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Mean Cordial Labelling of Graph

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ABSTRACT

Let f be a map from V(G) to{0,1,2}. Consider the number of vertices and edges respectively labeled with x (X= 0,1,2). A mean cordial graph is a graph with a mean cordial labelling. We investigate mean cordial labeling behavior of Paths, Cycles, Stars, Complete graphs, Combs and some more standard graphs.The graphs considered here are finite, un directed and simple. V(G) and E(G) denote the vertex and edge sets of a graph G, respectively. The cardinality of V(G) and E(G) are respectively called order and size of G Labelled graphs are used in radar, circuit design, communication network, astronomy, cryptography etc.

I. **Introduction:**

In computer science technology applications such as database design, software engineering, circuit design, networks, and data mining, graph theory is critical for automatic graph synthesis. Over 200 graph labelling strategies have been explored in hundreds of research articles due to the engagement of scholars over the last 60 years. Harary is our go-to source for basic terms and notations. Cahit created cordial labelling, and Ponraj et colleagues pioneered the mean cordial labelling of a graph. A.Nellai Murugan et al. introduced mean square cordial labelling and discussed it for various particular graphs. They've also spoken about mean square cordial labelling for several tree and cycle graphs. Dhanalakshmi et al. studied mean square cordial labelling and its rough approximations in relation to several cyclic and acyclic networks.

On Sunday afternoons in the eighteenth century, the virtuous citizens of Konigsberg, Germany's Eastern Purssia, would occupy themselves with "The Seven Bridges of Konigsberg." As illustrated in Figure 1.1, two islands C and D were joined to each other and to the banks of the magnificent Pregel River by seven bridges (a). “Kneiphoff” is the name of the island C.The goal was to start at any position on the ground, walk across each bridge once, and then return to the starting point. Because no one else could do it, it was no surprise in 1736 when the famous Swiss mathematician L. Euler demonstrated that it was impossible by expressing the land regions and bridges in Figure 1.1(a) with an analogous system of points and lines in Figure 1.1(b), resulting in a graph picture. As a result of the modest amusement chattering, today's huge and widely branched-out graph theory was discovered.

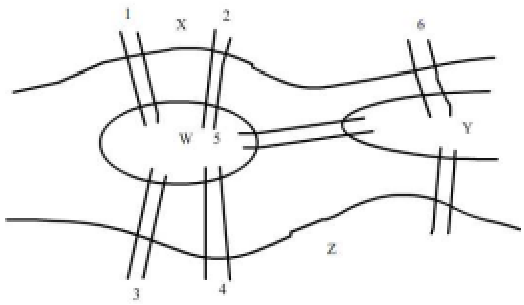


Figure 1.1(a)

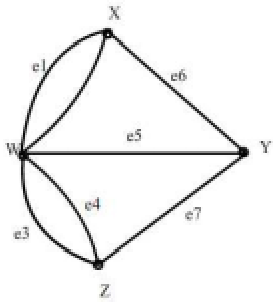


Figure 1.1(b)

II.PRELIMINARIES

What is Graph?

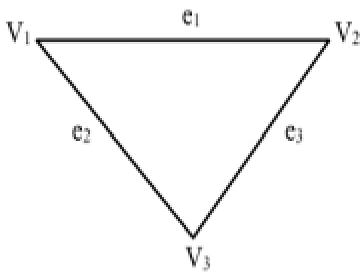
Regrettably, some individuals use the term "graph" quite loosely, so you won't know what kind of graph they're referring to unless you ask.We expect you to apply the terms carefully when you finish this chapter, not haphazardly.To motivate the various definitions, we’ll begin with some definitions and examples.

Definition 1.1:

A graph is an arranged triple G=(V(G),E(G),(G)) consists mainly of a non empty set V(G) of vertices, a set E(G) nonoverlapping from V(G) of edges, and an incident functions () associated with each edge of G, as well as an apparently random pair of vertices of G.

When e is an edge and u and v are vertices, and G (e) = uv, e is said to be join u and v, and the vertices u and v are called e's end points. Its name is Graph.

Example:



$G = (V(G), E(G))$

$V(G) = \{v_1, v_2, v_3\}$ & $E(G) = \{e_1, e_2, e_3\}$;

$\psi G(e_1) = v_1, v_2$, $\psi G(e_2) = v_1, v_3$, $\psi G(e_3) = v_2, v_3$

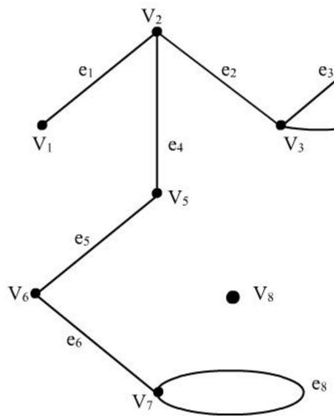
Definition 1.2:

Assume G is a graph. Parallel edges are defined as two (or more) edges of G that have the same end vertices.

Definition 1.3:

An isolated vertex of G is one that is not connected to any other vertex by any edge. Adjacent or neighbouring vertices are those that are connected by an edge. The neighbourhood set of v is the set of all neighbours of a fixed vertex v of G.

Example:



$G = (V(G), E(G)$

$V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$

$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

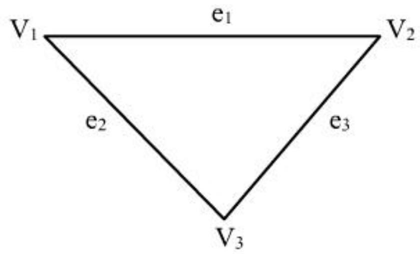
The e_1 and e_2 are adjacent to V_2

The edge e_8 is called *Loop*, edges e_5 and e_1 are *Parallel lines*, and V_8 is a *Isolated point.*

Definition 1.4:

If there are no loops and no parallel edges in a graph, it is called simple.

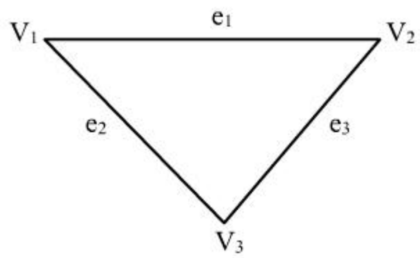
Example:



Definition – 1.5:

A finite graph is one with a finite number of vertices and edges as well as a finite number of edges. Otherwise, it's a never-ending graph.

Example:

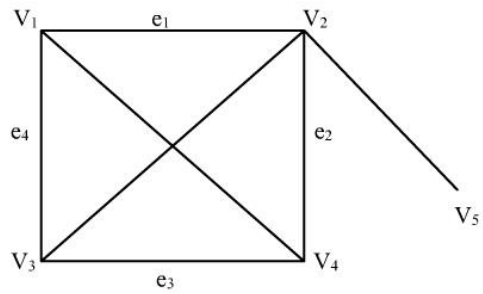


A finite graph with 3 vertices and 3 edges.

Definition – 1.6:

The total number of edges incident to each vertex is defined as a vertex's degree. Degrees of vertex is the term for it.

Example:



deg(v_1)=3; deg(v_2)=4; deg($valignl3$)=3; deg(v_4)= 3;deg($valignl5$)=1

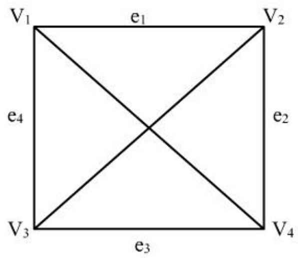
Definition- 1.7:

When all of G's vertices have the same degree r, (G)= r, and G is referred to as a regular graph of degree r. If d(v)=k for all v V, a graph is said to be k- regular. It's known as a Regular graph.

Definition-1.8:

A complete graph is a simple graph with one edge connecting each pair of different vertices. Thus, assuming there are no loops or parallel edges, a graph with n vertices is complete if it has as many edges as possible. k n denotes the entire network with n vertices.

Example:



deg(v_1)=3; deg(v_2)=3; deg($valignl3$)=3; deg(v_4)= 3

Definition – 1.9:

A *linear graph G =(V,E)*consists of a collection of items$v$={v_1, v_2,....} called vertices and another set E={e_1, e_2,...}whose constituents are edges

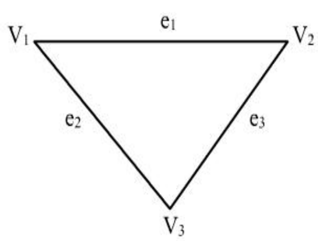
Definition – 1.10:

A loop is indeed an edge whose end points are the same. Multiple edges are defined as two or more edges that connect the same pair of vertices.

Definition – 1.11:

The degree, d(v 1) of a vertex v 1 is the number of edges incident on it, with self loops counted twice.

Example:



$$d(v_1) = d(v_2) = d(v_3) = 2$$

Definition- 1.12:

Two points of intersection If G has an u to v path, u and v of G are connected. If a pair of vertices in a graph G are connected, the graph is said to be connected. This is referred to as a connected graph.

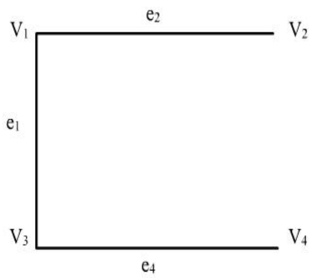
Definition-1.13:

The diameter of a graph G is the largest distance between any two of its vertices, and it is given by d (G).

Definition-1.14:

A tree is a linked graph that does not contain any cycles.

Example:

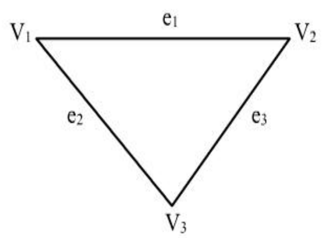


Tree with 4 vertices.

Definition-1.15:

A cycle is defined as a path that starts and finishes at the same vertices. It is denoted by C n.

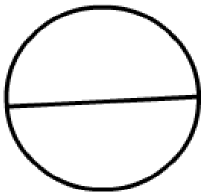
Example:



Definition-1.16:

A cycle chord is an edge that links two vertices but is not itself a cycle edge.

Example:



A cycle with one chord u_1u_2.

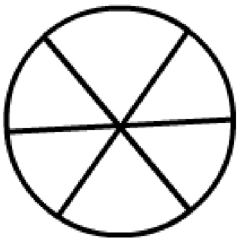
Definition-1.17:

If two chords of a cycle C n(n5) create a triangle with an edge of the cycle C n, they are said to be twin chords.

Definition-1.18:

A Wheel graph has n vertices (n 4) and is built by connecting a single vertex to all of C n's vertices.

Example:



Wheel graph (W_7)

III.MEAN CHORDIAL LABELING BEHAVIOUR

Theorem 2.1:

A connected mean cordial graph is made up of subgraphs.

Proof:

Assume G is a (p,q) graph. Make three copies of the letter K. Let G 1, G 2, and G 3 denote the first, second, and third copies of K, respectively. Let uV(G 1), vV(G 2), and wV(G 3) be the variables. Let G* be the graph with V(G)=V(G

1)V(G 2)V(G 3) and V(G)=V(G 1)V(G 2)V(G 3) andV(G)=V(G1)V(G 2)V(G1)

$E(G\ ) = E(G_1) \cup E(G_2) \cup E(G_3) \cup$ {uv,vw}

G* is clearly a super graph of G. Assign the label 0 to all of G1's vertices, 1 to all of G2, and 2 to all of G3's vertices.

Then $v_f(0) = v_f(1) = v_f(2) = p$ and

$$e(0) = (p|\ \ )$$

As a result, this labelling is a G* imply amicable labelling.

Theorem 2.2:

Any Path P_n is a mean cordial.

Proof:

Let P_n be the Path . $u_1, u_2, u_3,....u_n$.

Case (1): n ≡ 0(mod 3)

Let *n=3t*. Define $f(u_i)$=2, 1≤ $i \le t$, $f(u_{t+1})$=1, 1≤ $i \le t$, $f(u_{2t+i})$=0, 1≤ $i \le t$.

Then $v_f(0)=v_f(1)=v_f(2)=t$,and $e_f(0)=t-1, e_f(1)=e_f(2)=t$.

Therefore f is a mean cordial labelling.

Case (2): n ≡ 1(mod 3)

Let *n=3t+1*. Assign labels to the vertices u_i(1≤ $i \le n$-1) as in *case (1).* Then assign the label *0* to the vertex u_n. Here $v_f(0)=v_f(1)=v_f(2)= t$ and $e_f(0)=t$, $e_f(1)=e_f(2)=t$.

Case (3): n ≡ 2(mod 3)

Let *n=3t+2*. Assign labels to the vertices u_i (1≤ $i \le n$-1) as in case (2). Then assign the label *1* to the vertex u_n. Here $v_f(0)=v_f(1)=t+1, v_f(2)=t, e_f(1)=t+1$ and $e_f(0)=e_f(2)=t$.

Theorem 2.3:

The Star $K_{1,n}$ is a mean cordial if and onlf $n \le 2$.

Proof:

Let $V(K_{1,n})=\{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n})=\{uu_i : 1 \leq i \leq n\}$.

For $n \leq 2$, the result follows from Theorem 2.2. Assume $n > 2$.

Case (1): $f(u)=0$

Then $f(u)+f(v) \leq 2$ for all edge uv. This forces $e_f(2)=0$ a contradiction.

Case (2): $f(u)=2$

In this case $e_f(0)=0$, again a contradiction.

Case (3): $f(u)=1$

Here also $e_f(0)=0$, a contradiction.

Hence $K_{1,n}$ is not a mean cordial for all $n > 2$.

Theorem 2.4:

The cycle C_n is mean cordial if and only if n ≡ 1,2(mod 3).

Proof:

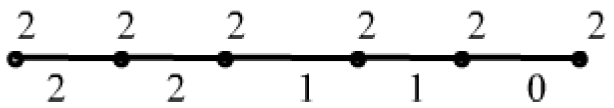
Let C_n be the cycle $u_1u_2.....u_nu_1$

Case(1): n ≡ 0 (mod 3)

Let $n=3t$. Then $v_f(0)=v_f(1)=v_f(2)=t$. In this case $e_f(0) \leq t-1$ contradiction

Case (2): n ≡ 1 (mod 3)

Let $n=3t+1$. Define $f(u_i)=0, 1 \leq i \leq t+1, f(u_{t+1+i})=1, 1 \leq i \leq t, f(u_{2t+1+i})=2, 1 \leq i \leq t$.



Then $v_f(0)=t+1$, $v_f(1)=v_f(2)=t$ and $e_f(1) = t+1$, $e_f(0) = e_f(2) = t$.

Case (3): n ≡ 2 (mod 3)

Let $n=3t+2$. Define $f(u_i)=0, 1 \leq i \leq t+1, f(u_{t+1+i})=1, 1 \leq i \leq t, f(u_{2t+1+i})=2, 1 \leq i \leq t+1$.

Then $v_f(1)=t$, $v_f(0)=v_f(2)=t+1$ and $e_f(1) = t+1$, $e_f(0) = e_f(2) = t$.

Theorem 2.5:

The Wheel W_n is not a mean cordial graph for all $n \geq 3$

Proof:

Let $W_n=C_n+K_1$ where C_n is the cycle $u_1u_2....uu_1$ and $V\{K_1\}=u$. If possible let there be a mean cordial labelling f.

Case(1): n ≡ 0 (mod 3)

Let $n=3t$. Then the size of the wheel is $6t$.

Subcase (i): $f(u)=0$,

Here $e_f\ (0) \leq t + t - 1 \leq 2t - 1$, a contradiction.

Subcase (ii): $f(u)=1$ or 2.

In this case $e_f\ (0) \leq t$, again a contradiction.

Case(2): n ≡ 1 (mod 3)

Let $n=3t+1$. Then $e_f\ (0) \leq 2t\text{-}1$ or $e_f\ (0) \leq t$ according as $f\ (u)=0$, or $f\ (u)\neq 0$, this is a contradiction.

Case(3): n ≡ 2 (mod 3)

Similarly to case(i), we get a contradiction.

Theorem 2.6:

$S(K_{1,n})$ is mean cordial, where $S(G)$ denotes subdivision of G.

Proof:

Let $V\ (\ S\ (K_{\ 1,n}\ ))\ =\{u,\ u_i\ ,v_i : 1 \leq i \leq n\}$ and

$E\ (\ S\ (K_{\ 1,n}\ ))\ =\{u\ u_i\ ,\ u_i\ v_i : 1 \leq i \leq n\}$

Case(i): n ≡ 0 (mod 3)

Let $n=3t$.

Define $f(u)=0$, $f(u_i)=0, 1\leq i \leq t$,

$f(u_{t+1})=1, 1\leq i \leq 2t$, , $f(v_i\ )=0, 1\leq i \leq t$,

$f(v_{t+i}\ )=2, 1\leq i \leq 2t$,Then $v_f\ (0)=2t+1$,

$v_f\ (1)=v_f\ (2)=2t$ and $e_f\ (0)=e_f\ (1)=2t$, $e_f\ (2)=2t$.

Case(ii): n ≡ 1 (mod 3)

Let $n=3t+1$. Assign labels to the vertices u, u_i and $v_i(1\leq i \leq n\text{-}1)$ as in case (i). Then assign the label 1 and 2 to the vertices $u_n$and v_n respectively.

Here $v_f\ (0)=v_f\ (1)=\ v_f\ (2)=2t+1$ and $e_f\ (0)=2t$,

$e_f\ (1)=\ e_f\ (2)=2t+1$.

Case(iii): n ≡ 2 (mod 3)

Let $n=3t+2$. Assign labels to the vertices u, u_i and v_i $(1\leq i \leq n - 1)$ as in case (ii). Then assign the label 0 and 2 to the vertices u_n and v_n respectively.

Here $v_f(0)=v_f(2)=2t+2, v_f(1)=2t+1$ and

$e_f(1)=2t+2$, $e_f(0)= e_f(2)=2t+1$.

Theorem 2.7:

The comb $P_n \Theta K_1$ is mean cordial where $G_1 \Theta G_2$ denotes the corona of G_1 and G_2.

Proof:

Let P_n be the path $u_1u_2....u_n$.

Let $V(P_n \Theta K_1)=V(P_n)\{v_i : 1 \leq i \leq n \}$,

$E(P_n \Theta K_1)=E(P_n)\{u_i v_i : 1 \leq i \leq n \}$.

Case (i): n ≡ 0 (mod 3)

Let $n=3t$.

$f(u_i)=2, 1\leq i \leq t , f(u_{t+1})=1, 1\leq i \leq t$

$f(u_{2t+i})=0, 1\leq i \leq t , f(v_i)=2, 1\leq i \leq t$

$f(v_{t+i})=1, 1\leq i \leq t , f(v_{2t+i})=0, 1\leq i \leq t$

Then $v_f(0)= v_f(1)= v_f(2) =2t$ and

$e_f(1)=e_f(2)=2t$, $e_f(0)= 2t - 1$,

Hence f is a mean cordial labelling.

Case(ii): n ≡ 0 (mod 3)

Let $n=3t+1$. Assign labels to the vertices u_i and and v_i $(1\leq i \leq n - 1)$ as in case (i). Then assign the label 0 and 1 to the vertices u_n and v_n respectively.

Here $v_f(0)=v_f(1)=2t+1, v_f(2)=2t$ and

$e_f(0)= e_f(2)=2t$, and $e_f(1)=2t+1$.

Hence f is a mean cordial labelling.

Case(iii): n ≡ 2 (mod 3)

Let $n=3t+2$. Assign labels to the vertices u_i and v_i $(1\leq i \leq n - 2)$ as in case (i). Then assign the label 0, 2 and 0, 1 to the vertices u_{n-1}, u_n and v_{n-1}, v_n respectively.

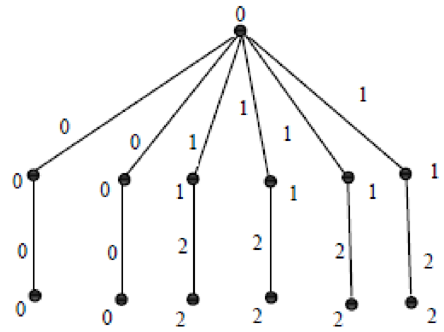
Here $v_f(0)=2t+2$, $v_f(1)=v_f(2)=2t+1$ and

$e_f(0)=e_f(1)=e_f(2)=2t+1$.

Hence f is a mean cordial labelling.

Theorem 2.8:

$P_n \Theta 2K_1$ is mean cordial.



S($K_{1,6}$)

Proof:

Let P_n be the Path $u_1u_2....u_n$

Let v_i and w_i be the pendant vertices which are adjacent to u_1, $1\le i\le n$.

Case(i): n is even.

Define $f(u_i)=0, 1\le i\le n/2$

$f\left(u_{\frac{n}{2}+i}\right)=1$, $1\le i\le n/2$

$f(v_i)=0$, $1\le i\le n/2$

$f(w_i)=1$, $1\le i\le n/2$

$f\left(v_{\frac{n}{2}+i}\right)=2$, $1\le i\le n/2$

$f\left(w_{\frac{n}{2}+i}\right)=2$, $1\le i\le n/2$

Then $v_f(0)=v_f(1)=v_f(2)=n$ and

$e_f(0)=n-1, e_f(1)=e_f(2)=n$

Hence f is a mean cordial labelling.

Case(ii): n is even.

Define $f(u_i)=0, 1\le i\le (n-1)/2$

$f\left(u_{\frac{n-1}{2}+i}\right)=1$, $1\le i\le (n+1)/2$

$f(v_i)=0$, $1\leq i \leq (n-3)/2$

$f(w_i)=1$, $1\leq i \leq (n-3)/2$

$f\left(v_{\frac{n-1}{2}}\right)=f\left(w_{\frac{n-1}{2}}\right)=0$

$f\left(v_{\frac{n+1}{2}}\right)=1, f\left(w_{\frac{n+1}{2}}\right)=2$

$f\left(v_{\frac{n+1}{2}+i}\right)=f\left(w_{\frac{n+1}{2}+i}\right)=2$, $1\leq i \leq (n-1)/2$

Then $v_f(0)=v_f(1)= v_f(2)=n$ and

$e_f(0)=n-1, e_f(1)= e_f(2)=n$

Hence f is a mean cordial labelling.

Theorem 2.9:

The $K_{2,n}$ is a mean cordial iff $n \leq 2$.

Proof:

Let $V(K_{2,n})=\{u, v, u_i : 1 \leq i \leq n\}$ and

$E(K_{2,n})=\{uu_i, vu_i : 1 \leq i \leq n\}$. $K_{2,1}$ and $K_{2,2}$ are mean cordial by Theorem 2.2 and 2.4 respectively. Assume $n > 2$.

Suppose f is a mean cordial labelling of $K_{2,n}$.Clearly either $f(u)=0$ or $f(v)=0$. Without loss of generality we can assume $f(u)=0$ so that $f(v) \neq 0$

Case(i): n ≡ 0 (mod 3)

Let $n=3t$. Then $e_f(0)=t$ or $t-1$, a contradiction since the size of $K_{2,n}$is $6t$.

Case(ii): n ≡ 1 (mod 3)

Let $n=3t+1$. Here $ce_f(0)=t$, again a contradiction.

Case(iii): n ≡ 2 (mod 3)

Let $n=3t+2$. Here $e_f(0)=t$ or $t+1$, again a contradiction to the size of $K_{2,n}$.

IV.Conclusion

The concept of mean cordial labelling was proposed in this study, and the mean cordial labelling behaviour of a few common graphs was investigated. The authors believe that investigating the mean cordial labelling behaviour of graphs generated from conventional graphs using the graph operation will be both fascinating and fruitful.

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