, for any x, y, z ∈ X.

The following corollary is easily proved by induction.

Corollary 3.15:

Every Pythagorean Fuzzy ideal A of X satisfies the following property:

(3.5) $(\forall x, a_1, \cdots, a_n \in X)((\cdots(x-a_1)-\cdots)-a_n = 0 \Rightarrow M_A(x) \geq \bigwedge_{k=1}^{n} M_A(a_k)$ and $N_A(x) \leq \bigvee_{k=1}^{n} N_A(a_k))$.

Definition 3.16

Let A and B be Pythagorean Fuzzy sets of a set X. The union of A and B is defined to be a Pythagorean Fuzzy set

$$A\tilde{\cup}B = \{\langle x, M_{A\cup B}(x), N_{A\cup B}(x)\rangle | x \in X\},$$

where$M_{A\cup B}(x) = \max\{M_A(x),M_B(x)\}$, $N_{A\cup B}(x) = \min\{N_A(x),N_B(x)\}$, for all $x \in X$. The intersection of Aand B is defined to be a Pythagorean Fuzzy set

$$A\tilde{\cap}B = \{\langle x, M_{A\cap B}(x), N_{A\cap B}(x)\rangle | x \in X\},$$

where$M_{A\cap B}(x) = \min\{M_A(x),M_B(x)\}$, $N_{A\cap B}(x) = \max\{N_A(x),N_B(x)\}$, for all $x \in X$.

Theorem 3.17

The intersection of two Pythagorean Fuzzy ideals of X is also a Pythagorean Fuzzy ideal of X.

Proof:

Let A and B be Pythagorean Fuzzy ideals of X. For any $x \in X$, we have $M_{A\cap B}(0) = \min\{M_A(0),M_B(0)\} \geq \min\{M_A(x),M_B(x)\} = M_{A\cap B}(x)$ and $N_{A\cap B}(0) = \max\{N_A(0),N_B(0)\} \leq \max\{N_A(x),N_B(x)\} = N_{A\cap B}(x)$. Let x, y ∈ X. Then we have

$$\begin{aligned} M_{A\cap B}(x) &= \min\{M_A(x),M_B(x)\} \\ &\geq \min\{\min\{M_A(x-y),M_A(y)\},\min\{M_B(x-y),M_B(y)\}\} \\ &= \min\{\min\{M_A(x-y),M_B(x-y)\},\min\{M_A(y),M_B(y)\}\} \\ &= \min\{M_{A\cap B}(x-y),M_{A\cap B}(y)\} \end{aligned}$$

and

$$\begin{aligned} N_{A\cap B}(x) &= \max\{N_A(x),N_B(x)\} \\ &\leq \max\{\max\{N_A(x-y),N_A(y)\},\max\{N_B(x-y),N_B(y)\}\} \\ &= \max\{\max\{N_A(x-y),N_B(x-y)\},\max\{N_A(y),N_B(y)\}\} \\ &= \max\{N_{A\cap B}(x-y),N_{A\cap B}(y)\}. \end{aligned}$$

Hence $A\tilde{\cap}B$ is a Pythagorean Fuzzy ideal of X.

Corollary 3.18

If $\{A_i \mid i \in \mathbb{N}\}$ is a family of Pythagorean Fuzzy ideals of X, then so is $\tilde{\cap}_{i\in\mathbb{N}}A_i$.

Proposition 3.19

Let A be a Pythagorean Fuzzy ideal of X. Then $X_M = \{x \in X | M_A(x) = M_A(0)\}$ and $X_N = \{x \in X | N_A(x) = N_A(0)\}$ are ideals of X.

Proof:

Clearly, $0 \in X_M$. Let $x-y, y \in X_M$. Then $M_A(x-y) = M_A(0)$ and $M_A(y) = M_A(0)$. It follows from (3.4) that $M_A(x) \geq \min\{M_A(x-y),M_A(y)\} = M_A(0)$. By (3.2), we get $M_A(x) = M_A(0)$. Hence $x \in X_M$. Therefore X_M is an ideal of X. By a similar way, X_N is an ideals of X.

Let $f : X \to Y$ be a function of sets. If

$$P = \{\langle y, M_P(y),N_P(y)\rangle | y \in Y\}$$

is a Pythagorean Fuzzy set of a set Y , then the preimage of P under f is defined to be a Pythagorean Fuzzy set

$$f^{-1}(P) = \{\langle x, f^{-1}(M_P)(x), f^{-1}(N_P)(x)\rangle | x \in X\}$$

of X, where $f^{-1}(M_P)(x) = M_P(f(x))$ and $f^{-1}(N_P)(x) = N_P(f(x))$ for all $x \in X$.

Theorem 3.20

Let $f : X \to Y$ be a homomorphism of subtraction algebras. If $P = \{\langle y, M_P(y), N_P(y)\rangle | y \in Y\}$ is a Pythagorean Fuzzy subalgebra of Y , then the preimage of P under f is a Pythagorean Fuzzy subalgebra of X.

Proof:

Let $f^{-1}(P)$ be the preimage of P under f. For any x, y ∈ X, we have

$$f^{-1}(M_P(x-y)) = M_P(f(x-y)) = M_P(f(x)-f(y))$$

$$\geq \min\{M_P(f(x)),M_P(f(y))\}$$

$= \min\{f^{-1}(M_P)(x),f^{-1}(M_P)(y)\}$

and

$$f^{-1}(N_P(x-y)) = N_P(f(x-y)) = N_P(f(x)-f(y))$$

$$\leq \max\{N_P(f(x)),N_P(f(y))\}$$

$$= \max\{f^{-1}(N_P)(x),f^{-1}(N_P)(y)\}.$$

Hence $f^{-1}(P)$ is Pythagorean Fuzzy subalgebra of X.

Let $f : X \to Y$ be an onto function of sets. If A is a Pythagorean Fuzzy set of X, then the image of A under f is defined to be a Pythagorean Fuzzy set

$f(A) = \{\langle y, f(M_A)(y), f(N_A)(y)\rangle | y \in Y\}$
of Y, where $f(M_A)(y) = \bigvee_{x\in f^{-1}(y)} M_A(x)$ and $f(N_A)(y) = \bigwedge_{x\in f^{-1}(y)} N_A(x)$.

Theorem 3.21
For an onto homomorphism $f : X \to Y$ of subtraction algebras, let A be a Pythagorean Fuzzy set of X such that (3.6) $(\forall C \subseteq X)(\exists x_0 \in C)(M_A(x_0) = \bigvee_{z\in C} M_A(z), N_A(x_0) = \bigwedge_{z\in C} N_A(z))$. If A is a Pythagorean Fuzzy subalgebra of X, then the image of A under f is a Pythagorean Fuzzy subalgebraof .

Proof:
Let f(A) be the image of A under f. Let $a,b\in Y$. Then $f^{-1}(a) \neq \emptyset$ and $f^{-1}(b) \neq \emptyset$ in X. By (3.6), there exist $x_a \in f^{-1}(a)$ and $x_b \in f^{-1}(b)$ such that
$M_A(x_a) = \bigvee_{z\in f^{-1}(a)} M_A(z)$, $N_A(x_a) = \bigwedge_{z\in f^{-1}(a)} N_A(z)$,
$M_A(x_b) = \bigvee_{w\in f^{-1}(b)} M_A(w)$, $N_A(x_b) = \bigwedge_{w\in f^{-1}(b)} N_A(w)$.
Thus
$f(M_A)(a-b) = \bigvee_{x\in f^{-1}(a-b)} M_A(x) \geq M_A(x_a - x_b) \geq \min\{M_A(x_a), M_A(x_b)\}$
$= \min\{\bigvee_{z\in f^{-1}(a)} M_A(z), \bigvee_{w\in f^{-1}(b)} M_A(w)\}$
$= \min\{f(M_A)(a), f(M_A)(b)\}$,

and

$f(N_A)(a-b) = \bigwedge_{x\in f^{-1}(a-b)} N_A(x) \leq N_A(x_a - x_b) \leq \max\{N_A(x_a), N_A(x_b)\}$
$= \max\{\bigwedge_{z\in f^{-1}(a)} N_A(z), \bigwedge_{w\in f^{-1}(b)} N_A(w)\}$
$= \max\{f(N_A)(a), f(N_A)(b)\}$.
Hence f(A) is a Pythagorean Fuzzy subalgebra of Y .

References

J. C. Abbott, Sets, Lattices and Boolean Algebras, Allyn and Bacon, Boston, 1969.
S. S. Ahn and Y. H. Kim, Quotient subtraction algebras by an int-soft ideal, J. Comput. Anal. Appl. 25 (2018), 728-737.
K. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and Systems 20 (1986), 87–96. [4] Y. B. Jun and H. S. Kim, On ideals in subtraction algebras, Sci. Math. Jpn. Online, e-2006 (2006), 1081-1086.
Y. B. Jun, H. S. Kim and E. H. Roh, Ideal theory of subtraction algebras, Sci. Math. Jpn. Online, e-2004 (2004), 397-402.
Y. B. Jun, Y. H. Kim and K. A. Oh, Subtraction algebras with additional conditions, Commun. Korean Math. Soc. 22 (2007), 1–7.
Y. B. Jun, F. Smarandache and H. Bordbar, Neutrosophic N -structures applied to BCK/BCI-algebras, Information, (to submit).
R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Soft set, IRJMETS, 2021, Volume 3(2), 905-914.
R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, IRJASH, 2021, volume 3, 62-82.
R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic Generalized semi-closed sets, 123-131.
R. Radha, A. Stanis Arul Mary, Improved Correlation Coefficients of Quadripartitioned Neutrosophic Pythagorean sets for MADM, 142-153.
R. Radha, A. Stanis Arul Mary. Heptapartitioned Neutrosophic sets, IRJCT, 2021, volume 2,222-230.
R. Radha, A. Stanis Arul Mary, F. Smarandache. Quadripartitioned Neutrosophic Pythagorean soft set, International journal of Neutrosophic Science, 2021, volume 14(1), 9-23.
R. Radha, A. Stanis Arul Mary, F. Smarandache. Neutrosophic Pythagorean soft set, Neutrosophic sets and systems, 2021, vol 42,65-78.
R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean resolvable and irresolvable spaces (Communicated)
R. Radha, A. Stanis Arul Mary, Bipolar Pentapartitioned Neutrosophic set and it's Generalized Semi-closed sets, International Journal of Research Publications and Reviews vol (2) issue (8) (2021) Page 1130 - 1137.
B. M. Schein, Difference Semigroups, Comm. Algebra 20 (1992), 2153-2169.
F. Smarandache, Neutrosophy, Neutrosophic Probablity, Sets, and Logic, Amer. Res. Press, Rehoboth, USA, 1998.
L. A. Zadeh, Fuzzy sets, Inform. Control 8 (1965), 338-353.
B. Zelinka, Subtraction Semigroups, Math. Bohemica 120 (1995), 445-447.