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Ordinary Differential Equations for Mathematical Modeling Approach of Food Intake and Body Weight

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ABSTRACT

In this article, we examine models for characterising international conflicts, which werefirst developed by Lewis Fry Richardson and describe the interaction between twonations or alliances that believe war is imminent. To avoid substantial weight loss orgrowth, food intake and energy expenditure are frequently managed. Hormones andnutrients (glucose, triglycerides) are two of the most essential elements that influencefood intake. Leptin is also linked to leptin resistance, which is characterised by adiminished ability to manage food intake and is usually associated with obesity. Humanstarvation investigations have employed mathematical models of body weight dynamicsto improve clinical weight loss methods or to explore an experimentally inaccessiblephenomenon.

Keyword: Ordinary Differential Equation,Non- Linear Differential Equation,Homogeneous Linear Differential Equation,Phase Portrait.

1. INTRODUCTION

Ordinary differential equations are used in a variety of domains, including physics, electronics, and population dynamics. This is a useful tool for examining the relationship between different dynamic quantities. The renowned predator-prey model, which represents the conflict between many carnivorous and vegetarian species, is used as a real-life example in the teaching of ODE.

War is frequently depicted in vivid and incisive ways by artists, which touch our senses. Because he lost his right hand at the Battle of Curupayti, Argentinian artist Candido Lopez painted "After the Battle of Curupayti" with his left hand. Curupayti was a battleground during the Great War in La Plata (1865-1870), also known as the Triple Alliance War. The war between Paraguay, Uruguay, Argentina, and Brazil is considered one of the worst disasters in modern history.

The technique taken in this work differs from that of an artist in that it employs a scientific tool - mathematical modelling - to rationally analyse international relationships such as trade agreements.It is common knowledge that a variety of elements influence the course and outcome of a battle. Many countries are attempting to reduce the stress of a prospective conflict or to fight more effectively in order to reduce the suffering and potential economic losses associated with war. LF Richardson, FW Lanchester, PM Morse, and GE Kimball were among the first to develop mathematical models of hostilities or battles for military or economic objectives.

In this work, we examine models for characterising international conflicts, which were first developed by Lewis Fry Richardson and describe the interaction between two nations or alliances that believe war is imminent. We provide a mathematical history backdrop for instructional purposes. Lewis Fry Richardson (1881-1953) was a forerunner in applying a mathematical model to the study of arms races and international affairs. Richardson spent half of his life studying the mathematics of armed conflict in the hopes of reducing aggressiveness through a quantitative understanding of the causes and dynamics of war.He undertook a comprehensive quantitative analysis of war and devoted half of his life to the study of armed conflict mathematics. He compiled a large quantity of data regarding battles in his book, Statistics of Deadly Quarrels, which covered everything from World War I to Chicago gang warfare. In 1919, he wrote The mathematical psychology of war, Generalized foreign politics, and Arms and insecurity, which summarised his efforts in conflict analysis.

He developed mathematical models of arms races based on differential equations, assuming that if one country raises its armaments, the other will do the same. The first country, in order, responds by stockpiling additional armaments. This type of arms race, according to Richardson, can be represented by a pair of differential equations.

The work that is most intriguing examines or calculates the coefficients and stability of a system of differential equations. Some coefficients, such as the measurement of each nation's pleasure or discontent, cannot be estimated.

The model created by Richardson is, in our opinion, an outstanding application of ODEs (ordinary differential equations) and is beneficial for practising ODEs. As a typical example, one would expect this type of model to be included in textbooks.First, a simplified model of an arms race is presented in this work. Additional aspects that influence the relationship between two states or alliances, such as the cost of arms, grievances between nations, and ambitions, are taken into account in more realistic models.

2. BASIC DEFINITIONS

DEFINITION 2.1: An equation involving derivatives or differentials of one or more dependent variables with respect to one or more independent variables is called a **differential equation.**

EXAMPLE:

$$dy = (x + \sin x)dx$$

DEFINITION 2.2:

A DE involving derivatives with respect to a single independent variable is called an **ordinary differential equation**.

DEFINITION 2.3:

The order of the highest order derivative involved in a differential equation is the called the **order of the differential equation**.

DEFINITION 2.4:

A DE is called **linear** if (1) every dependent variable and every derivative involved occurs in the first degree only, and (2) no products of dependent variables and/or derivatives occur. A differential equation which is not linear is called a **non- linear differential equation.**

EXAMPLE:

$dy = (x + \sin x)dx \ \ is\ a$ linear equation.

$\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + (\frac{dx}{dt})^5 = e^t is\ a$ non-linear equation.

DEFINITION 2.5:

A linear differential equation of the form

$$x^n \frac{d^n y}{d^{x^n}} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n y = X$$

$$(x^n D^n + a_1 x^{n-1} D^{n-1} + \cdots + a_n)y = X$$

Where $a_{1,} a_{2,\dots} a_n$ are constants, and X is either a constant or a function of x only is called**a homogeneous linear differential equation.**

DEFINITION 2.6:

Thus we consider the equation

$$y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = b(x) \qquad (2.1)$$

As in the case when $a_1, \dots a_n$ are constants we designate the left side of equation (2.1) by$L(y)$.Thus

$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y,$ (2.2)

And equation (1) becomes simply $L(y) = b(x)$. If $b(x) = 0$ for all x on I we say $L(y) = 0$ is a **homogeneous equation,** whereas if $b(x) \neq 0$ for some x in I, the equation $L(y) = b(x)$ is called a ***non-homogeneous equation.***

DEFINITION 2.7:

A **phase portrait** is a geometric representation of the trajectories of a dynamical system in thephase plane. Each set of initial conditions is represented by a different curve or point.

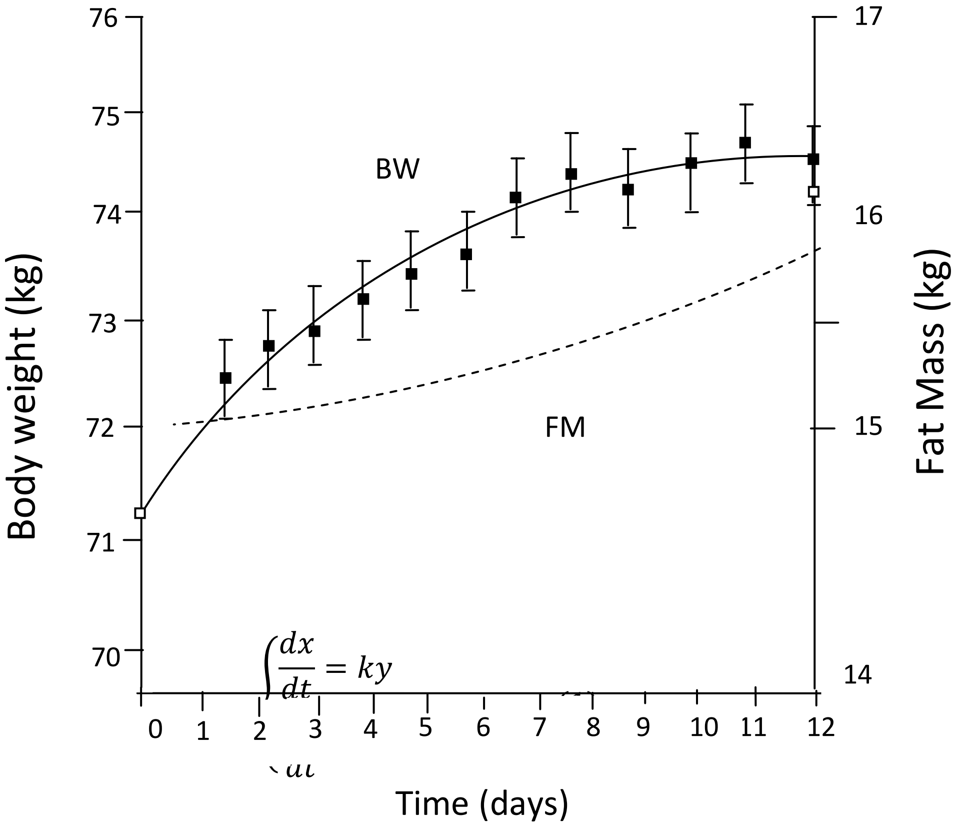
3. ORDINARY DIFFERENTIAL EQUATIONS FOR MATHEMATICAL MODELING APPROACH OF FOOD INTAKE AND BODY WEIGHT

Mechanisms governing body weight regulation have been extensively explored experimentally and conceptually in the last 25 years, owing to an increase in the number of overweight and obese people around the world. This is accomplished through the management of the energy balance's components: energy intake and expenditure. The discovery of hormones that increase or decrease food intake, such as leptin in 1994 [Zhang et al., 1994] and ghrelin in 1999 [Higgins et al., 2007], paved the way for a better knowledge and management of body weight regulation processes. Understanding and addressing the causes of body weight dysregulation is critical for public health.Because experimental procedures, particularly in humans, have limits, interdisciplinary approaches such as mathematics and computational modelling can supplement tests, aid therapeutic interventions (diet alteration, surgery, and/or medications), or lead future investigations. Hall proposed a mathematical model in 2010 based on [Hall,2006], which combined macronutrient balance, body composition, and energy expenditure adaption [Hall, 2010b].This model can anticipate the impact of food changes on fuel selection (see Figure1.16) and energy expenditure, which adjusts to the diet and results in changes in body weight and composition, and takes fluid dynamics into consideration (intracellular and extracellular). Energy intake corresponds to the sum of macronutrient energy (carbohydrates, proteins, and fats), while energy expenditure is separated into the following components:

- Feeding's thermogenic effect, which varies based on the dietary content
- Adaptive thermogenesis and physical activity, depending on body weight (depending on intake),
- Resting metabolic rate, which takes into account the amount of energy required.

Body composition changes as a result of imbalances between macronutrient intake and use. Fat intake, de novo lipogenesis, ketone synthesis, and fat oxidation all affect fat mass. Glycogen, protein, bone mass, and extracellular water are all components of fat-free mass, which are all affected by dietary salt changes. Human experiments were used to calibrate this sophisticated model, and the predictions are congruent with prior feeding research. This model works for both obese and non-obese people, and the sole input is the amount of food consumed throughout the duration of the tests.It is also possible to assess energy intake by tracking the evolution of body weight [Hall,2010b].

Guo and Hall proposed a mathematical model of energy metabolism in mice in addition to existing human models [Guo and Hall, 2009, 2011]. This model, which is based on the first law of thermodynamics, considers fat mass, fat-free mass, energy expenditure, and metabolic fuel selection (law of energy conservation). A link between fat mass and fat-free mass fluctuations is determined from a system of differential equations in the following form, similar to Forbes body composition function for humans [Forbes, 1987]:



$$\frac{dx}{dt} = ky$$

It is easy to obtain the solutions for the system (A) which we give as follows:

$$x(t) = \sqrt{\frac{k}{l}}(Ae^{\sqrt{kl}t} - Be^{-\sqrt{kl}t}$$

$$y(t) = Ae^{\sqrt{kl}t} + Be^{-\sqrt{kl}t}$$

Given the initial condition,

$$x(0) = x_{0,}$$

$$y(0) = y_0$$

We can obtain

$$A = \frac{1}{2}(y_0 + \sqrt{\frac{l}{k}}x_0)$$

$$B = \frac{1}{2}(y_0 - \sqrt{\frac{l}{k}}x_0)$$

Let $x(0) = 16.2$

$y(0) = 71.2$

$A = \frac{1}{2}(71.2 + \sqrt{\frac{0.9}{0.9}}16.2)$

$$= \frac{1}{2}(71.2 + 16.2)$$

$$A = 43.7$$

$$B = \frac{1}{2}(71.2 + \sqrt{\frac{0.9}{0.9}}16.2)$$

$$= \frac{1}{2}(71.2 - 16.2)$$

$$B = 27.5$$

$$x(t) = \sqrt{\frac{0.9}{0.9}}(Ae^{t(0.9)} - Be^{-t(0.9)}$$

$$x(t) = 43.7e^{0.9t} - 27.5e^{-0.9t}$$

$$y(t) = 43.7e^{.9t} + 27.5e^{-0.9t}$$

4. CONCLUTION

More research into mathematical modelling of international relationships, we believe, could be done. Military, industry, and other industries can all benefit from this type of modelling. These mathematical models are also realistic applications of ordinary differential equations (ODEs) from an educational standpoint, leading to the suggestion that they be included in ODE textbooks as flexible and vivid examples to show and learn differential equations.Ghrelin production is suppressed by food consumption, but leptin production increases according to fat mass. Leptin, ghrelin, and glucose, in relation to their biological functions as food intake regulators, govern hunger.We demonstrated that our model can predict the evolution of body weight and food intake seen experimentally in both restricted and ad libitum food groups pretty accurately.We hypothesised that certain groups will have leftover food, which was demonstrated experimentally, and that the pattern of food availability has an impact on both body weight dynamics and total food consumption.Using a memory of food intake to assume that food intake causes an adaptation in the rate of energy expenditure produces better results than using a constant rate of energy expenditure, showing the impact of adaptations due to caloric restriction. As a result of meal patterns and the time it takes for energy expenditure to adapt, we found that one group can have a greater mean body weight than the others despite having a lower total food intake.

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