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Pythagorean Fuzzy Preopen Sets

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A B S T R A C T

The concept of Fuzzy sets was introduced by Zadeh in 1965. After the introduction of Intuitionistic Fuzzyset by Atanassov in 1986, R. R. Yager generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean Fuzzy set. The main focus of this research paper is to introduce the concept of preopen set and preclosed sets in Pythagorean Fuzzy topological spaces. We studied and established more relations between the pre interior and pre closure in the Pythagorean Fuzzy topological concepts.

Keywords: Pythagorean Fuzzy preopen set, Pythagorean Fuzzy topological space, Pythagorean Fuzzy set, Pythagorean Fuzzy preclosed sets, interior and closure of Pythagorean Fuzzy space.

1. Introduction

Intuitionistic Fuzzy set was first introduced by K. T. Atanassov in 1983. After that he introduced the concept of Intuitionistic sets as generalization of Fuzzy sets. The concept of generalized topological structures in Fuzzy topological spaces using Intuitionistic Fuzzy sets was introduced by D. Coker. D. Coker introduced the concept of Intuitionistic Fuzzy sets, Intuitionistic Fuzzy topological spaces, Intuitionistic topological spaces and Intuitionistic Fuzzy points. R. R. Yager generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean Fuzzy set. G. Sasikala, M. Navaneethakrishnan introduced the concept of Intuitionistic semiopen sets, Intuitionistic **α**-open sets and Intuitionistic preopen sets. In this paper, we focus on the main concept of Pythagorean Fuzzy preopen sets in Pythagorean Fuzzy topological spaces.

2. Preliminaries

Definition 2.1:
An Pythagorean Fuzzy set (PFS in short) A in X is an object having the form $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ where the functions $\lambda_A: X \to [0,1]$ and $\mu_A: X \to [0,1]$ denote the degree of membership (namely $\lambda_A(a)$) and the degree of non-membership (namely $\mu_A(a)$) of each element $a \in X$ to the set A respectively, $0 \leq \lambda_A^2(a) + \mu_A^2(a) \leq 1$ for each $a \in X$.

Definition 2.2:
Let A and B be PFSs of the forms $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ and $B = \{< a, \lambda_B(a), \mu_B(a) > / a \in X\}$. Then

a) $A \subseteq B$ if and only if $\lambda_A(a) \leq \lambda_B(a)$ and $\mu_A(a) \geq \mu_B(a)$ for all $a \in X$
b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
c) $\overline{A} = \{< a, \mu_A(a), \lambda_A(a) > / a \in X\}$
d) $A \cap B = \{< a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) > / a \in X\}$
e) $A \cup B = \{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > / a \in X\}$
f) $\emptyset = \{< a, \emptyset, X > / a \in X\}$ and $X = \{< a, X, \emptyset > / a \in X\}$
g) $\overline{X} = \emptyset$ and $\overline{\emptyset} = X$

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Definition 2.3:

An Pythagorean Fuzzy topology by subsets of a non-empty set X is a family $\tau$of PFSs satisfying the following axioms.

a) $\emptyset, X \in \tau$

b) $G_1 \cap G_2 \in \tau$ for every $G_1, G_2 \in \tau$ and

c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$

The pair (X, τ) is called an Pythagorean Fuzzy topological space (PFTS in short) and any PFS G in τ is called an Pythagorean Fuzzy open set (PFOS in short) in X. The complement $\overline{A}$ of an Pythagorean Fuzzy open set A in an PFTS (X, τ) is called an Pythagorean Fuzzy closed set (PFCS in short).

Definition 2.4:

Let (X, τ) be an PFTS and $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ be an PFS in X. Then the interior and the closure of A are denoted by PFint(A) and PFcl(A) and are defined as follows.

PFcl(A) = $\cap\{K|K$ is an PFCS and $A \subseteq K\}$ and

PFint(A) = $\cup\{G|G$ is an PFOS and $G \subseteq A\}$

Also, it can be established that PFcl(A) is an PFCS and PFint(A) is an PFOS, A is an PFCS if and only if PFcl(A) = A and A is an PFOS if and only if PFint(A) = A. We say that A is PF- dense if PFcl(A) = X.

Lemma 2.5:

Let (X, τ) be an PFTS and let A and B be an Pythagorean Fuzzy subset of X. Then the following hold.

1) PFcl($\emptyset$) = $\emptyset$ and PFcl(X) = X
2) A is an PFCS if and only if A = PFcl(A)
3) PFcl(PFcl(A)) = PFcl(A)
4) A $\subseteq$ B implies that PFcl(A) $\subseteq$PFcl(B)
5) PFcl(A $\cap$ B) $\subseteq$PFcl(A) $\cap$ PFcl(B)
6) PFcl(A$\cup$B) = PFcl(A) $\cup$PFcl(B)

Lemma 2.6:

Let (X, τ) be an PFTS and let A and B be an Pythagorean Fuzzy subset of X. Then the following hold.

1) PFint($\emptyset$) = $\emptyset$ and PFint(X) = X
2) A is an PFOS if and only if A = PFint(A)
3) PFint(PFint(A)) = PFint(A)
4) A $\subseteq$ B implies that PFint(A) $\subseteq$PFint(B)
5) PFint(A $\cap$ B) = PFint(A) $\cap$ PFint(B)
6) PFint(A$\cup$B) $\supseteq$PFint(A) $\cup$PFint(B)

Lemma 2.7:

Let (X, τ) be an PFTS and let A and B be two PFS in (X, τ). IF B is an PFOS, then PFcl(A) $\cap$ B $\subseteq$PFcl(A $\cap$ B). So if G is PFCS and H is any PFS, then PFint(H $\cup$ G) $\subset$PFint(H) $\cup$ G.

Lemma 2.8:

For any Pythagorean Fuzzy set A in (X, τ), we have X – PFint(A) = PFcl(X – A) and X – PFcl(A) = PFint(X – A).

Definition 2.9:

Let (X, τ) be an PFTS and $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$. A is an Pythagorean Fuzzy semiopen set (PFSOS in short) in an PFTS (X, τ) if there is an Pythagorean Fuzzy open set $G \neq < X, \emptyset, X >$ such that $G \subset A \subset$PFcl(A). Clearly, every PFOS is an PFSOS and $\emptyset$ and X are PFSOS. Also, from the definition, it follows that the closure of every PFOS is an Pythagorean Fuzzy semiopen set. The complement of every PFSOS is said to be anPythagorean Fuzzy semiclosed set (PFSCS in short).

The semiinterior and the semiclosure of an PFS A is denoted by PFSint(A) and PFScl(A) and are defined as follows.

PFScl(A) = $\cap\{K|K$ is an PFSCS and $A \subseteq K\}$

PFSint(A) = $\cup\{G|G$ is an PFSOS and $G \subseteq A\}$.

It can be established that PFScl(A) is the smallest PFSCS contained in all PFSCS containing A and PFSint(A) is the largest PFSOS contained in A, A is anPFSCS if and only if PFScl(A) = A and A is an PFSOS if and only if PFSint(A) = A.

Definition 2.10:

Let (X, τ) be an PFTS and A be an PFS. A is said to be an Pythagorean Fuzzyα-open set (in short PFαOS) if A $\subseteq$PFint(PFcl(PFint(A))).

The family of all Pythagorean Fuzzyα-open set is denoted by PFαOS. The complement of an PFαOS is called an Pythagorean Fuzzyα-closed set (in short PFαCS).

Definition 2.11:

Let X be a nonempty set and p$\in$ X, a fixed element. Then the PFS p defined by $p = < X, \{p\}, \{p\}^c >$ is called an Pythagorean Fuzzy point (PFP in short).

3. Pythagorean Fuzzy Preopen Sets

In this section, we have defined Pythagorean Fuzzy preopen (PFPO in short) sets in a Pythagorean Fuzzy topological space (X, τ) and studied its characterizations and properties.

Definition 3.1:
Let (X, τ) be an PFTS and A be an PFS. A is said to be PFPO if A⊂PFint(PFcl(A)).

Example 3.2:
Let X = {a,b} and let τ = {∅,A,1} be a Pythagorean Fuzzy topology on X, where A={< a,0.2,0.3 >, < b,0.4,0.7 >}, τ^c= {0,X,A^c}. Then PFcl(A) = A^c and PFint(A^c) = A. So A ⊂PFint(PFcl(A)). Therefore, A is an PFPO set in (X, τ). If B = {(a, 0.2, 0.6), (b, 0.5, 0.2)}. Then PFcl(B) = A^c and PFint(PFcl(B)) = A. So B ⊈PFint(PFcl(B)) = A. Therefore, B is not an PFPO set in (X, τ).
The following Theorem 3.3 shows that the arbitrary union of PFPO sets is an PFPO set.

Theorem 3.3:
Let (X, τ) be an PFTS and {A_α| α ∈ I} be a family of PFPO sets in (X, τ). Then ∪{A_α|α ∈ I} is an PFPO set.
Proof: Let {A_α| α ∈ I} be a family of Pythagorean Fuzzy preopen sets in (X, τ). Then for each α, A_α⊂PFint(PFcl(A_α)). Since A_α ⊂∪ A_α, PFint(PFcl(A_α)) ⊂PFint(PFcl(∪ A_α)) and also A_α⊂PFint(PFcl(∪ A_α)). Hence ∪ A_α⊂PFint(PFcl(∪ A_α)) which shows that ∪ A_α is an PFPO set.

Definition 3.4:
Let (X, τ) be an PFTS**.** If A and B are any two PFPO sets in (X, τ), one of which is anPFO set, then A ∩ B is an PFPO.
Proof:
Let A and B be two Pythagorean Fuzzy sets in (X, τ). Suppose A is an PFPO set and B is an PFO set. Then A ⊆PFint(PFcl(A)). Now
A ∩ B ⊂PFint(PFcl(A) ∩ B) = PFint(PFcl(A)) ∩ PFint(B),
= PFint(PFcl(A) ∩ B),
⊂PFint(PFcl(A∩ B)), by Lemma 2.7
Therefore, A ∩ B is anPFPO set.

The converse of the Theorem need not be true.

Theorem 3.5:
Let (X, τ) be an PFTS, A be an PFPO set and B be an Pythagorean Fuzzy set such that B ⊆ A ⊆ PFcl(B). Then B is also an PFPO set.
Proof: A ⊆PFcl(B) implies that PFcl(A) ⊆PFcl(B) and so A ⊆PFint(PFcl(A)) ⊆PFint(PFcl(B)) and so B ⊆PFint(PFcl(B)) . Therefore, B is an PFPO set.

Definition 3.6:
Let (X, τ) be an PFTS and A = < X,A^1, A^2> be an Pythagorean Fuzzy set. A is said to be Pythagorean Fuzzypreclosed (PFPC in short) if it is the complement of an PFPO set.

Lemma 3.7:
Let (X, τ) be an PFTS and A = < X,A^1, A^2> be an Pythagorean Fuzzy set. Then A is PFPC if and only if PFcl(PFint(A))⊆ A.
Proof:
A is PFPC if and only if $\overline{A}$ is PFPO if and only if $\overline{A}$ ⊂PFint(PFcl($\overline{A}$)) if and only if $\overline{\mathrm{PFint}(\mathrm{PFcl}(\overline{A}))}$⊂ A if and only if PFcl[$\overline{\mathrm{PFcl}(\overline{A})}$]⊂ A if and only if PFcl(PFint($\overline{\overline{A}}$)) ⊂ A if and only if PFcl(PFint(A)) ⊂ A, by Lemma 2.8.

Definition 3.8:
Let (X, τ) be an PFTS and A be an Pythagorean Fuzzy set. Then A is said to be an Pythagorean Fuzzy regular open (PFRO in short) set if A = PFint(PFcl(A)). A is said to be an Pythagorean Fuzzy regular closed (PFRC in short) set if A = PFcl(PFint(A)). By Lemma 2.8, it follows that A is an PFRO set if and only if $\overline{A}$ is an PFRC set. The following Theorem 3.10 gives some properties of PFRO and PFRC sets.

Theorem 3.9:
Let (X, τ) be an PFTS and A be an Pythagorean Fuzzy set. Then the following hold.
a) PFcl(A) is an PFRC if A is an PFO set.
b) If F is an PFC set, then PFint(F) is an PFRO set.
c) PFint(PFcl(A)) is an PFRO set.
d) PFint(PFcl(A)) is an PFRC set.
Proof:
(a) Let A be Pythagorean Fuzzy open set. Then A= PFint(A). Now, PFcl(A) = PFcl(PFint(A)) ⊂PFcl(PFint(PFcl(A))). Thus PFcl(A) ⊂PFcl(PFint(PFcl(A))). Again PFcl(PFint(PFcl(A))) ⊂PFcl(A). Thus PFcl(PFint(PFcl(A))) = PFcl(A).
(b) If F is a PFC, then X − F is PFO and so PFcl(X − F) is Pythagorean Fuzzyregularclosed by (a). Therefore X − PFcl(X − F) = PFint(F) is an PFRO set.
(c) By (b), since PFcl(A) is an PFC set,PFint(PFcl(A)) is an PFRO set.
(d) By (a), since PFint(A) is an PFO set, PFcl(PFint(A)) is an PFRC set.

The following Theorem gives characterizations of PFPO sets in terms of PFRO sets.

Theorem 3.10:
Let (X, τ) be an PFTS and A be an PFPO set. Then PFcl(A) is an PFRC set.
Proof:
A is an PFPO set implies that A ⊆PFint(PFcl(A)) which implies that PFcl(A) ⊆PFcl(PFint(PFcl(A))) and so PFcl(A) = PFcl(PFint(PFcl(A))) which implies that PFcl(A) is an PFRC set.

4. Pythagorean Fuzzy Pre-closure and Pythagorean Fuzzy Pre-interior Operators

Let (X, τ) be an PFTS and A = < X, A^1,A^2> be an PFS. Then the Pythagorean Fuzzy preinterior and the Pythagorean Fuzzy preclosure of a subset A are denoted by PFPint(A) and PFPcl(A), and are denoted as follows.
PFPcl(A) = ∩{K|K is an PFPC and A ⊆ K}
PFPint(A) = ∪{G|G is an PFPO and G ⊆ A}.
It can be established that PFPcl(A) is the smallest PFPC set containing A and PFPint(A) is the largest PFPO set contained in A, A is an PFPC if and only if PFPcl(A) = A and A is an PFPO if and only if PFPint(A) = A. The following Theorems 4.1, 4.2 and 4.3 gives some properties of the preclosure and preinterior operators, the proof of which are omitted.

Theorem 4.1:
Let (X, τ) be an PFTS and let A and B be Pythagorean Fuzzy sets. Then the following hold.
1) PFPcl(∅) = ∅ and PFPcl(X) = X
2) A is an PFPCS if and only if A = PFPcl(A)
3) PFPcl(PFPcl(A)) = PFPcl(A)
4) A ⊆ B implies that PFPcl(A) ⊆PFPcl(B)
5) PFPcl(A ∩ B) ⊆PFPcl(A) ∩ PFPcl(B)
6) PFPcl(A∪B) = PFPcl(A) ∪PFPcl(B)

Theorem 4.2:
Let (X, τ) be an PFTS and let A and B be an Pythagorean Fuzzy sets. Then the following hold.
1) PFPint(∅) = ∅ and PFPint(X) = X
2) A is an PFPOS if and only if A = PFPint(A)
3) PFPint(PFPint(A)) = PFPint(A)
4) A ⊆ B implies that PFPint(A) ⊆PFPint(B)
5) PFPint(A ∩ B) = PFPint(A) ∩ PFPint(B)
6) PFPint(A∪B) ⊇PFPint(A) ∪PFPint(B)

Lemma 4.3:
Let (X, τ) be an PFTS and let A be an PFS. Then x ∈ PFPcl(A) if and only if every PFPO set U containing x, intersects A.
Proof: The proof of the theorem follows from [Lemma 2.3]

Theorem 4.4:
Let (X, τ) be an PFTS and let A be an Pythagorean Fuzzy set. Then the following hold.
1. PFPint(A) = A ∩ PFint(PFcl(A))
2. PFPcl(A) = A ∪PFcl(PFint(A))
3. PFαcl(A) = A ∪PFcl(PFint(PFcl(A)))
4. PFαint(A) = A ∩ PFint(PFcl(PFint(A)))

Proof:
(i) Since PFPint(A) is an PFPO set, PFPint(A) ⊂PFint(PFcl(A)) and so PFPint(A) ⊂ A ∩ PFint(PFcl(A)). Conversely, let x ∈PFint(PFcl(A)). If U is any PFO set containing x, then U ∩PFint(PFcl(A)) is an PFO set containing x. Since x ∈ PFcl(A), U ∩ PFint(PFcl(A) ∩ A) ≠∅ [9] which implies that x ∈ PFcl(PFint(PFcl(A)) ∩ A). Therefore, PFint(PFcl(A))⊂PFcl(PFint(PFcl(A)) ∩ A) and so, PFint(PFcl(A)) ⊂PFint(PFcl(PFint(PFcl(A)) ∩ A)). Since A ∩PFint(PFcl(A)) ⊂PFint(PFcl(A)) ⊂PFint(PFcl(PFint(PFcl(A)) ∩ A)), A ∩ PFint(PFcl(A)) is an PFPO set contained in A it follows that A∩ PFint(PFcl(A)) ⊂PFPint(A). Hence PFPint(A) = A ∩ PFint(PFcl(A)).
(ii) Now PFcl(PFint(A ∪ PFcl(PFint(A))))⊆PFcl(PFint(A)) ∪ PFcl(PFint(A)) = PFcl(PFint(A)) ⊆ A ∪ PFcl(PFint(A)), by Lemma 2.7, A ∪ PFcl(PFint(A)) is an PFPC containing A. Therefore, PFPcl(A) ⊂ A ∪ PFcl(PFint(A)). Conversely , since PFPcl(A) is PFPC, we have PFcl(PFint(A)) ⊂PFcl(PFint(PFPcl(A))) ⊂PFPcl(A) and hence A ∪ PFcl(PFint(A)) ⊂PFPcl(A). Therefore, PFPcl(A) = A ∪ PFcl(PFint(A)). The proof of (iii) and (iv) are similar to the proof of (i) and (ii) respectively.

Theorem 4.5:
Let (X, τ) be an PFTS and let A be an Pythagorean Fuzzy set. Then

a) PFPcl($\overline{A}$) =$\overline{\text{PFPint(A)}}$

b) PFPint($\overline{A}$) =$\overline{\text{PFPcl(A)}}$

Proof:

(a) PFPcl($\overline{A}$) = (X – A) ∪ PFcl(PFint(X – A)) = (X – A) ∪ (A – (PFint(PFcl(A)))) = X – (A ∩ (PFint(PFcl(A)))) = PFPint(A).

(b) The proof is similar to (a).

The following Theorem 4.6 gives characteristics of PF – dense sets.

Theorem 4.6:

Let (X, τ) be an PFTS and let A be an Pythagorean Fuzzy set. Then the following are equivalent.

a. A is PF – dense
b. PFPcl(A) = X
c. If B is any Pythagorean Fuzzy preclosed set such that A ⊂ B, then B = X
d. Every nonempty Pythagorean Fuzzy preopen set has a nonempty intersection with A.
e. PFPint(X− A) = ∅

Proof:

(a) ⇒ (b). Suppose x ∉PFPcl(A). Then there exists an Pythagorean Fuzzy preopen set G containing x such that G ∩ A ≠ ∅. Since G is a nonempty Pythagorean Fuzzy preopen set, there is a nonempty Pythagorean Fuzzy open set H such that H ⊂ G and so H ∩ A = ∅, a contradiction. Therefore, PFPcl(A)=X.

(b) ⇒ (c). If B is any Pythagorean Fuzzy preclosed set such that A ⊂ B, then X = PFPcl(A) ⊂PFPcl(B) = B which implies that B = X.

(c) ⇒ (d). If G is a nonempty Pythagorean Fuzzy preopen set such that G ∩ A = ∅, the A ⊂ X – G and X – G is Pythagorean Fuzzy preclosed. By (c), it follows that G = ∅, a contradiction.

(d) ⇒ (e). Suppose that PFPint(X – A) ≠ ∅. Then PFPint(X – A) is a nonempty Pythagorean Fuzzy preopen set such that PFPint(X – A) ∩ A ≠ ∅, a contradiction to the hypothesis.

(e) ⇒ (a). PFPint(X – A) ≠ ∅ implies that X – PFPint(X– A) = X. Thus PFPcl(A) = X. Hence PFcl(A) = X which shows that A is PF- dense.

The following Theorem 4.7 gives characterizations of PFPC sets.

Theorem 4.7:

Let (X, τ) be an PFTS and let A be an Pythagorean Fuzzy set. Then the following are equivalent.

a) A is an PFPC set.
b) PFint(PFcl(A)) ⊆ A
c) PFcl(PFint($\overline{A}$)) ⊇$\overline{A}$
d) $\overline{A}$is an Pythagorean Fuzzy preopen set.

Proof:

(a) ⇒ (b). If A is an Pythagorean Fuzzy preclosed set, then X – A is an PFPO set and so X – A ⊂PFint(PFcl(A)). Therefore, X – PFint(PFcl(A)) ⊂ A which implies that PFcl(PFint(A)) ⊂ A.

(b) ⇒ (c). By (b), PFint(PFcl(A)) ⊆ A and so X – A ⊂ X – PFint(PFcl(A)) which implies that PFcl(PFint($\overline{A}$)) ⊇$\overline{A}$.

(c) ⇒ (d). By definition, $\overline{A}$ is anPythagorean Fuzzy preopen set.

(d) ⇒ (a). By definition, A is an PFPC.

The following Theorems 4.8 and 4.9 gives some properties of the operators PFPint and PFPcl.

Theorem 4.8:

For every PFO set U in an PFTS (X, τ) and every A ⊂ X, we have PFint(PFcl(U ∩ A)) = PFint(PFcl(U)) ∩ PFint(PFcl(A)).

Proof:

If U is an PFO set, [U ∩PFint(PFcl(A))] = PFint(U ∩ PFcl(A)) ⊂ U ∩ PFcl(A). By Lemma 2.7, we have U ∩PFcl(A) ⊂PFcl(U ∩ A). Hence PFcl(U ∩ PFint(PFcl(A))) ⊂PFcl(U ∩ A). Since PFint(PFcl(A)) is an PFPO set, we have PFint(PFcl(U)) ∩PFint(PFcl(A)) ⊂PFcl(U) ∩ PFint(PFcl(A)) ⊂PFcl(U ∩ PFint(PFcl(A))). Therefore, PFint(PFcl(U) ∩PFint(PFcl(A)) ⊂PFcl(U) ∩ PFcl(A) ⊂PFcl(U ∩ A) which implies that PFint(PFcl(U) ∩PFint(PFcl(A)) ⊂PFint(PFcl(U ∩ A)).

On the other hand, PFint(PFcl(U ∩ A)) ⊂PFint(PFcl(U) ∩PFcl(A)) = PFint(PFcl(U) ∩PFint(PFcl(A)). Hence PFint(PFcl(U ∩ A)) = PFint(PFcl(U) ∩PFint(PFcl(A)).

Theorem 4.9:

Let A and B be subsets of an PFTS (X, τ). Then PFcl(PFint(PFcl(A ∩ B))) = PFcl(PFint(PFcl(A))) ∪PFcl(PFint(PFcl(B))).

Proof:

We know that PFint(PFcl(A ∪ B)) = PFint(PFcl(A) ∪ PFcl(B)) ⊂PFcl(A) ∪ PFint(PFcl(B)) ⊂PFcl(A) ∪ PFcl(PFint(PFcl(B))). Since PFcl(PFint(PFcl(B))) is an PFC set, we get PFint(PFcl(A ∪ B)) ⊂PFint(PFcl(A) ∪ PFcl(PFint(PFcl(B)))) ⊂PFint(PFcl(A)) ∪ PFcl(PFint(PFcl(B))), by Lemma 2.7 and so PFint(PFcl(A ∪ B)) ⊂PFcl(PFint(PFcl(A))) ∪ PFcl(PFint(PFcl(B))). On the other hand, we havePFcl(PFint(PFcl(A ∪ B))) = PFcl(PFint(PFcl(A) ∪ (PFcl(B)))) ⊃ PFcl(PFint(PFcl(A)) ∪ PFint(PFcl(B))) = PFcl(PFint(PFcl(A))) ∪ PFcl(PFint(PFcl(B))). Hence PFcl(PFint(PFcl(A ∪ B))) = PFcl(PFint(PFcl(A))) ∪ PFcl(PFint(PFcl(B))).

The following Theorem 4.10 shows that the family of α- open sets are exactly the sets which are both semiopen and preopen.

Theorem 4.10:
For any subset A of anPythagorean Fuzzy topological space (X, τ), the following conditions are equivalent.

a) $A \in \tau_\alpha (X)$
b) $A \in PFSO(X) \cap PFPO(X)$
c) There exists an PFO set U such that $U \subset A \subset PFint(PFcl(U))$

Proof:
(a) ⇒ (b). Suppose $A \in \tau_\alpha$. Then $A \subset PFint(PFcl(PFint(A)))$ and so $A \subset PFcl(PFint(A))$. Also, $A \subset PFint(PFcl(PFint(A)))$ implies that $A \subset PFint(PFcl(A))$. Therefore, $A \in PFSO(X) \cap PFPO(X)$.
(b) ⇒ (c). Since $A \in PFSO(X)$, there exists an PFO set U such that $U \subset A \subset PFcl(U)$. Therefore $PFcl(A) = PFcl(U)$ and $PFint(PFcl(A)) = PFint(PFcl(U))$. Since $A \in PFPO(X)$, it follows that $A \subset PFint(PFcl(A))$. Hence $U \subset A \subset PFint(PFcl(U))$.
(c) ⇒ (a). Suppose $U \subset A \subset PFint(PFcl(U))$ where U is an PFO set. Thus $U \subset PFint(A) \subset PFint(PFcl(U))$ and $PFcl(U) \subset PFcl(PFint(A)) \subset PFcl(PFint(PFcl(U))) \subset PFcl(U)$. Here $PFcl(U) = PFcl(PFint(A))$ and $PFint(PFcl(U)) = PFint(PFcl(PFint(A)))$ which implies that $A \subset PFint(PFcl(PFint(A)))$. Therefore $A \in \tau_\alpha(X)$.

The following Theorem 4.11 gives the relation between the closure and interior operators.

Theorem 4.11:
For any subset A of a Pythagorean Fuzzy topological space (X, τ), the following hold.

a) $PF\alpha cl(PFint(A)) = PFcl(PFint(A))$
b) $PFcl(PF\alpha int(A)) = PFcl(PFint(A))$
c) $PFint(PF\alpha cl(A)) = PFint(PFcl(A))$
d) $PF\alpha int(PFcl(A)) = PFint(PFcl(A))$

Proof:
(a) By Theorem 4.4, $PF\alpha cl(PFint(A)) = PFint(A) \cup PFcl(PFint(PFcl(PFint(A)))) = PFint(A) \cup PFcl(PFint(A)) = PFcl(PFint(A))$.
(b) Since $\tau \subset \tau_\alpha$, $PFint(A) \subset PF\alpha int(A)$ for all $A \subset X$ and so $PFcl(PFint(A)) \subset PFcl(PF\alpha int(A))$. On the other hand, $PF\alpha int(A) = A \cap PFint(PFcl(PFint(A)))$ by Theorem 4.4 and so $PFcl(PF\alpha int(A)) = PFcl(A \cap PFint(PFcl(PFint(A)))) \subset PFcl(A) \cap PFcl(PFint(PFcl(PFint(A)))) \subset PFcl(A) \cap PFcl(PFint(A))$. Therefore, $PFcl(PF\alpha int(A)) \subset PFcl(PFint(A))$. Hence, $PFcl(PF\alpha int(A)) = PFcl(PFint(A))$.
(c) $PFint(PF\alpha cl(A)) = PFint(A \cup PFcl(PFint(PFcl(A)))) = PFint(A) \cup PFint(PFcl(PFint(PFcl(A)))) = PFint(A) \cup PFint(PFcl(A)) = PFint(PFcl(A))$.
(d) By Theorem 4.4, $PF\alpha int(A) = A \cap PFint(PFcl(PFint(A))$ and so $PF\alpha int(PFcl(A)) = PFcl(A) \cap PFint(PFcl(PFint(PFcl(A)))) = PFcl(A) \cap PFint(PFcl(A)) = PFint(PFcl(A))$.

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