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Tangent Similarity Measures of Pythagorean Fuzzy Sets

M.Sowmiya[a], A. Stanis Arul Mary[b]

[a]*PG Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore,India(TN),mk.sowmiya2000@gmail.com*
[b]*Assistant Professor, Department of Mathematics,Nirmala College for Women, Coimbatore,India(TN),stanisarulmary@gmail.com*

A B S T R A C T

In this paper, a new tangent similarity measure between two Pythagorean fuzzy sets [PFS] was proposed and its properties were studied. Also, using the tangent similarity measure and weighted tangent similarity measures of Pythagorean fuzzy set we have given a solution to the Automobile problem.

Keywords: Pythagorean fuzzy set, Tangent similarity measure, Weighted tangent similarity measure.

1.Introduction

Similarity measure is an essential research topic in the current fuzzy, Pythagorean, neutrosophic and different hybrid environments .Fuzzy sets were introduced by L.A. Zadeh in 1965.Zadeh's idea of fuzzy set evolved as a new tool having the ability to deal with uncertainities in real life problems and discussed only membership function. After the extensions of fuzzy set theory Atanassov generalised this concept and introduced a new set called intuitionistic fuzzy set(IFS) in 1986 ,which can be describe the non-membership grade of an imprecise event along with its membership grade under a restriction that the sum of both membership and non-membership grades does not exceed 1. IFS has its greatest use in practical multiple attribute decision making problems. In some practical problems. In some practical problems, the sum of membership and non-membership degree to which an alternative satisfying attribute provided by decision maker (DM) may be bigger than 1.

Yager was decided to introduced the new cocept known as Pythagorean fuzzy sets. Pythagorean fuzzy sets has limitation that their square sum is less than or equal to 1.

Recently , Ye presented the correlation coefficient of single-valued neutrosophic sets (SVNSs) and the cross-entropy measure of SVNSs and applied them to single-valued neutrosophic decision making problems. Then , Ye proposed similarity measures between interval neutrosophic sets and their applications in multicriteria decision making. Ye also proposed three vector similarity measures for SVNSs and instance of SVNSs and interval valued neutrosophic set, including the Jaccard , Dice , and cosine similarity and applied them to multi-criteria decision making problems with simplified neutrosophic information. Pramanik and Mondal proposed cotangent similarity measure of rough neutrosophic sets and its application to application to automobile problem. Pramanik and Mondal also proposed weighted fuzzy similarity measure based on tangent function and its application to automobile problem. Pramanik and Mondal proposed tangent similarity measures between intuitionistic fuzzy sets and studied some of its properties and applied it for automobile problem. Broumi and Smarandache defined Hausdorff distance measure between two neutrosophic sets . Broumi and Smarandache extended the concept of cosine similarity measure of SVNSs into INSs and applied it to pattern recognition.

In this paper propose tangent similarity measures for Pythagorean fuzzy sets [PFS] . We also proposed similarity measures for automobile problem.

2.Preliminaries

Definition 2.1

Let E be a universe. An intuitionistic fuzzy set A in E is defined as object of following form

A={ <x,$\boldsymbol{M_A}(\boldsymbol{x})$,$\boldsymbol{N_A}(\boldsymbol{x})$>:x$\boldsymbol{\epsilon}$E}

Where $\boldsymbol{M_A}$:E→[0,1] ,$\boldsymbol{N_A}: \boldsymbol{E} \to [\mathbf{0}, \mathbf{1}]$ define the degree of membership and degree of non-membership of element x$\boldsymbol{\epsilon}$E respectively

0≤ $\boldsymbol{M_A}(\boldsymbol{x}) + \boldsymbol{N_A}(\boldsymbol{x}) \leq \mathbf{1}$ for any x$\boldsymbol{\epsilon}$E

Here, $\boldsymbol{M_A}(\boldsymbol{x})$ and $\boldsymbol{N_A}(\boldsymbol{x})$ is the degree of membership and non-membership of the element x respectively.

Definition 2.2

Let X be universal set .Then a Pythagorean fuzzy set A which is set of ordered pairs over X

A={<x,$\boldsymbol{M_A}(\boldsymbol{x})$,$\boldsymbol{N_A}(\boldsymbol{x})$> | x$\boldsymbol{\epsilon}$X}

Where $\boldsymbol{M_A}$:X→[0,1] , $\boldsymbol{N_A}: \boldsymbol{X} \to [\mathbf{0}, \mathbf{1}]$ denote the respectively degree of membership and degree of non-membership of element x$\boldsymbol{\epsilon}$X to the set A which is a set subset of X and

0≤ $\left(\boldsymbol{M_A}(\boldsymbol{x})\right)^{\mathbf{2}} + \left(\boldsymbol{N_A}(\boldsymbol{x})\right)^{\mathbf{2}} \leq \mathbf{1}$ for any x$\boldsymbol{\epsilon}$E

$\boldsymbol{M_A}(\boldsymbol{x})$ and $\boldsymbol{N_A}(\boldsymbol{x})$ is the degree of membership and non-membership of the element x respectively.

Definition 2.3

Let A and B be Pythagorean Fuzzy sets in a topological space X of the form A={<x,$\boldsymbol{M_A}(\boldsymbol{x})$,$\boldsymbol{N_A}(\boldsymbol{x})$> | x$\boldsymbol{\epsilon}$X}, B={<x,$\boldsymbol{M_B}(\boldsymbol{x})$,$\boldsymbol{N_B}(\boldsymbol{x})$> | x$\boldsymbol{\epsilon}$X}

A∪B={x,max($\boldsymbol{M_A}(\boldsymbol{x})$,$\boldsymbol{M_B}(\boldsymbol{x})$) ,min($\boldsymbol{N_A}(\boldsymbol{x})$,$\boldsymbol{N_B}(\boldsymbol{x})$)| x$\boldsymbol{\epsilon}$X }

A∩B={x,min($\boldsymbol{M_A}(\boldsymbol{x})$,$\boldsymbol{M_B}(\boldsymbol{x})$) ,max($\boldsymbol{N_A}(\boldsymbol{x})$,$\boldsymbol{N_B}(\boldsymbol{x})$)| x$\boldsymbol{\epsilon}$X }

A^C **={(x,**$N_A(x)$**,**$M_A(x)$**) | x**$\boldsymbol{\epsilon}$**X}**

3.Tangent Similarity Measures for Pythagorean Fuzzy Sets

3.1 Definition

Let P = {(x, $M_P(x), N_P(x)): x \in X$} and
Q = {(x, $M_Q(x), N_Q(x): x \in X$} be two Pythagorean fuzzy numbers . Now tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as follows

$T_{PFS}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}[\,1 - tan(\frac{\pi}{8}[|M_P^2(x_i) - M_Q^2(x_i)|+|N_P^2(x_i) - N_Q^2(x_i)|])]$

3.2 Preposition

The defined tangent similarity measure $T_{PFS}(P,Q)$ between Pythagorean fuzzy sets P and Q satisfies the following properties

1) $0 \leq T_{PFS}(P,Q) \leq 1$;
2) $T_{PFS}(P,Q) = 1$ iff P = Q;
3) $T_{PFS}(P,Q) = T_{PFS}(Q,P)$;
4) If O is a PFS set in X and P ⊆ $Q \subseteq O$ then

$T_{PFS}(P,O) \leq T_{PFS}(P,Q)$ and $T_{PFS}(P,O) \leq T_{PFS}(Q,O)$.

Proof

1) As the membership ,indeterminacy and non-membership functions of the Pythagorean fuzzy sets and the value of the tangent function also is within [0,1].

Hence $0 \leq T_{PFS}(P,Q) \leq 1$.

2) For any two Pythagorean fuzzy sets P and Q if P = Q, this implies

$$M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i)$$

Hence $|M_P^2(x_i) - M_Q^2(x_i)| = 0, |N_P^2(x_i) - N_Q^2(x_i)| = 0,$

Thus $T_{PFS}(P,Q) = 1$.

Conversely, if $T_{PFS}(P,Q) = 1$, then $|M_P^2(x_i) - M_Q^2(x_i)| = 0, |N_P^2(x_i) - N_Q^2(x_i)| = 0,$ since tan(0) = 0. So that $M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i),$

Hence P = Q.

3) The Proof is obvious

4) If P ⊆ Q ⊆ O

then $M_P(x_i) \leq M_Q(x_i) \leq M_o(x_i)$, $N_P(x_i) \geq N_Q(x_i) \geq N_0(x_i)$,

$$|M_P^2(x_i) - M_Q^2(r_i)| \leq |M_P^2(r_i) - M_O^2(x_i)|,$$
$$|M_Q^2(x_i) - M_O^2(x_i)| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$|N_P^2(x_i) - N_Q^2(x_i)| \leq |N_P^2(x_i) - N_o^2(x_i)|,$$
$$|N_Q^2(x_i) - N_O^2(x_i)| \leq |N_P^2(x_i) - N_O^2(x_i)|,$$

Thus,

$T_{PFS}(P,O) \leq T_{PFS}(P,Q)$ and $T_{PFS}(P,O) \leq T_{PFS}(Q,O)$

Since tangent function is increasing in the interval $[0, \frac{\pi}{4}]$.

3.3 Definition

Let P = {(x, $M_P(x), N_P(x)): x \in X\}$ and

Q = {(x, $M_Q(x), N_Q(x): x \in X\}$ be two Pythagorean numbers . Now weighted tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as follows

$$T_{PFS}(P,Q) = \sum w_i \ [1 - tan(\frac{\pi}{8}[|M_P^2(x_i) - M_Q^2(x_i)| + |N_P^2(x_i) - N_Q^2(x_i)|])]$$

Where $w_i \in [0,1], i = 0,1,2 \dots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$.

3.4 Preposition

The defined weighted tangent similarity measure $T_{PFS}(P,Q)$ between Pythagorean fuzzy set P and Q satisfies the following properties

1) $0 \leq T_{PFS}(P,Q) \leq 1$;
2) $T_{PFS}(P,Q)$ = 1 iff P = Q;
3) $T_{PFS}(P,Q) = T_{PFS}(Q,P)$;
4) If O is a Pythagorean fuzzy set in X and P ⊆ Q ⊆ O then $T_{PFS}(P,O) \leq T_{PFS}(P,Q)$ and $T_{PFS}(P,O) \leq T_{PFS}(Q,O)$.

Proof

1) As the membership, indeterminacy and non-membership function of the Pythagorean fuzzy sets and the value of the tangent function also is within [0,1] and Where $w_i \in [0,1], i = 0,1,2 \dots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$.

Hence $0 \leq T_{PFS}(P,Q) \leq 1$.

2) For any two Pythagorean fuzzy sets P and Q if P = Q, this implies $M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i)$

Hence

$$|M_P^2(x_i) - M_Q^2(x_i)| = 0, |N_P^2(x_i) - N_Q^2(x_i)| = 0$$

Thus $T_{PFS}(P,Q) = 1$.

Conversely, if $T_{PFS}(P,Q) = 1$,

then $|M_P^2(x_i) - M_Q^2(x_i)| = 0, |N_P^2(x_i) - N_Q^2(x_i)| = 0,$ since tan(0) = 0. So we can write $M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i),$

Hence P = Q.

3) The Proof is obvious

4) If O is a Pythagorean fuzzy set in X and P ⊆ Q ⊆ O

To prove:

$T_{PFS}(P,O) \leq T_{PFS}(P,Q)$ and $T_{PFS}(P,O) \leq T_{PFS}(Q,O)$

If P ⊆ Q ⊆ O then $M_P(x_i) \leq M_Q(x_i) \leq M_O(x_i)$, $N_P(x_i) \geq N_Q(x_i) \geq N_O(x_i)$,

$$\sum w_i = 1.$$

$$|M_P^2(x_i) - M_Q^2(x_i)| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$|M_Q^2(x_i) - M_O^2(x_i)| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$|N_P^2(x_i) - N_Q^2(x_i)| \leq |N_P^2(x_i) - N_O^2(x_i)|,$$
$$|N_Q^2(x_i) - N_O^2(x_i)| \leq |N_P^2(x_i) - N_O^2(x_i)|,$$

Thus,

$T_{PFS}(P,O) \leq T_{PFS}(P,Q)$ and $T_{PFS}(P,O) \leq T_{PFS}(Q,O)$

Since tangent function is increasing in the interval $[0, \frac{\pi}{4}]$.

4. Decision Making Based on Tangent Similarity Measures

Let $A_1, A_2, \dots A_m$ be a discrete set of candidates, $C_1, C_2, .. C_n$ be the set of criteria for each candidate and $B_1, B_2, \dots, B_k$ are the alternatives of each candidate. The decision -maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performance of candidates $A_i(i = 1,2, \dots m)$ against the criteria $C_j(j = 1,2 \dots, n)$.The values associated with the alternatives for MADM problem can be presented in the following decision matrix(see Tab 1 and Tab 2). The relation between candidates and attributes are given in Tab 1. The relation between attributes and alternatives are given in the Tab 2.

Tab 1 : The relation between candidates and attributes

R_1	C_1	C_2	…	C_n
A_1	d_{11}	d_{12}	…	d_{1n}
A_2	d_{21}	d_{13}	…	d_{2n}
…	…	…	…	…
A_m	d_{m1}	d_{m2}	…	d_{mn}

Tab 2 : The relation between attributes and alternatives

R_2	B_1	B_2	…	B_k
C_1	δ_{11}	δ_{12}	…	δ_{1k}
C_2	δ_{21}	δ_{22}	…	δ_{2k}
…	…	…	…	…
C_n	δ_{n1}	δ_{n2}	…	δ_{nk}

Here d_{ij} and δ_{ij} are all Pythagorean Fuzzy numbers.

The steps corresponding to Pythagorean number based on tangent and cotangent functions are presented following steps.

Step 1: Determination of the relation between candidates and attributes

The relation between candidate $A_i(i = 1,2, \dots m)$ and the attribute $C_j(j = 1,2 \dots n)$ is presented in Tab 3.

Tab 3 : The relation between candidates and attributes in terms of Pythagorean fuzzy numbers

R_3	C_1	C_2	…	C_n
A_1	(a_{11}, b_{11})	(a_{12}, b_{12})	…	(a_{1n}, b_{1n})
A_2	(a_{21}, b_{21})	(a_{22}, b_{22})	…	(a_{2n}, b_{2n})
…	…	…	…	…
A_m	(a_{m1}, b_{m1})	(a_{m2}, b_{m2})	…	(a_{mn}, b_{mn})

Step 2: Determination of the relation between attributes and alternatives

The relation between attributes $C_i(i = 1,2, \dots n)$ and the alternatives $B_t(t = 1,2 \dots k)$ is presented in Tab 4.

Tab 4 : The relation between attributes and alternatives in terms of Pythagorean fuzzy sets

R_4	B_1	B_2	…	B_n
C_1	(c_{11}, d_{11})	(c_{12}, d_{12})	…	(c_{1k}, d_{1k})
C_2	(c_{21}, d_{21})	(c_{22}, d_{22})	…	(c_{2k}, c_{2k})
…	…	…	…	…
C_n	(c_{n1}, d_{n1})	(c_{n2}, c_{n2})	…	(c_{nk}, d_{nk})

Step 3: Determination of the relation between attributes and alternatives

Determine the similarity measure between the Tab 3 and Tab 4 using $T_{PFS}(P,Q)$ $T_{PFS}(P,Q)$, $COT_{PFS}(P,Q)$ and $COT_{PFS}(P,Q)$.

Step 4:Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of the similarity measures. Highest value reflects the best alternative.

Step 5: End

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5. Example

In day to day life new upcoming models are arriving in the automobile field which leads to confusion to conclude the best one .

For example

R={R_1,R_2, R_3,R_4} be a set of Respondents

B={Cost,Mileage,Colour} be a set of benefits

A={Yamaha, Hero,Bajaj,TVS} be a set of automobiles

The solution strategy to examine respondent which will provide truth membership,indeterminate and false membership for each respondent regarding relation between respondent and different benefits (Table i)

The Tangent similarity measure between $S_1$and S_2 (Table iii)

Table (i) Relation between Respondent and benefits

S_1	Cost	Mileage	Colour
R_1	(0.5,0.4)	(0.3,0.6)	(0.7,0.3)
R_2	(0.8,0.3)	(0.4,0.8)	(0.1,0.6)
R_3	(0.1,0.3)	(0.2,0.4)	(0.7,0.2)
R_4	(0.4,0.2)	(0.3,0.5)	(0.3,0.2)

Table (ii) The relation between Benefits and Automobile

S_2	Yamaha	Hero	Bajaj	TVS
R_1	(0.4,0.1)	(0.1,0.2)	(0.3,0.2)	(0.1,0.3)
R_2	(0.4,0.3)	(0.3,0.4)	(0.2,0.1)	(0.4,0.5)
R_3	(0.7,0.1)	(0.2,0.7)	(0.4,0.3)	(0.2,0.5)

Table (iii) The Tangent similarity measure between S_1 and S_2

Tangent similarity measure	Yamaha	Hero	Bajaj	TVS
R_1	**0.9133**	0.8106	0.8669	0.8542
R_2	0.7398	0.8148	0.7633	**0.8460**
R_3	**0.941**	0.8639	0.9132	0.8840
R_4	0.9091	0.9023	**0.937**	0.9304

Weight information

W=$(w_1,w_2,w_3)^T$ =$(0.25,0.35,0.4)^T$ such that $\sum_{j=1}^{n} w_j$=1

Table (iv) The weighted tangent similarity measure between S_1 and S_2

Weighted tangent similarity measure	Yamaha	Hero	Bajaj	TVS
R_1	**0.9168**	0.7979	0.8648	0.8469
R_2	0.7322	0.8297	0.7671	**0.8608**
R_3	**0.9465**	0.8407	0.9065	0.8649
R_4	0.8972	0.8934	**0.9342**	0.9297

The highest correlation measure reflects the best automobile selection

Therefore,R_1 selects Yamaha

R_2 selects TVS

R_3 selects Yamaha

R_4 selects Bajaj

6. Conclusion

In this paper, we have proposed tangent similarity measures for Pythagorean fuzzy sets and proved some of its properties.
We proposed tangent similarity measures for Pythagorean fuzzy sets can be used in the field of practical decision making pattern recognition ,medical diagnosis ,data mining clustering analysis.

References

K.Atanassov,Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1986) 87-96.

S. Broumi, F. Smarandache, More on Intuitionistic Neutrosophic Soft Sets, Computer Science andInformation Tech-nology, 1(2013), 257 – 268.

S. Broumi, F. Smarandache, Cosine similarity measures of intervaluedneutrosophicsets,Neutrosophic Sets Syst, 5 (2014), 15-20.

K.Mondal, S.Pramanik, Intuitionistic fuzzy similarity measure based on tangent function and itsapplication to multi-attribute decision, Global Journal of Advanced Research, 2 (2015), 464-471.

S.Pramanik and K.Mondal, Cotangent similarity measure of rough neutrosophic sets and its application in medical diagnosis, Journal of New Theory, 4 (2015),464-471.

S.Pramanik and K.Mondal, Weighted fuzzy similarity measure based on tangent function and itsapplication to medical diagnosis, International Journal of InnovativeResearch in Science, Engineeringand Technology, 4 (2015), 158-164.

R. Radha, A. Stanis Arul Mary. PentapartitionedNeutrosophicpythagorean Soft set, IRJMETS, 2021 ,Volume 3(2),905-914.

R. Radha, A. Stanis Arul Mary. PentapartitionedNeutrosophic Pythagorean Set, IRJASH, 2021, volume 3, 62-82.

R.Radha, A. Stanis Arul Mary, PentapartitionedNeutrosophic Generalized semi-closed sets, 123-131.

R.Radha, A.Stanis Arul Mary, Improved Correlation Coefficients of QuadripartitionedNeutrosophicPythagoreanPythagorean sets for MADM,142-153.

R. Radha, A. Stanis Arul Mary. Heptapartitionedneutrosophic sets, IRJCT, 2021 volume 2,222-230.

R. Radha, A. Stanis Arul Mary, F. Smarandache. QuadripartitionedNeutrosophic Pythagorean soft set, International journal of Neutrosophic Science, 2021,volume14(1),9-23.

R. Radha, A. Stanis Arul Mary, F. Smarandache. Neutrosophic Pythagorean soft set, Neutrosophic sets and systems, 2021,vol 42,65-78.

R. Radha ,A.Stanis Arul Mary, Pentapartitionedneutrosophicpythagorean resolvable and irresolvable spaces(Communicated)

R. Radha ,A.Stanis Arul Mary , Bipolar PentapartitionedNeutrosophic set and it's GeneralizedSemi-closed Sets IJRPR Vol (2) Issue (8) (2021) Page 1130-1137

F. Smarandache, Degree of dependence and independence of the (sub)components of fuzzy setand Neutrosophic set. Neutrosophic Sets Syst., 11(2016), 95–97.

F.Smarandache , A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, NeutrosophicSet,Neutrosophic Probability; American Research Press: Rehoboth, DE, USA, 1999.

R.R. Yager,Pythagorean Fuzzy Subsets,In:Proc Joint IFSA World Congress and NAFIPS AnnualMeeting,Edmonton,Canada, (2013),57-61.

J.Ye, Similarity measure between interval neutrosophic sets and their applications in multiciteriadecision making ,Journal of intelligent and fuzzy systems 26(2014),165–172.

J.Ye, single-valued neutrosophic cross-entropy for multicriteria decision making problems, AppliedMathematical Modelling,38 (2014), 1170-1175.

J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valuedneutrosophic environment, International Journal of

General Systems, 42(4)(2013), 386–394.

H.M.Zhang, Z.S.Xu and Q.Chen, Clustering method of intuitionistic fuzzy sets, Control Decision, 22(2007), 882-888.